SFU Economics Econ 802, Prof. Karaivanov

FINAL EXAM (December 4, 2014)

This is a closed book examination. To get full credit you need to answer ALL questions. Please **always explain** how you obtained your answers – no credit will be given for correct answers without explanation. **There is a total of 120 POINTS.**

I. TRUE/FALSE - no points will be given for correct answers without explanation (5 pts each, 30 pts in total)

1. If preferences are strictly convex, then there always exists a utility function representing them.
2. Let $x^0 = (x_1^0, x_2^0)$ maximize $u(x_1, x_2)$ subject to $p^0.x \leq y$. This consumer is willing to pay at least $(p_1^0 - p_1^1)x_1^0$ for a reduction in the price of good 1 to p_1^1 (the price of good 2 is held constant).
3. A risk-averse person would always buy full insurance against a negative shock to his/her wealth.
4. Profit maximization implies cost minimization.
5. A competitive cost-minimizing firm has the production function $f(x_1, x_2) = x_1 + x_2$ and uses positive amounts of both inputs. If the price of input 1 doubles and the price of input 2 triples, then the firm's costs of producing a given amount of output would more than double.
6. The First Welfare Theorem implies that any Walrasian equilibrium is Pareto efficient.

II. PROBLEMS

1. Consumer choice (20 pts)

A consumer has preferences over **three** goods and the following **indirect** utility function:

$$v(\mathbf{p}, y) = \frac{y^3}{p_1 p_2 p_3}$$

where $p_1, p_2, p_3 > 0$ are the goods' prices and $y > 0$ is the consumer's income.

(a) Derive the Marshallian demand functions, $x_i^*(\mathbf{p}, y)$, $i = 1, 2, 3$. Show that they are homogeneous of degree 0 in $(\mathbf{p}, y)$.

(b) Derive the expenditure function, $e(\mathbf{p}, u)$ and the Hicksian demand functions, $\hat{x}_i(\mathbf{p}, u)$, $i = 1, 2, 3$ for any feasible utility level u.

(c) Using the results in (a)–(b), verify the Slutsky equation for good 2 with respect to a change in p_1.

(d) Prove that $\sum_{j=1}^{3} \frac{\partial \hat{x}_i(\mathbf{p},u)}{\partial p_j} p_j = 0$ for any $i = 1, 2, 3$.

2. Cost minimization (20 pts)

Consider a firm which operates two plants with strictly convex cost functions $c_i(q_i)$ where q_i is the quantity of output produced in plant $i = 1, 2$. Define $C(q)$ to be the minimized total costs of producing a given total output q. That is:

$$C(q) \equiv \min_{q_1, q_2} \; c_1(q_1) + c_2(q_2) \text{ s.t. } q_1 + q_2 = q$$

(a) Write down the first order conditions of the above problem.

(b) Show that at the total cost-minimizing output allocation, (q_1^*, q_2^*) **marginal costs** are equalized across the two plants and are equal to $C'(q)$. Explain the economic intuition.

(c) Suppose now that each plant maximizes its own profit $\pi_i = pq_i - c_i(q_i)$ taking the price of output p as given. Show that if $p = C'(q)$ for a given $q > 0$, then the plants' individual profit-maximizing output choices result in an aggregate output of q.

3. Risky housing (25 pts)

John has $\$w$ dollars. He is buying a house for price p, where $p > w$ and making a downpayment $d \in [0, p)$. This means he must take a loan of size $p - d$ from the bank. Suppose that, after John has bought the house, with probability $\pi \in (0,1)$ the house price falls to θp where $\theta \in (0,1)$ and with the remaining probability, $1 - \pi$ it stays equal to p. John has a strictly increasing and strictly concave utility of wealth function, u.

(a) Suppose that the bank charges gross interest rate R no matter what happens to the house price (this means if John borrows \$1 he must pay back $\$R$). Explain in words why John's expected utility over wealth can be written as,

$$\pi u(w - d + \theta p - R(p-d)) + (1-\pi)u(w - d + p - R(p-d)) \quad (*)$$

(b) Suppose the bank wants to break even on John's loan and its opportunity cost of funds is 1. That is, for every dollar lent, the bank wants to receive exactly \$1 back. What rate R should the bank charge if John is able to pay back his loan in both states of the world? What rate R should the bank charge if John can only pay back if the house price does not fall? (assume that if John can't pay back the bank receives zero)

Suppose now the bank can charge two different rates, R^L and R^H corresponding to the case of the house price falling to θp and the case of a stable price. The bank still wants to break even, so we must have $\pi R^L + (1-\pi)R^H = 1$.

(c) Find the optimal R^L and R^H that maximize John's expected utility subject to the bank breaking even and supposing that John is able to pay back in both states of the world. *[use the expression in (*) but allow the R's to differ].* Show that John is fully insured against the house price fall. Explain the intuition.

(d) Are the values of R^L and R^H in (c) smaller or larger than 1? Show that the difference $R^H - R^L$ decreases in θ. Explain why.

4. GE (25 pts)

Consider an exchange economy with two goods, with quantities denoted by a and b and two consumers, A and B. A's preferences are represented by $u_A(a^A, b^A) = \sqrt{a^A b^A}$ and B's preferences are represented by $u_B(a^B, b^B) = a^B b^B + b^B$. A has an endowment of $e_a^A = 11$ units of good 1 and $e_b^A = 0$ units of good 2. B has an endowment of $e_a^B = 0$ units of good 1 and $e_b^B = 3$ units of good 2.

(a) Draw the Edgeworth box and the endowment allocation. Label your graph clearly. Write down the set of all feasible allocations.

(b) Explain why the set of all Pareto efficient allocations can be found by solving the following problem:

$$\max_{a^A, b^A} \; u_B(e_a^A + e_a^B - a^A, e_b^A + e_b^B - b^A)$$
$$\text{s.t. } u_A(a^A, b^A) = \bar{u}$$

for any utility level $\bar{u}$ that is feasible for consumer A. Solve the above problem and write down the set of all PE allocations in this economy.

(c) Normalize $p_2 = 1$. **Define** and **solve** for a competitive equilibrium (CE) in this economy. Are A and B better off at the CE allocation relative to their endowment allocation?

(d) Can you find initial endowments for which A consumes $(11, 3)$ in a competitive equilibrium? Explain why or why not. *You can assume that the conditions for the Second Welfare Theorem hold without proof.*