Exam #1 Answer Key

Economics 435: Quantitative Methods

Spring 2005

1 Regression with standardized variables

a)

$$\hat{\beta}_0 = 0$$

b)

$$R^2 = \hat{\beta}_1^2$$

c)

$$c\hat{o}rr(\tilde{y}, \tilde{x}) = \hat{\beta}_1$$

d) Well, since we have shown that $\hat{\beta}_1 = c\hat{o}rr(\tilde{x}, \tilde{y})$, and all correlations are between -1 and $+1$, the answer to the question is "no."

2 Heteroskedasticity: A simple example

a) There are many ways to show this, but here's one:

$$\begin{aligned}
\hat{\beta}_1 &= \frac{c\hat{o}v(y,x)}{v\hat{a}r(x)} \\
&= \frac{1/n \sum_{i=1}^n (y_i - \bar{y})(x_i - \bar{x})}{1/n \sum_{i=1}^n (x_i - \bar{x})^2} \\
&= \frac{\sum_{i=1}^n y_i(x_i - \bar{x}) - \bar{y}\sum_{i=1}^n (x_i - \bar{x})}{n\bar{x}(1-\bar{x})} \\
&= \frac{\sum_{i=1}^n x_i y_i (1-\bar{x}) + (1-x_i) y_i (0 - \bar{x})}{n\bar{x}(1-\bar{x})} \\
&= \frac{(1-\bar{x})\sum_{i=1}^n x_i y_i - \bar{x}\sum_{i=1}^n (1-x_i) y_i}{n\bar{x}(1-\bar{x})} \\
&= \frac{\sum_{i=1}^n x_i y_i}{n\bar{x}} - \frac{\sum_{i=1}^n (1-x_i) y_i}{n(1-\bar{x})} \\
&= \bar{y}_1 - \bar{y}_0
\end{aligned}$$

For $\hat{\beta}_0$, we simply note that $\bar{y} = (1-\bar{x})\bar{y}_0 + \bar{x}\bar{y}_1$, and substitute.

b) Assumptions SLR1-SLR4 are met, so the OLS estimators are unbiased and consistent.

c)

$$\begin{aligned}
var(\hat{\beta}_0) &= var(\bar{y}_0) \\
&= \frac{\sigma_0^2}{n(1-\bar{x})} \\
var(\hat{\beta}_1) &= var(\bar{y}_1) + var(\bar{y}_0) - 2cov(\bar{y}_1, \bar{y}_0) \\
&= \frac{\sigma_1^2}{n\bar{x}} + \frac{\sigma_0^2}{n(1-\bar{x})} \\
cov(\hat{\beta}_0, \hat{\beta}_1) &= cov(\bar{y}_0, \bar{y}_1 - \bar{y}_0) \\
&= cov(\bar{y}_0, \bar{y}_1) - var(\bar{y}_0) \\
&= \frac{\sigma_0^2}{n(1-\bar{x})}
\end{aligned}$$

3 Hypothesis testing

a) Our best chance for rejecting the null is if all of our observations are greater than zero ($\bar{y}_n = 1$) of if all observations are less than zero ($\bar{y}_n = 0$). If this is the case then $|t| = \sqrt{n}$. With a sample size of two, the largest possible value for $|t|$ is $\sqrt{2} = 1.414 < 1.96$. In general, we will have no chance of rejecting the null if $\sqrt{n} < 1.96$ or if $n < 1.96^2 = 3.84$. In other words we need a sample of size four or greater to have a chance at rejecting the null.

b) I would define:

$$t = \sum_{i=1}^{n} I(x_i > 10)$$

Under the null hypothesis:

$$\Pr(t = 0) = 1$$

So we can reject the null at the 5% significance level (actually, at any significance level) if $t > 0$.

In other words, we can reject the null if we ever see a value of x bigger than 10, and we cannot reject the null otherwise. This makes sense, hopefully.

4 Craps

a)

$$\begin{aligned}
\Pr(x = 2) &= 1/36 \\
\Pr(x = 3) &= 2/36 \\
\Pr(x = 4) &= 3/36 \\
\Pr(x = 5) &= 4/36 \\
\Pr(x = 6) &= 5/36 \\
\Pr(x = 7) &= 6/36 \\
\Pr(x = 8) &= 5/36 \\
\Pr(x = 9) &= 4/36 \\
\Pr(x = 10) &= 3/36 \\
\Pr(x = 11) &= 2/36 \\
\Pr(x = 12) &= 1/36
\end{aligned}$$

b) The probabilities are:

$$\begin{aligned}
\Pr(x \in \{7,11\}) &= 8/36 \\
\Pr(x \in \{2,3,12\}) &= 4/36 \\
\Pr(x = 4) &= 3/36 \\
\Pr(x = 5) &= 4/36 \\
\Pr(x = 6) &= 5/36 \\
\Pr(x = 8) &= 5/36 \\
\Pr(x = 9) &= 4/36 \\
\Pr(x = 10) &= 3/36
\end{aligned}$$

c) The probabilities are:

$$\begin{aligned}
\Pr(x = 4 | x \in \{4,7\}) &= \frac{3/36}{3/36 + 6/36} = 1/3 \\
\Pr(x = 5 | x \in \{5,7\}) &= \frac{4/36}{4/36 + 6/36} = 2/5 \\
\Pr(x = 6 | x \in \{6,7\}) &= \frac{5/36}{5/36 + 6/36} = 5/11 \\
\Pr(x = 8 | x \in \{8,7\}) &= \frac{5/36}{5/36 + 6/36} = 5/11 \\
\Pr(x = 9 | x \in \{9,7\}) &= \frac{4/36}{4/36 + 6/36} = 2/5 \\
\Pr(x = 10 | x \in \{10,7\}) &= \frac{3/36}{3/36 + 6/36} = 1/3
\end{aligned}$$

d) The probability is:

$$\begin{aligned}
\Pr(\text{win}) &= 8/36 + 3/36 * 1/3 + 4/36 * 2/5 + 5/36 * 5/11 + 5/36 * 5/11 + 4/36 * 2/5 + 3/36 * 1/3 \\
&= \frac{244}{495} \\
&\approx 49.3\%
\end{aligned}$$

e) For every \$1 bet, the casino's profit is \$1 with 50.7% probability and -\$1 with 49.3% probability. The expected profit per \$1 bet is thus approximately 1.4 cents.