

(Non-)Existence of Equilibria in Multi-Community Models*

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Summary

This paper reconsiders equilibrium existence in models with migration and voting over local public goods. We show that under some straightforward assumptions on preferences and income distributions, the basic structure of multi-community models (perfect mobility, majority rule, single crossing property) implies that no equilibrium with jurisdictions conducting different policies can exist. Stratification equilibria – with sorting of the population according to income classes – are therefore not as natural as is sometimes suggested. Mechanisms that can serve to support stratification (tight housing markets, returns to scale in the provision of publicly consumed goods) are discussed.

Keywords: Local Public Goods, Income Taxation, Migration, Fiscal Federalism

JEL-Classification: H71, H73

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1 Introduction

Positive theories on the provision of local public goods in multi-community models have flourished in recent years. The main theme of these models is Tiebout's notion that mobile households select jurisdictions to live in according to their preferences over fiscal policies. This mechanism leads to a regional clustering of households with similar tastes and, if tastes are related to income or ethnic groups, to stratification of classes or races.¹

This paper reconsiders the theory of migration and voting over local public good provision. We show that under some straightforward assumptions on the preferences and the income distribution, stratified equilibria in which jurisdictions have distinct local policies and households sort according to income classes do not exist. As a consequence, the only equilibrium displays opposite characteristics: all jurisdictions conduct the same policy and share the same population structure, i.e., they are effectively homogeneous with respect to important economic and demographic variables. Our non-existence result arises because the local majority rule outcomes that are induced by sorting and the requirement that poorer households should not prefer to live in higher-income communities are inconsistent with each other. Hence, in the absence of additional mechanisms that prevent migration from poor into wealthy jurisdictions (housing markets, scale effects of populations size, positive migration cost), stratification may not emerge as frequently as is sometimes suggested.

Our basic framework incorporates the main properties of many Tiebout models with residential and political choice: (1) there are two or more jurisdictions among which households are perfectly mobile, (2) a continuum of households can be ranked according to preferences over fiscal policies (income), (3) local policies are determined by majority rule of the local inhabitants, and (4) local public spending is financed by a proportional income tax levied according to the residency principle. We also assume that there is (5) no scale effect in local public spending (local public goods are private in nature) and (6) no housing market or zoning. This setup corresponds to the seminal work of Westhoff [29] which differs only in its consideration of pure local public goods, i.e., scale effects in population size. Rose-Ackerman [24] extends Westhoff's framework by incorporating a housing market. The same approach is used in Epple et al. [10],[11], who also allow for publicly provided private goods.² The analysis of Fernandez and Rogerson [15] and Fernandez [14] is closest to our model in that housing markets and economies of scale in public spending are disregarded. All of these deal with stratification equilibria, but only Westhoff analytically proves their existence for general income distribu-

¹For a comprehensive survey of the literature, see Ross and Yinger [25].

²A similar non-convexity arises from peer-group effects between different households, which have been studied by de Bartholomé [1].

tion functions.³ Hansen and Kessler [19] show for the special case of local redistribution that stratification emerges only if jurisdiction differ significantly in their geographical size, which translates into sufficiently tight (equilibrium) housing markets; otherwise, stratified equilibria are impossible. The present paper generalizes the latter result to a broader class of local public goods. In doing so, we classify potential stratification equilibria according to preference-based characteristics of public goods, namely the degree of substitutability of public for private goods determining individual preferences for fiscal policies.

The remainder of the paper is organized as follows. The model is presented in Section 2. In Sections 3.1 and 3.2., we develop necessary conditions for stratification and relate them to the fundamentals of the model (preferences and the income distribution). Section 3.3. then shows that under some straightforward assumptions on those, the conditions are inconsistent with each other. In the final Section 4, we extensively discuss which assumptions are crucial for (non-)stratified equilibria.

2 The Model

The economy is populated by a continuum of households with mass normalized to unity. Households have identical preferences over the consumption of a private composite good c and a publicly provided private good g . Their utility function $u(c, g)$ is strictly increasing, twice continuously differentiable and strictly quasi-concave. In addition, we make the following assumptions.

A1 : $u(c, g)$ is homothetic.

A2 : $c', g' > 0, c, g \geq 0 : u(c', g') > u(0, g), u(c', g') > u(c, 0)$.

A1 imposes a regularity of preferences with respect to different income levels.⁴ A2 is made for technical reasons.

The households are differentiated by their exogenously given income y which is distributed according to a continuous distribution function $F(y)$ with density $f(y) > 0, y \in [\underline{y}, \bar{y}] \subseteq R_0^+$.

There are $j = 1, \dots, J$ politically independent jurisdictions and each household lives in one of them. We denote the measure of households with income y living in jurisdiction j by $f_j(y)$

³Epple et. al. [11] show existence in a model with a housing market and a uniform distribution of income. As the authors themselves, note, however, it is necessary to assume that there is some (possibly small) fixed cost associated with public good provision, so that no single household can live alone in a community.

