

FINAL EXAM (April 10, 2017)

This is a closed book examination. Please write your student ID number on your exam. Using any electronic devices except a basic calculator is **not permitted** and will be considered cheating.

3 HOURS. 150 POINTS IN TOTAL

I. TRUE or FALSE – no points given for answers without justification (5 pts each)

1. If a consumer has convex preferences, then her optimal consumption bundle is on her budget line.
2. For a competitive firm, marginal revenue equals the output price.
3. Long-run profits cannot be smaller than short-run profits, except when the firm makes a loss.
4. A quantity tax makes the equilibrium quantity bought by consumers different from the quantity sold by producers.
5. An allocation is Pareto efficient if no consumer can be made better off.

Problem 1 (35 pts) (Explain your answers!)

Max consumes cheap coffee c , with price $p_1 = 1$ and sandwiches s , with price p_2 . His utility function is $u(c, s) = 10s - \frac{s^2}{2} + c$ and his income is $m = 50$.

(a) Find Max's demand functions for coffee and sandwiches. Argue that if $50 \leq p_2(10 - p_2)$, then Max consumes no coffee. How many sandwiches would he consume in that case?

(b) Suppose $p_2 = 5$. Find Max's optimal consumption bundle and plot his demand curve for sandwiches.

(c) Suppose there are *100 consumers in total*, all with the same preferences and income as Max. Show that the *market demand* for sandwiches is $q^D(p_2) = 1000 - 100p_2$. If the market supply for sandwiches is $q^S(p_2) = 300p_2 - 1000$, find the equilibrium price and quantity.

Suppose that, starting from the equilibrium in part (c), the government imposes a tax of \$4 per sandwich. Everything else stays the same.

(d) Find the after-tax equilibrium quantity, the price paid by consumers, the price received by producers, the consumers' surplus, the tax revenue, and the deadweight loss of the tax.

(e) Find the income and substitution effects of the price change caused by the tax on Max's consumption of coffee and sandwiches. How much extra income would Max need to be able to afford the consumption bundle he chose before the tax? If Max received this extra income as a lump-sum subsidy, would Max choose his before-tax consumption bundle? Explain why or why not.

Problem 2 (35 pts) (Explain your answers!)

A chair-making firm has the production function $f(x_1, x_2) = \sqrt{x_1} + \sqrt{x_2}$ where x_1 is the number of workers and x_2 is the quantity of wood used. The output price is p , the wage is $w_1 = 1$ and the price of wood is $w_2 = 2$.

(a) Suppose that, in the short-run, the firm must employ 9 workers, that is, x_1 is fixed at 9. Solve for the firm's short-run cost function and conditional input demands for producing y units of output. For what p (if any) would the firm shut down?

(b) Find the firm's short-run profit function and input demands for any $p > 0$.

Now suppose the firm can vary both inputs freely.

(c) Find the firm's long-run cost function and conditional input demands for producing y units of output. What is the firm's long-run supply function?

(d) Find the firm's long-run profit function and input demands for any $p > 0$. Show that the firm's short-run and long-run profits are equal at $p = 6$. Why?

Problem 3 (25 pts) (Explain your answers!)

Suppose that, due to licensing regulations, initially there are 30 firms in the cake industry, each with average cost function $AC(y) = 3y^2 - 9y + 9$ where y is output. Market demand is $150 - 10\sqrt{p}$ where p is the cake price.

- (a) Show that the marginal cost function of each cake firm is $MC(y) = 9(y - 1)^2$.
- (b) What is the minimum output price above which a cake firm would produce a positive output?
- (c) Find the industry supply function and the short-run industry equilibrium price and quantity when the number of firms is **fixed at 30**. Do firms make a profit or a loss?
- (d) Suppose now the government allows **free entry and exit** in the industry but simultaneously imposes a \$1.75 tax on each cake sold. What would be the resulting long-run industry equilibrium price paid by consumers? The price received by producers? The quantity of cakes sold? The number of firms in the market? [*fractions are ok*]

Problem 4 (30 pts) (Explain your answers!)

Consider an exchange economy with two goods, 1 and 2 and two consumers, A and B. The goods' prices are $p_1 = 1$ and p_2 . Person A treats goods 1 and 2 as *perfect substitutes* (for him, each unit of good 1 is just as good as each unit of good 2, no matter what their quantities). Person B's *demand function* for good 1 is $x_1^{B*} = \frac{m^B}{1+p_2}$ and her *inverse demand function* for good 2 is $p_2 = \frac{m^B}{x_2^{B*}} - 1$ where m^B is B's income. A's endowments are $w_1^A = 6$ and $w_2^A = 0$ and B's endowments are $w_1^B = 0$ and $w_2^B = 4$.

- (a) Show that B's quantities demanded of each good, (x_1^{B*}, x_2^{B*}) are obtained from the *perfect complements* utility function $\min\{x_1, x_2\}$. If A's income is m^A what are A's demand functions?
- (b) Draw the Edgeworth box and the endowment allocation W . Draw A and B's indifference curves through W . Label your graph clearly.
- (c) Find A and B's incomes m^A and m^B obtained from selling their endowments.
- (d) Describe (using words or a graph) and solve for the competitive equilibrium in this exchange economy. [*Hint: show that the equilibrium price of good 2 must be $p_2^* = 1$*]
- (e) [*harder*] Is allocation M at which B consumes the bundle $(1, 1)$ Pareto efficient? What about allocation N at which A consumes the bundle $(1, 1)$?