Exam #1 Answer Key

Economics 435

Spring 2004

1 Airplanes

a) The probability is $p_L q_L + p_R q_R$.

b)

$$
\begin{aligned}
E(\hat{p}_L) &= \frac{p_L(1-q_L)}{1-p_L q_L - p_R q_R} \\
E(\hat{p}_R) &= \frac{p_R(1-q_R)}{1-p_L q_L - p_R q_R} \\
E(\hat{c}) &= p_L q_L + p_R q_R
\end{aligned}
$$

c)

$$
\begin{aligned}
E(\hat{c}|L-reinforced) &= p_R q_R \\
E(\hat{c}|R-reinforced) &= p_L q_L
\end{aligned}
$$

d) No. The answer to question (b) gives only three equations, and there are four unknowns. So we cannot identify the individual probabilities. In words, we don't know whether more planes returned with damage on the left because more planes received damage on the left or because planes that received damage on the right were more likely to crash.

e) Based on the answer to question (c), and the assumption that $q_L = q_R$, we should reinforce the left if $p_L > p_R$. In addition, the assumption that $q_L = q_R$ implies that $\frac{\hat{p}_L}{\hat{p}_R} = \frac{p_L}{p_R}$. Since $\hat{p}_L > \hat{p}_R$, it follows that $p_L > p_R$. Therefore the RAF should reinforce the left side.

f) Based on the answer to question (c), and the assumption that $p_L = p_R$, we should reinforce the left if $q_L > q_R$. In addition, the assumption that $p_L = p_R$ implies that $\frac{\hat{p}_L}{\hat{p}_R} = \frac{1-q_L}{1-q_R}$. Since $\hat{p}_L > \hat{p}_R$, it follows that $1 - q_L > 1 - q_R$, or $q_R > q_L$. Therefore the RAF should reinforce the right side.

g) Reinforce the right on 50 of them and the left on the other 50. Assign planes to the "left-reinforced" and "right-reinforced" groups randomly. If more right-reinforced planes come back than left-reinforced planes, then reinforce the right side. If more left-reinforced planes come back, reinforce the right side.

2 Income and life expectancy

a) The null hypothesis is

$$H_0 : \beta_2 = 0$$

The alternative is:

$$H_1 : \beta_2 \neq 0$$

The test statistic is:

$$t = \frac{\hat{\beta}_2}{\hat{sd}(\hat{\beta}_2}$$

Under the null, its asymptotic distribution is $N(0, 1)$ implying we should reject the null if $|t| > 1.96$. The actual value of the test statistic is:

$$\frac{-0.0353}{0.0058} = 6.08$$

So we reject the null. The relationship is not linear.

b) The effect will be $(1.9375 + 2 * (-0.0353) * GDP)\Delta GDP$

c) The change will be $1.9375 * (4.733 - 0.490) - 0.0353 * (4.733^2 - 0.490^2) =$ 7.43 years. So moving up to the median income is predicted to increase life expectancy to around 46.43 years.

3 The mode

a) Using the hint, we note that:

$$M = \arg \max_{z \in K} E[I(x = z)]$$

Applying the analog principle, we get

$$\hat{M} = \arg \max_{z \in K} \frac{1}{n} \sum_{i=1}^{n} I(x_i = z)$$

In other words, pick $\hat{M}$ to be the value of x that appears most frequently in the sample.

b) This estimator is biased. Suppose that $n = 1$, $K = \{0, 1\}$, and $f(1) = 0.75$, $f(2) = 0.25$. Note that the population mode is 1. Now we find the expected value of $\hat{M}$. With probability 0.75, we will have $x_1 = 1$ and thus $\hat{M} = 1$. With probability 0.25, we will have $x_1 = 0$ and thus $\hat{M} = 0$. Therefore, for a sample of size 1, $E(\hat{M}) = 0.75 \neq M$.

c) This estimator is consistent. Let the parameter vector $(\theta_1, \theta_2, \ldots, \theta_k) = (f(K_1), f(K_2), \ldots, f(K_k))$ and let $\hat{\theta}_i = \sum_{i=1}^{n} I(x_i = K_i)$. First, note that as a consequence of the law of large numbers,

$$\text{plim } \hat{\theta}_i = \theta_i$$

Applying the Slutsky theorem,

$$\begin{aligned}
\text{plim } \hat{M} &= \text{plim } \max(\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_k) \\
&= \max(\text{plim } \hat{\theta}_1, \text{plim } \hat{\theta}_2, \ldots, \text{plim } \hat{\theta}_k) \\
&= \max(\theta_1, \theta_2, \ldots, \theta_k) \\
&= M
\end{aligned}$$

d) No. Consider the example we used for part (b) of the question. No matter what value for n, the distribution of $\hat{M}$ will take on one of only two points. This will still be true if we subtract $E(\hat{M})$ and divide by $SD(\hat{M})$. So this distribution cannot converge to a continuous distribution like the $N(0, 1)$.