⁴In particular, A1 ensures that the indifference curves of differently endowed households cross only once in the policy space, which is necessary to characterize equilibrium outcomes. See also the literature cited in the Introduction, and single community models like Glomm and Ravikumar [17], [18], and Epple and Romano [8]. The homotheticity assumption is stronger than is necessary for our results (see Section 4) but simplifies the exposition considerably.

and by $\alpha_j = \int_{\underline{y}}^{\bar{y}} f_j(y) dy$ the size of the overall population that resides in j . The jurisdictions supply the local public good in quantity g_j such that each household living in jurisdiction j consumes exactly that quantity. The costs for the provision of the public goods are financed by local income taxation: each jurisdiction levies a proportional income tax with rate t_j on all its inhabitants' incomes. Thus, tax revenues in jurisdiction j are given by $t_j \bar{Y}_j$, where $\bar{Y}_j = \frac{1}{\alpha_j} \int_{\underline{y}}^{\bar{y}} y f_j(y) dy$ is average income in j . Without loss of generality we assume that one unit of income can be transformed into one unit of the local public good. Since the publicly provided good is private in nature (public service), local expenditures are $g_j \alpha_j$. We call a policy (t_j, g_j) *feasible* if the local budget constraint, which we can express in per-capita terms to read

$$g_j \leq t_j \bar{Y}_j \quad j = 1, \dots, J, \quad (1)$$

holds. In each jurisdiction, (t_j, g_j) is determined by majority vote of the residents.

Households choose a jurisdiction to live in, vote on the local tax rate and the quantity of the local public good, and consume. An equilibrium in our model is defined as follows:

Definition. An intercommunity equilibrium is a vector of fiscal policies $\{(t_j^*, g_j^*)\}_{j=1, \dots, J}$ and a distribution of households over jurisdictions $\{f_j^*(y)\}_{j=1, \dots, J}$ with $\alpha_j^* > 0$ and $\sum_j^J f_j^*(y) = f(y)$, $\forall y$ such that

1. given policies $\{(t_j^*, g_j^*)\}_{j=1, \dots, J}$, each household chooses its residency optimally (external equilibrium), and
2. given residential choices $\{f_j^*(y)\}_{j=1, \dots, J}$, local policies are feasible and preferred to any other feasible policy by a majority of the inhabitants in each jurisdiction (internal equilibrium).

Observe that since each jurisdiction must have a positive population in equilibrium, political outcomes are well defined. Furthermore, policies are determined taking residential choices as given so that tax competition aspects between jurisdictions are eliminated.⁵

3 Equilibrium Analysis

3.1 Residential Choice

Jurisdictions are identical except for the fiscal policies (t_j, g_j) , and hence migration is a choice of the preferred policy from the set $\{(t_j, g_j)\}_{j=1, \dots, J}$. Despite the fact that utility functions

⁵Both requirements and our equilibrium definition are standard in the literature. See, e.g., Westhoff [29], Rose-Ackerman [24], Nechyba [23] and Epple et al. [10], [11]. As has been shown by Fernandez and Rogerson [15], this equilibrium notion can be rationalized by a sequential game in which households first settle in a jurisdiction and local policies are determined subsequently. An alternative formulation where voters foresee policy-induced migration is employed in Epple and Romer [13].

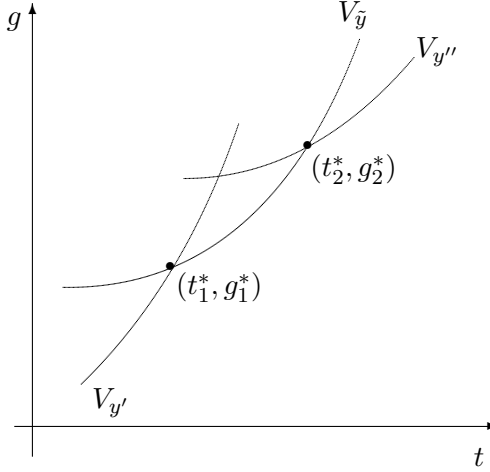


Figure 1: Preferences in policy space

are homothetic, a household's preferred policy will generally vary with income: let $V(y, t, g) \equiv u((1-t)y, g)$ denote the indirect utility function of a household with income y . The marginal rate of substitution between the local public good and the tax rate is defined as $M(y, t, g)$. Thus,

$$M(y, t, g) = \left. \frac{dg}{dt} \right|_{v=\bar{v}} = \frac{u_c}{u_g} y, \quad (2)$$

where subscripts indicate partial derivatives. $M(\cdot)$ changes in income according to the following result (see Appendix).

Lemma 1. *Let σ denote the elasticity of substitution between the private good and the public good along an indifference curve. Then,*

$$\text{sign} \frac{\partial M(y, t, g)}{\partial y} = -\text{sign} \{1 - \sigma\}. \quad (3)$$

We call the private and the public good *complements* if $\sigma < 1$ and *substitutes* if $\sigma > 1$ for all $y \in [y, \bar{y}]$ and (t, g) . Thus, Lemma 1 states that the slope of an indifference curve through any point of the policy plane displayed in Figure 1 decreases (increases) in household income if the goods are complements (substitutes). In the figure, points (t_1^*, g_1^*) and (t_2^*, g_2^*) depict two alternative policy bundles and $V_y, y \in \{y', \tilde{y}, y''\}$, denote the indirect indifference curves of a household with income y . If the goods are complements ($\sigma < 1$), Figure 1 is based on $y' < \tilde{y} < y''$. If the goods are substitutes ($\sigma > 1$), Figure 1 describes the case $y' > \tilde{y} > y''$. All households with incomes smaller (larger) than \tilde{y} prefer to live in jurisdiction 1 and all households with incomes larger (smaller) than \tilde{y} prefer to live in jurisdiction 2, if the goods are complements (substitutes). From Lemma 1, the property that indifference curves of two

households with different incomes cross at most can thus be obtained by making one of the following *alternative* assumptions:

$$\mathbf{A3.1} : \partial M(\cdot)/\partial y < 0 \Leftrightarrow \sigma < 1, \quad \forall y \forall (t, g).$$

$$\mathbf{A3.2} : \partial M(\cdot)/\partial y > 0 \Leftrightarrow \sigma > 1, \quad \forall y \forall (t, g).$$

