Midterm

Economics 435: Quantitative Methods

Fall 2011

Please answer the question I am asking - no more and no less - and remember that the correct answer is usually short.

1 Warmup

Let $D_n = x_1, \ldots, x_n$ be a sample of size n in which the x's are independent but not identically distributed. Specifically, even numbered observations are drawn from the $N(\mu_E, \sigma^2)$ distribution, while odd-numbered observations are drawn from a the $N(\mu_O, \sigma^2)$ distribution. Let

$$\bar{x}_n = \frac{1}{n}\sum_{i=1}^{n} x_i$$

a) Assume that n is an even number. Is $\bar{x}_n$ is an unbiased estimator of $\theta = \frac{\mu_E + \mu_O}{2}$?

b) Find $var(\bar{x}_n)$.

c) Is $\bar{x}_n$ is a consistent estimator of θ?

2 The Mincer human capital model

Suppose we have a random sample of size n on the variables $(LNWAGE, EDUC, AGE)$ where $LNWAGE_i$ is person i's log wage, $EDUC_i$ is the number of years spent in formal education, and AGE_i is current age. We use these variables to construct an additional variable:

$$EXPER_i = (AGE_i - EDUC_i - 6)$$

which measures person i's years of work experience[1] under the assumption that person i started school at age 6, did not work while in school, and has worked ever since.

A simple version of the Mincer human capital model or Mincer wage model takes the form:

$$LNWAGE_i = \beta_0 + \beta_1 EDUC_i + \beta_2 EXPER_i + u_i \tag{1}$$

where:

$$E(u_i | EDUC_i, EXPER_i, AGE_i) = 0$$

The coefficient β_1 in this regression is often interpreted as the rate of return to investments in schooling, and β_2 is interpreted as the returns to experience. We usually expect both β_1 and β_2 to be positive.

[1] This measure of experience is sometimes called *potential* experience, to distinguish it from actual work experience

Finally, let's assume that:

$$cov(AGE_i, EDUC_i) = 0$$

That is, older people are no more nor less educated than younger people. This might be a reasonable assumption as long as our sample excludes people below 30 (who may not have had the opportunity to complete their schooling) and over 55 (who grew up at a time of lower educational opportunity).

a) Let $\hat{\beta}_1$ be the coefficient on $EDUC$ in an OLS regression of $LNWAGE$ on $EDUC$. Find its probability limit, if it exists. If it does not exist, briefly explain why.

b) Let $\hat{\beta}_1$ be the coefficient on $EDUC$ in an OLS regression of $LNWAGE$ on $(EDUC, AGE)$. Find its probability limit, if it exists. If it does not exist, briefly explain why.

c) Let $\hat{\beta}_1$ be the coefficient on $EDUC$ in an OLS regression of $LNWAGE$ on $(EDUC, AGE, EXPER)$. Find its probability limit, if it exists. If it does not exist, briefly explain why.

d) Assuming that the Mincer model is correct, would the regressions in (a) and (b) generally lead us to overestimate β_1, underestimate β_1, or get it about right?

3 Vector autoregressions

Note: I have corrected the typo (m_{t-2} instead of m_{t-1}) in the midterm I handed out.

Last week, Chris Sims and Tom Sargent were awarded this year's Nobel prize in economics. Both Sims and Sargent are empirically-oriented macroeconomists. Sims is most well known for introducing something to empirical macroeconomics called the "vector autoregression" (VAR). We will work with a simple example of a VAR that addresses an important identification problem in macroeconometrics: measuring the effect of monetary policy on output is complicated by the fact that monetary policy both (probably) affects output and responds to output.

Let y_t be log real GDP per capita at time t, let m_t be the log of the money supply at time t, and suppose[2] that:

$$y_t = a_0 m_t + a_1 y_{t-1} + a_2 m_{t-1} + u_t \quad (2)$$
$$m_t = b_0 y_t + b_1 y_{t-1} + b_2 m_{t-1} + v_t \quad (3)$$

where u_t is an unexpected real shock and v_t is an unexpected nominal shock that meet the following conditions:

$$E(u_t | y_{t-1}, m_{t-1}) = 0$$
$$E(v_t | y_{t-1}, m_{t-1}) = 0$$
$$cov(u_t, v_t) = 0$$

That is, future real and nominal shocks are uncorrelated with each other, and unpredictable using current information.

Without further assumptions the parameters of this model are not identified. A common solution to this problem is to add the plausible assumption that output responds to monetary policy with a lag of at least one period. That is:

$$a_0 = 0$$

This simple assumption will allow us to identify the model parameters.

[2] You may notice that there is no intercept in this model. That is a common simplification in macroeconometrics, because the variables have usually been detrended, deseasonalized, and otherwise transformed so that they are all mean-zero.

a) Prove that:

$$m_t = \pi_1 y_{t-1} + \pi_2 m_{t-1} + \epsilon_t$$

where

$$\begin{aligned} \pi_1 &= b_0 a_1 + b_1 \\ \pi_2 &= b_0 a_2 + b_2 \\ \epsilon_t &= b_0 u_t + v_t \\ E(\epsilon_t | y_{t-1}, m_{t-1}) &= 0 \end{aligned}$$

b) Let $\sigma_u^2 = var(u_t)$, $\sigma_\epsilon^2 = var(\epsilon_t)$ and $\sigma_{u\epsilon} = cov(u_t, \epsilon_t)$. Prove that $\sigma_{u\epsilon} = b_0 \sigma_u^2$

c) Suppose you do the following.

1. Estimate an OLS regression of y_t on (y_{t-1}, m_{t-1}). Let $(\hat{a}_1, \hat{a}_2)$ be the coefficient estimates from that regression.
2. Estimate an OLS regression of m_t on (y_{t-1}, m_{t-1}). Let $(\hat{\pi}_1, \hat{\pi}_2)$ be the coefficient estimates from that regression.
3. Calculate the sample variances $(\hat{\sigma}_u^2, \hat{\sigma}_\epsilon^2)$ and covariance $\hat{\sigma}_{u\epsilon}$ of the residuals from these two regressions.

Our prior results (along with a few technical assumptions) imply that:

$$(\hat{a}_1, \hat{a}_2, \hat{\pi}_1, \hat{\pi}_2, \hat{\sigma}_u^2, \hat{\sigma}_\epsilon^2, \hat{\sigma}_{u\epsilon}) \to^p (a_1, a_2, \pi_1, \pi_2, \sigma_u^2, \sigma_\epsilon^2, \sigma_{u\epsilon})$$

Taking this result as given, find consistent estimators of b_0, b_1, and b_2 in terms of $(\hat{a}_1, \hat{a}_2, \hat{\pi}_1, \hat{\pi}_2, \hat{\sigma}_u^2, \hat{\sigma}_\epsilon^2, \hat{\sigma}_{u\epsilon})$.

d) Prove that your estimators are consistent.