SFU Economics Econ 802, Prof. Karaivanov

FINAL EXAM (December 5, 2018)

This is a closed book examination. To obtain full credit you must answer ALL questions. Please **always explain** how you obtained your answers – no points will be given for correct answers without explanation. **There is a total of 120 POINTS.**

I. TRUE/FALSE - no points given for correct answers without explanation (5 pts each, 30 pts in total)

1. A person who always chooses a random point on her budget line cannot be maximizing her utility.
2. Suppose that, for exogenous reasons, the housing price in Vancouver goes up. The total effect of this price increase is larger for people considering moving to Vancouver than for home owners considering leaving Vancouver.
3. The indirect utility function $v(p, y)$ is increasing in the price of a Giffen good.
4. If a person is risk-neutral, then maximizing his expected utility is equivalent to maximizing his expected monetary payoff.
5. The conditional input demands are homogeneous of degree one in input prices.
6. In a 2-person, 2-goods economy, if the endowment allocation is in the core then it is also a competitive equilibrium allocation.

II. PROBLEMS

1. Consumer choice (20 pts)

A consumer has strictly monotonic preferences defined over two goods: "food" with price p_1 and "expenditure on all other goods" (so $p_2 = 1$). Her demand function for food is $f^*(p, y) = \frac{y}{3p_1}$, where y is income.

(a) find the Marshallian demand for "all other goods", $d^*(p, y)$. Show that $f^*(p, y)$ and $d^*(p, y)$ are Marshallian demands for Cobb-Douglas utility of the form $f^\alpha d^{1-\alpha}$ and find α.

(b) find the indirect utility function $v(p, y)$ and verify Roy's identity for food, that is, show that $f^*(p, y) = -\frac{\partial v}{\partial p_1} / \frac{\partial v}{\partial y}$.

(c) find the consumer's Hicksian demand for food for any feasible utility level u.

(d) suppose the government puts tax $t > 0$ per unit of food bought, which raises its price from p_1 to $p_1 + t$. At the new prices $(p_1 + t, 1)$, what is the minimum additional income Δy, that the consumer would need to be still able to afford her original optimal bundle (chosen at prices $(p_1, 1)$ and income y)?

(e) if the government gives the consumer an *income subsidy* equal to the amount Δy from (d) while keeping the tax t, would the tax revenue collected by the tax be sufficient to pay for the subsidy? Explain why or why not.

2. A firm with minimum scale (25 pts)

A firm produces output y using the **single-input** production function:

$$y = \begin{cases} f(k - \bar{k}) \text{ if } k > \bar{k} \\ 0 \text{ if } k \leq \bar{k} \end{cases}$$

where $\bar{k} > 0$ and f is strictly increasing and strictly concave with $f(0) = 0$. Let $r > 0$ be the input price and $p > 0$ be the output price.

(a) Plot the production function. Write down the firm's cost minimization problem and solve for the firm's cost function $c(r, y)$ for any r and any $y > 0$. Prove that $c(r, y)$ is increasing in r and y. Is the cost function homogeneous of any degree in r? What about in y? Discuss briefly the economic intuition.

(b) Write down the firm's profit maximization problem and define its profit function $\pi(p,r)$ for any p and r. Would this firm always produce a positive output? Explain. For $f(k-\bar{k}) = \sqrt{k-\bar{k}}$ find a condition on the parameters p, r and $\bar{k}$ under which the firm will produce zero output.

(c) Prove that the profit function $\pi(p,r)$ from (b) is increasing in p. Is the profit function homogeneous of any degree in (p,r)? Discuss briefly the economic intuition.

(d) Let $f^{-1}(y)$ be the inverse of the function f. Prove and explain why for $k > \bar{k}$ the firm's profit function must satisfy

$$\pi(p,r) = \max_{y>0} \; py - rf^{-1}(y) - r\bar{k}$$

3. Arrow securities (20 pts)

Mark has strictly increasing and strictly convex utility $u(x)$ defined over monetary outcomes. There are n states of the world which occur with probabilities p_i respectively, where $p_i \in (0,1)$ and $i = 1,..n$. At price \$$\alpha$ per unit, Mark can buy any amount, x_i of asset i for any $i = 1,..n$. Each asset i pays \$1 in state i and zero in any other state *[such assets are called Arrow securities]*. For example, \$$(5\alpha)$ invested in asset $i = 2$ pays \$5 in state of the world $i = 2$ and \$0 in each other state $j \neq 2$.

(a) write Mark's *expected utility* maximization problem of investing total amount w across the n assets, before the state of the world is known.

(b) define $V(p,w)$ as the maximum expected utility that Mark can achieve for given w and probabilities p_i. Prove that $V(p,w)$ is *convex* in the probabilities p.

(c) for $u(x) = \ln x$, solve for Mark's optimal purchases x_i^* of each asset i. Compute the certainty equivalent of Mark's asset portfolio $\{x_1^*, ..., x_n^*\}$.

4. General equilibrium (25 pts)

Consider an exchange economy with two goods, 1 and 2 and two persons, 1 and 2. Person 1's *expenditure function* is $e(p,u) = 2\sqrt{up_1p_2}$. Person 2's Marshallian demand functions are $x_j^{2*}(p,y) = \frac{y}{p_1+p_2}$ for $j = 1,2$. Person 1's endowments are $e_1^1 = 4$ and $e_2^1 = 2$ and person 2's endowments are $e_1^2 = 2$ and $e_2^2 = 2$.

(a) show that person 2's Marshallian demands correspond to preferences for *perfect complements*. What type of preferences does person 1 have? *[hint: derive his Marshallian demands]*.

(b) normalize $p_1 = 1$ and verify that Walras' Law holds for any $p_2 > 0$.

(c) draw the Edgeworth box and the endowment allocation E. Sketch 1 and 2's indifference curves at E. Label your graph clearly.

(d) plot the set of all Pareto efficient allocations in this economy (provide explanations). Is allocation M at which person 2 consumes $(3,3)$ in the core of the economy?

(e) **define** and **solve** for a competitive equilibrium (CE) in this exchange economy.

(f) starting from the CE allocation in (e), a third consumer arrives in the economy, with the same preferences as person 2 and zero amount of each good. To welcome person 3, person 1 gives her, free of charge, one unit of each good (from his own allocation). Solve for a competitive equilibrium in the new three-person economy.