Intuitively, households with higher incomes demand more of the public good, *ceteris paribus*. However, this income effect may be offset by a price (substitution) effect, caused by the increase in the price of public service that is associated with rising income if taxation is redistributive in nature. It can be shown that the income effect always dominates if the elasticity of substitution falls short of the income elasticity of demand for the public good (A3.1). If the reverse holds, the price effect dominates (A3.2).⁶

For expositional simplicity, we begin our analysis with the case of two communities. The following proposition states a necessary equilibrium condition if they conduct different policies, assuming (w.l.o.g.) that community 1 provides less of the public good.

Proposition 1. *If either under A3.1 or A3.2, two jurisdictions $j \in \{1, 2\}$ conduct different policies, then in equilibrium*

a) $(t_1, g_1) < (t_2, g_2)$,

b) $\exists \tilde{y} \in (\underline{y}, \bar{y})$ with $V(\tilde{y}, t_1, g_1) = V(\tilde{y}, t_2, g_2)$,

c) for $\sigma < 1$ ($\sigma > 1$), all households with $y < \tilde{y}$ live in jurisdiction 1 (2) and all households with $y > \tilde{y}$ live in jurisdiction 2 (1).

Proof. Consider Figure 1 and suppose $g_1 < g_2$ and $t_1 \geq t_2$. In this case, every household would strictly prefer to live in jurisdiction 2, contradicting the requirement that no community is empty in equilibrium. Hence, $(t_1, g_1) < (t_2, g_2)$. Parts b) and c) follow from Lemma 1 and $f(y) > 0, y \in [\underline{y}, \bar{y}]$. \square

The observation that households sort themselves according to income classes and communities will display a natural ordering with respect to policies is well known from the related literature and is frequently called *stratification* equilibrium. The proposition is a comprehensive description of stratification characteristics. In Epple and Romer [13], for instance, the local public good is pure redistribution, i.e. c and g are perfect substitutes ($\sigma \rightarrow \infty$). As a result, if a stratification equilibrium exists, the upper classes cluster together in jurisdictions

⁶Recall that under A1, g is a normal good and the income elasticity equals unity. See the proofs of Lemma 1 and Lemma 2 (below) in the Appendix for a formal argument, which is based on Kenny [21]. See also Epple and Romano [9] or Bergstrom and Goodman [4]. These, or alternative characterizations of the single crossing property are imposed by most papers of regional choice and local public goods.

with low government activity and poor households live together in jurisdictions with high government activity. The case of complementary goods ($\sigma < 1$) is treated, e.g., in Westhoff [29], Rose-Ackerman [24], Epple et al. [10] and Fernandez and Rogerson [15] and Fernandez [14]. With complements, wealthy households prefer to live in communities with a higher level of public good and higher taxes than the poor. The economic intuition for this behavior has already been laid out in the preceding discussion. Irrespective of the nature of the local public good, its provision entails redistribution from the rich to the poor: households with above-average income pay more in taxes than they receive in terms of g while households with lower than average income receive more than their tax liability. Thus, there is a natural reluctance of the rich toward living in communities with high taxes and high local public good supply, as they face a higher effective price of public consumption. The second effect, however, is working in the opposite direction. The preferences favor a modest mix of the goods c and g . Due to their higher consumption of the private good c , wealthy households strongly favor large quantities of the public good. In the case of substitutes, this allocative preference for higher g is overcompensated by the adverse redistributive component of the public sector (because substituting private for public consumption does not involve large utility losses). The converse is true in the case of complements, where the redistributive component of public good provision is dominated by the increase in demand associated with higher income.

3.2 Voting on Fiscal Policies

The fiscal policy which gains the political support of the majority of the residents is implemented in each jurisdiction. Recall that in making their political decision, voters take the residential decisions of all households (and hence, the local tax base) as given. The preferred policy bundle (t_j^y, g_j^y) of a voter with income y in jurisdiction j maximizes her indirect utility $V(y, t_j, g_j)$ subject to the local budget constraint (1). Graphically, the slope of a household's indifference curve, $M(\cdot)$, is equated to the slope of budget line at t_j^y and $g_j^y = t_j^y \bar{Y}_j$ in the (t, g) -space. Since the budget set is linear and the indifference curves \bar{V}_y are convex due to quasi-concavity of $u(\cdot)$,⁷ there is a unique t_j^y that solves $\max_{t_j} V((1 - t_j)y, t_j \bar{Y}_j)$. Using the homotheticity of preferences, the corresponding first-order conditions can be written as

$$\frac{u_g \left(\frac{1-t_j^y}{t_j^y} \frac{y}{\bar{Y}_j}, 1 \right)}{u_c \left(\frac{1-t_j^y}{t_j^y} \frac{y}{\bar{Y}_j}, 1 \right)} = \frac{y}{\bar{Y}_j}. \quad (4)$$

⁷For a formal proof, see Westhoff [29].

Hence, the preferred tax rate is a function of the ratio of the voter's income and the local average only, i.e. $t_j^y = t_j^y(\frac{y}{\bar{Y}_j})$. We also find (see Appendix)

Lemma 2.

$$\text{sign} \frac{dt_j^y(\cdot)}{dy} = \text{sign} \frac{dt_j^y(\cdot)}{d(y/\bar{Y}_j)} = \text{sign} \{1 - \sigma\}. \quad (5)$$

Therefore, the preferred tax rate of a voter increases (decreases) with her income and with the ratio of individual income to the community's average if the goods are complements (substitutes). Using (1), Lemma 2 also implies that the preferred level of the public good increases in a voter's income in the case of complements, and decreases otherwise. As $M(\cdot)$ equals the slope of (1) at (t_j^y, g_j^y) , the intuition is the same as the one given after Lemma 1 and Proposition 1.

Because preferences are single-peaked, the median voter theorem can be applied directly. Thus, the preferred policy of the median income household any jurisdiction beats all other policies in a pair wise competition.

Lemma 3. *The equilibrium tax rate t_j^* in jurisdiction j is the preferred tax rate of the local median income household y_j^m , i.e.,*

$$t_j^* = t_j^{y_j^m} \left(\frac{y_j^m}{\bar{Y}_j} \right). \quad (6)$$

Hence, the equilibrium tax rate in a jurisdiction depends only on the local ratio of median to mean income, which we denote by $\Phi_j \equiv y_j^m / \bar{Y}_j$.

3.3 Intercommunity Equilibrium

From Proposition 1 we know that in an equilibrium with different policies there will be one 'rich' jurisdiction inhabited by high income households and one 'poor' jurisdiction inhabited by low income households. In what follows we change the notation slightly, and refer to the 'rich' and the 'poor' jurisdiction with the indices r and p . Having addressed internal and external equilibrium separately, we are now in a position to combine both equilibrium requirements. As a first step in this direction is

Proposition 2. *In any equilibrium with $(t_r, g_r) \neq (t_p, g_p)$, $\Phi_p < \Phi_r$.*

Proof. Suppose $(t_r, g_r) \neq (t_p, g_p)$ in equilibrium. Then the equilibrium must have the characteristics described in Proposition 1, i.e. either $(t_r, g_r) < (t_p, g_p)$ (if $\sigma > 1$) or $(t_r, g_r) > (t_p, g_p)$ (if $\sigma < 1$). In case of $t_r < t_p$ apply Lemma 2 and 3 for $\sigma > 1$, hence $\Phi_r > \Phi_p$. For the case $t_r > t_p$ apply Lemma Lemma 2 and 3 for $\sigma < 1$, and $\Phi_r > \Phi_p$ follows. \square

Thus, irrespective of the character of the local public goods, the ratio of median to mean income must be larger in the rich community than in the poor community. Since this is only a necessary characteristic, we additionally need the boundary household \tilde{y} to be indifferent between living in either community to establish the existence of equilibrium.

The following result states a condition for this to be impossible.

Lemma 4. *For $\Phi_p \leq 1$ and $\Phi_r \leq 1$, there is no $\tilde{y} \in [\underline{y}, \bar{y}]$ with $V(\tilde{y}, t_p, g_p) = V(\tilde{y}, t_r, g_r)$.*

Proof. Denote the preferred policy of the voter with mean income in each jurisdiction by (\bar{t}_j, \bar{g}_j) , $j = p, r$, and note that this is also the private allocation an individual with mean income would realize when being ‘alone’. Since the tax price of a unit of the public good is unity for this individual in both jurisdictions and due to the homothetic preferences, $\bar{t}_p = \bar{t}_r$, and \bar{g}_j is proportional to the respective mean income.⁸ Consider the case of complements. If the income distribution is skewed to the right in the rich jurisdiction ($\Phi_r \leq 1$), the mean income type living there prefers more government activity than the median (decisive) type, i.e., $(\bar{t}_r, \bar{g}_r) > (t_r, g_r)$ by Lemma 2. Single-peakedness thus implies $V(\tilde{y}, t_r, g_r) > V(\tilde{y}, \bar{t}_r, \bar{g}_r)$. In the poor jurisdiction, mean income is also larger than median income ($\Phi_p \leq 1$). Due to single-peakedness, the boundary individual must thus be better off with the mean’s preferred allocation than with the median’s, $V(\tilde{y}, \bar{t}_p, \bar{g}_p) > V(\tilde{y}, t_p, g_p)$. But from $\bar{t}_p = \bar{t}_r$, and $\bar{g}_r > \bar{g}_p$ it follows that $V(\tilde{y}, t_r, g_r) > V(\tilde{y}, \bar{t}_r, \bar{g}_r) > V(\tilde{y}, \bar{t}_p, \bar{g}_p) > V(\tilde{y}, t_p, g_p)$. Hence, the boundary individual always prefers to live in the rich region. An analogous argument applies for the case of substitutes and is omitted here for simplicity. \square

Thus, provided the median to mean ratios in both jurisdictions are smaller than unity, a boundary household cannot be indifferent between the jurisdictions. The basic intuition is as follows. If the potential boundary household \tilde{y} resides in the poor jurisdiction, it has the highest income in this jurisdiction and therefore is a ‘net contributor’ to the local budget (in terms of the private consumption good). If the household moved to the wealthy jurisdiction, it would be the lowest income resident and, hence, a ‘net recipient’ of local public funds. In the case of pure redistribution, g is a pure monetary transfer and it follows immediately that the household would always prefer to live in the rich region. More generally, in the case of substitutes, the household can never be compensated by the higher public good supply in the poor jurisdiction if its marginal utility of private consumption exceeds the marginal utility of public consumption, i.e., as long as both jurisdictions impose inefficiently high taxes. Conversely, in the case of complements, the resulting utility difference cannot be compensated

⁸As we discuss in Section 4 below, the line of argument that follows also holds under less restrictive assumptions than A1.

by the lower tax rate in the poor jurisdiction if the marginal utility of private consumption falls short of the marginal utility of public consumption, i.e., as long as taxes in both jurisdictions are inefficiently low. For reasons laid out above, however, democratic decision making and their presumed composition causes communities to depart from (privately) efficient taxation in exactly this way. In particular, note from Lemma 2 that local tax rates are above (below) the efficient level in the case of substitutes (complements) if and only if the decisive median voter has less than average income.

To analyze whether the median to mean ratio in the rich community Φ_r can exceed unity, we have a closer look at the underlying income distribution $f(y)$. Denote the median income of the overall distribution as y^m and the average income as \bar{Y} . We make the following assumptions.

A4 : $y^m < \bar{Y}$.

A5 : $f'(\hat{y}) = 0 \Rightarrow f'(y) > 0$ ($f'(y) < 0$) for $y < \hat{y}$ ($y > \hat{y}$), $\hat{y} \in [y, \bar{y}]$.

A4 implies that the income distribution is skewed to the right. A5 states that the density function has at most one mode \hat{y} .⁹ Note that if there is no interior mode, A4 and A5 jointly imply that $f(y)$ decreases monotonically. Closer inspection of the curvature reveals

Lemma 5. *Under A4 and A5, $\Phi_r \leq 1$, $\forall \tilde{y} \in [y, \bar{y}]$.*

Proof. See Appendix.

Taking together Lemma 4 and Lemma 5 yields:

Proposition 3. *Under A4 and A5, $\exists \tilde{y} \in [y, \bar{y}]$ with $V(\tilde{y}, t_p, g_p) = V(\tilde{y}, t_r, g_r)$ and $\Phi_p < \Phi_r$.*

Combining Propositions 1, 2, and 3, we see that there cannot be indifference between the two jurisdictions of *any* household \tilde{y} if the equilibrium is stratified. Hence, there cannot be an equilibrium with $(t_r, g_r) \neq (t_p, g_p)$. It is easy to show that the requirements for a stratification equilibrium become stronger for more than two communities (see Appendix). This leads to

Theorem. *Under A1-A5, no stratification equilibrium exists for $J \geq 2$.*

We close this section by noting that the theorem does not imply overall non-existence of equilibrium. As is well known, multi-community models generally have a symmetric equilibrium where jurisdictions conduct identical policies and are characterized by the same relative

⁹While A4 is undisputed, there exist few examples of income distributions which are bimodal, e.g. the nationwide distribution of household incomes in Great Britain. Most nationwide distributions, however, independent of the measurement concept, appear to be unimodal [see Burkhauser et al. [6]]. In any case, A5 is only a sufficient condition for the following results.

household distribution (up to size differences). Each jurisdiction then displays the same economic and demographic patterns as the nation. It is straightforward to show that such an equilibrium exists in our model, starting from a distribution of households over jurisdictions with $y_j^m = y^m$ and $\bar{Y}_j = \bar{Y}$ for all j . Hence, while there cannot be equilibria with different policies and stratification, equilibria with opposite characteristics are supported.

4 Discussion and Conclusion

This paper has provided a range of straightforward conditions under which a stratified equilibrium does not exist. Although this non-existence problem is not new to the literature on multi-community models, we believe our analysis to be of interest because it highlights the basic forces that prevent stratification. If local taxation is redistributive in nature, there is a natural reluctance of the middle-income households to stay in poorer jurisdictions. At the same time, wealthy communities are attractive because their policy provides a better mix of private and public consumption for the middle class. Importantly, the latter property endogenously emerges from the same preference monotonicity that otherwise works for stratification under plausible assumptions on the income distribution.

In obtaining the result, we have exploited three key properties of our framework which otherwise closely mirrors existing models: homothetic preferences, constant returns to community scale, and the absence of a housing market.¹⁰ We now discuss each of these in turn. As to the restriction on preferences, homotheticity is not necessary for our finding. Instead, what generates non-existence is that preferences (in addition to single-crossing) display an income elasticity of the demand for public goods that does not strictly exceed one.¹¹ Whether this is a plausible assumption is of course an empirical question and will also depend on the nature of the public good under consideration. The demand for educational spending, for example, appears to satisfy this property (see Shapiro and Rubinfeld [27]).

In light of previous contributions that have shown existence of stratified equilibria, the

¹⁰In their analysis of various policy impacts on a stratification equilibrium with two jurisdictions, Fernandez and Rogerson [15] use a model that is closest to ours with $\sigma < 1$. The authors assume stratification, but do not impose homotheticity and consider only three income groups. The boundary household in their presumed equilibrium belongs to the middle income class and the median in the poor (rich) community is a poor (rich) household, thus implicitly generating a situation with $\Phi_p < 1 < \Phi_r$, which violates A4 and A5 in our paper.

¹¹To see this, recall that both Lemma 1 and Lemma 2 can be framed in terms of the difference between the income elasticity, η_g , and σ . That Proposition 2 must still hold can be shown by using an argument similar to the proofs of Lemma 1 and 2., i.e., although tax rates are now no longer a function of Φ_j only, single crossing and $\eta_g \leq 1$ ensure that policies can still be related to Φ . Finally, Lemma 4 uses $\eta_g = 1$ only to obtain $\bar{t}_r = \bar{t}_p$, but continues to hold for $\bar{t}_r \leq \bar{t}_p$, which is implied by $\eta_g \leq 1$ and $\bar{Y}_r > \bar{Y}_p$. The remaining propositions and lemmas do not use A1. See also Kessler and Hansen [22] who consider quasi-linear preferences and do not impose homotheticity.

absence of scale economies in public good provision is important. For instance, Westhoff [29] considers the case of pure local public goods and Epple et al. [11] analyze public services with a fixed (possibly small) cost component. Intuitively, returns to community size are able to prevent migration into rich communities because their policy bundle can be undesirable if they are sufficiently small. Existence can then be shown under quite general assumptions.¹² Yet, the resulting non-convexity also implies that if a unique stratified equilibrium exists, it will be unstable (see, e.g, the discussion in Epple et al.[10]). Also, while there are many local public goods that display economies of scale, there are others which are more private in character such as health care or education (see Bergstrom and Goodman [4] and Edwards [7]).

Naturally, a land market can help to support stratification as well. High housing prices in wealthy communities can keep middle class (boundary) households ‘at bay’, a mechanism absent in our model. Yet, it is also clear that housing markets and the induced rent differentials are not sufficient for stratified equilibria to emerge. Suppose for example that there are two communities of fixed size and each households consumes only one unit of housing. For concreteness, assume our framework otherwise applies and purely redistributive policies ($\sigma \rightarrow \infty$). Now consider first a situation where the sum of housing units in the economy is exactly equal to the population size. Obviously, the boundary household in a potential stratification equilibrium is then *exogenously* determined by the relative size of the two communities. A stratification equilibrium (presuming single crossing etc.) will exist if and only if at this exogenous partition of the population (\tilde{y}), the rich community votes for lower income taxes than the poor ($\Phi_r \geq \Phi_p$, see Proposition 2). To see this, note that in this simple case of redistribution, what matters for households are tax rates t_j and the differential between the transfer and the housing price $g_j - p_j$. But if \tilde{y} is such that $t_r > t_p$, neither $g_r - p_r > g_p - p_p$ nor $g_r - p_r \leq g_p - p_p$ can be an equilibrium. In the former case, everybody would want to switch¹³ while in the latter case, indifference requires $p_p > p_r$ and, hence, $g_p > g_r$ which is impossible due to the smaller tax (base) in the poor jurisdiction. With a right-skewed and unimodal income distribution, however, $\Phi_r > \Phi_p$ is possible only for \tilde{y} very large, i.e., the fraction of the population living in the rich community to be very small. This will be the case only if one of the jurisdictions is sufficiently small. If, in contrast, both jurisdictions are of similar size, the median to mean ration is always smaller in the upper tail of the distribution, and hence, stratification cannot arise. An analogous argument applies if the total available

¹²Epple et al. [11] also consider a housing market and Rose-Ackermann [24] shows that Westhoff’s results continue to hold if a housing market is present.

¹³If only (some of) the residents of one community would like to move, housing prices would adjust accordingly. Once there are households in both communities who are unhappy with their locations, however, a housing market cannot resolve the inconsistency.

space exceeds population size: although \tilde{y} is no longer fixed, stratification requires $t_r \leq t_p$ or \tilde{y} large. But we also know (Proposition 3) that migration to the rich community can be prevented only if $p_r > p_p$ or, expressed differently, the rich community is ‘crowded’ at \tilde{y} (the housing market is tight). Again, therefore, small or medium size differentials do not support stratification (see Hansen and Kessler [19]). More generally, housing markets can facilitate the existence of external equilibrium, but the internal equilibrium must still be consistent with the presumed distribution of the population *and* the equilibrium rent differential, which depends on supply (space) and demand (policy-induced utility differences between the communities). Whether this can be ensured depends on the housing supply function, the income distribution, and how housing prices affect voter’s preferred choice.

Finally, one should note that both economies of scale and housing markets foster the segregation of classes through the link that is created between community size (or composition) and the implicit ‘price’ differential between rich and poor communities. As indicated above for the case of housing markets, since stratification requires the differential to be large enough *in equilibrium*, migration cannot always be prevented. Other, more direct ways to avoid ‘the poor chasing the rich’ are also conceivable and may prove more effective. For example, rich communities may impose minimum income requirements or pass zoning laws to prevent in-migration.¹⁴ Similarly, they could adopt a constitutional rule for the admission of immigrants. As discussed in Jehiel and Scotchmer [20], those may lead to very different equilibrium allocations. Although a rule that requires the consent of a majority is essentially ineffective in their model, the unanimity rule is shown to block migration in equilibrium.¹⁵

Summarizing, while in many situations stratification is possible, there are many other settings under which one should not expect stratification to emerge.¹⁶ In particular, a housing market can support sorting of the population within metropolitan areas, but is not likely to play an important role if one considers more spacious jurisdictions. For example, there is

¹⁴For an analysis of zoning in a multi-community model, see Fernandez and Rogerson [16]. Both institutions lead to admission prices that vary positively with immigrants’ incomes. In fact, type-dependent pricing of access to a public (club) good can be observed in many contexts. Epple and Romano [9], for example, study a model with peer-group effects in education. Private schools can charge tuition fees that depend on parent’s incomes and the equilibrium is stratified.

¹⁵Their result that the equilibrium in which a majority can block immigration is the same as the free migration equilibrium can be explained as follows. Jehiel and Scotchmer consider a pure public good financed by a head tax. Consumption is therefore non-rival and spending increases with the population size. Since the induced policy change if ‘one more’ immigrant is admitted is of second-order, the median voter always supports immigration.

¹⁶Indeed, as is discussed in Epple and Platt [12] and de Bartholomé and Ross [2], the observed income variation within communities of many metropolitan areas in the U.S. is not consistent with strict sorting according to incomes. While Epple and Platt show that community heterogeneity can arise if households differ in tastes and incomes, de Bartholomé and Ross provide an explanation based on the explicit consideration of space (commuting cost to the inner city).

increasing evidence that jurisdictions in federal settings (such as states in the US, *Kantone* in Switzerland, *Länder* in Germany) converge with respect to per capita-income and taxation policies.¹⁷ A plausible explanation for this convergence is given by the traditional neoclassical growth model [Sala-i-Martin [26]], but an alternative (or complementary) reason is the endogenous harmonization of equilibrium policies and (average) incomes proposed here.¹⁸

Appendix

Proof of Lemmas 1 and 2.

Since Lemma 1 follows immediately from Lemma 2 (see below), it is convenient to prove the second result first: we show that preferred level of public good provision g of a voter in a given community varies positively (negatively) with y if and only if $\sigma < 1$ ($\sigma > 1$) for all (c, g) and y . Since $g_j^y = t_j^y \bar{Y}_j$ and \bar{Y}_j is fixed, Lemma 2 follows. To this end, consider the purchasing problem of an individual with income y for two goods, c and g . Normalize the price of c to one and let p be the price of g (in units of y and c). The demand for g varies with p according to the Slutsky equation, which reads in elasticity terms

$$\frac{\partial \log g}{\partial \log p} = \frac{\partial \log g}{\partial \log p} \Big|_{U=\bar{U}} - (1 - \gamma) \frac{\partial \log g}{\partial \log y}, \quad (7)$$

where $\gamma = c/y \in [0, 1]$ is the expenditure share of c . Note that for the compensated demand functions (using $dU = 0$),

$$\frac{\partial c}{\partial \log p} \Big|_{U=\bar{U}} + p \frac{\partial g}{\partial \log p} \Big|_{U=\bar{U}} = 0 \Rightarrow \gamma \frac{\partial \log c}{\partial \log p} \Big|_{U=\bar{U}} + (1 - \gamma) \frac{\partial \log g}{\partial \log p} \Big|_{U=\bar{U}} = 0. \quad (8)$$

By definition,

$$\sigma \equiv \frac{d \log(g/c)}{d \log(1/p)} \Big|_{U=\bar{U}} = \frac{\partial \log c}{\partial \log p} \Big|_{U=\bar{U}} - \frac{\partial \log g}{\partial \log p} \Big|_{U=\bar{U}}. \quad (9)$$

Combining (8) and (9), solving for $\partial \log g / \partial \log p \Big|_{U=\bar{U}}$ and substituting back into (7) yields

$$\frac{\partial \log g}{\partial \log p} = -\gamma\sigma - (1 - \gamma)\eta_g, \quad (10)$$

where η_g is the income elasticity of demand for g and the price effect is now expressed in terms of the elasticity of substitution, σ .

The remainder of the proof follows Kenny [21]. The demand for g depends on the price p and on income y . Given the local budget constraint, the unit price of the public good is

¹⁷See Barro and Sala-i-Martin [3] and their references.

¹⁸Sala-i-Martin [26] rejects the notion of convergence through migration referring to low migration rates. It is plausible (due to decreasing relative migration costs) and empirically confirmed [see, e.g. Borjas et al. [5]], however, that high income households have the highest mobility. Even small numbers of migrating households can therefore result in large changes of regional incomes.

y/\bar{Y}_j and proportional to the voters income. Using $d \log p = d \log y$ and (10), we find

$$\begin{aligned} \frac{d \log g}{d \log y} &= \frac{\partial \log g}{\partial \log y} + \frac{\partial \log g}{\partial \log p} \frac{d \log p}{d \log y} = \eta_g - \gamma \sigma - (1 - \gamma) \eta_g \\ &= \gamma(\eta_g - \sigma) \quad \begin{matrix} \geq 0 \\ \leq 0 \end{matrix} \Leftrightarrow \eta_g \begin{matrix} \geq \\ \leq \end{matrix} \sigma. \end{aligned}$$

Consequently, g_j^y (and, hence, t_j^y) is everywhere increasing in y if and only if the income elasticity of demand for public services everywhere exceeds the elasticity of substitution between public and private goods. Recalling that $\eta_g \equiv 1$ for homothetic functions proves Lemma 2. To show Lemma 1, it suffices to note that $M(y, t_j^y, g_j^y) = \bar{Y}_j$. For a small change in y , therefore,

$$\text{sign} \frac{\partial M(y, \cdot)}{\partial y} = -\text{sign} \frac{dM(y, t_j^y, t_j^y \bar{Y}_j)}{dt_j^y} \frac{dt_j^y}{dy} = -\text{sign} \frac{dt_j^y}{dy},$$

where the last equality uses the fact that t_j^y and $g_j^y = t_j^y \bar{Y}_j$ are varied in proportion to $M(\cdot)$, i.e., we move along the indifference curve $V = \bar{V}$, the slope of which is increasing in t (see, e.g., Westhoff [29]). Varying y and \bar{Y}_j (to cover the entire (t, g) -plane), we obtain

$$\frac{\partial M(y, t, g)}{\partial y} \leq 0 \quad \forall y \quad \Leftrightarrow \quad \eta_g \geq \sigma \quad \forall y,$$

which proves Lemma 1 since $\eta_g \equiv 1$. \square

Proof of Lemma 5.

Let $F(y)$ be some twice continuously differentiable distribution function on $[y, \bar{y}]$ and let the associated density function $f(y)$ satisfy assumptions A4 and A5. We show that the truncated distribution $F_r(y) = [F(y) - F(\tilde{y})]/[1 - F(\tilde{y})]$, defined on $y \in [\tilde{y}, \bar{y}]$, is characterized by $\Phi_r \leq 1 \Leftrightarrow y_r^m \leq \bar{Y}_r \forall \tilde{y} \in [y, \bar{y}]$. Observe first that

$$\bar{Y}_r = \frac{1}{1 - F(\tilde{y})} \int_{\tilde{y}}^{\bar{y}} y f(y) dy = \frac{1}{1 - F(\tilde{y})} \left\{ \bar{y} - F(\tilde{y}) \tilde{y} - \int_{\tilde{y}}^{\bar{y}} F(y) dy \right\}$$

by partial integration. Hence,

$$y_r^m \leq \bar{Y}_r \quad \Leftrightarrow \quad G(\tilde{y}) \equiv \int_{\tilde{y}}^{\bar{y}} F(y) dy + F(\tilde{y}) \tilde{y} - \bar{y} + [1 - F(\tilde{y})] y_r^m \leq 0, \quad (11)$$

Clearly, $G(\bar{y}) = 0$ and $G(\underline{y}) \leq 0$ by A4 so that (11) is satisfied at $\tilde{y} = \underline{y}$ and $\tilde{y} = \bar{y}$. Therefore, (11) holds if $G(\tilde{y})$ has no local maxima on (\underline{y}, \bar{y}) . We proceed to show that $G''(\tilde{y}) > 0$ whenever $G'(\tilde{y}) = 0$.

Taking the derivative of $G(\tilde{y})$, using $F(y_r^m) - F(\tilde{y}) = \frac{1}{2}[1 - F(\tilde{y})]$ and $\partial y_r^m / \partial \tilde{y} = \frac{1}{2}[f(\tilde{y})/f(y_r^m)]$, one obtains

$$G'(\tilde{y}) = 0 \quad \Leftrightarrow \quad F(y_r^m) - F(\tilde{y}) = f(y_r^m)(y_r^m - \tilde{y}),$$

i.e., the area under the density function between \tilde{y} and y_r^m must be equal to a rectangle with height $f(y_r^m)$ and length $y_r^m - \tilde{y}$. For this to be the case, the unimodal density function $f(y)$ cannot be monotonic over $[\tilde{y}, y_r^m]$. Hence, we must have $\tilde{y} < \hat{y} < y_r^m$ and $f'(y_r^m) < 0$ at any such point. Furthermore, $f(y)$ cannot attain a minimum at $y = y_r^m$, implying $f(y_r^m) > f(\tilde{y})$. Taking the second derivative of G , evaluated at $G'(\tilde{y}) = 0$, we find

$$G''(\tilde{y}) = -f'(y_r^m)(y_r^m - \tilde{y})\frac{\partial y_r^m}{\partial y} + f(y_r^m) - f(\tilde{y}) > 0 \quad \text{for} \quad f'(y_r^m) < 0 \text{ and } f(y_r^m) > f(\tilde{y}).$$

$G'(\tilde{y}) = 0$ therefore implies $G''(\tilde{y}) > 0$, which completes the proof. \square

Proof of the Theorem.

Suppose there exists a fully stratified equilibrium with an arbitrary number of jurisdictions $J \geq 2$. Now consider the two wealthiest jurisdictions, say 1 and 2, and note that since the equilibrium is stratified, the (conditional) income distribution $f_c(y)$ of households living there must still satisfy A4 and A5. Applying Proposition 3 to those two communities, we see that there cannot be an indifferent household, a contradiction. Hence, the two jurisdictions must be equivalent with $(t_1^*, g_1^*) = (t_2^*, g_2^*) > 0$, $\bar{Y}_1^* = \bar{Y}_2^* = \bar{Y}_c$ from (1) and $y_1^m = y_2^m$ from Lemma 2, which together with $f_1^*(y) + f_2^*(y) = f_c(y)$ implies $y_1^{m*} = y_2^m = y_c^m$. The two jurisdictions implement the same policy as a single community with an income distribution $f_c(y)$. Next, consider this (joint) community and the next richest, 3. Again, Proposition 3 leads to a contradiction and by the same argument as above, the richest three jurisdictions must therefore be effectively identical. Repeating this line of reasoning for the remaining communities completes the proof. \square

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