

Midterm Answer Key

Economics 435: Quantitative Methods

Fall 2009

1 Properties of expected values

a) We start with the definition, and then use a little algebra along with the linearity of expectations theorem:

$$\begin{aligned} \text{cov}(x, y) &= E((x - E(x))(y - E(y))) \\ &= E(xy - xE(y) - yE(x) + E(x)E(y)) \\ &= E(xy) - E(xE(y)) - E(yE(x)) + E(E(x)E(y)) \\ &= E(xy) - E(x)E(y) - E(x)E(y) + E(x)E(y) \\ &= E(xy) - E(x)E(y) \end{aligned}$$

b) First, we note that:

$$E(xu|x) = xE(u|x) = x \times 0 = 0$$

Then we note that by the law of iterated expectations:

$$E(u) = E(E(u|x)) = E(0) = 0$$

and

$$E(xu) = E(E(xu|x)) = E(0) = 0$$

By the result established in (a):

$$\begin{aligned} \text{cov}(x, u) &= E(xu) - E(x)E(u) \\ &= 0 - E(x)0 \\ &= 0 \end{aligned}$$

c) First note that:

$$\begin{aligned} E(xy) &= E(E(xy|x)) \\ &= E(xy|x=1) \Pr(x=1) + E(xy|x=0) \Pr(x=0) \\ &= E(1 \times y|x=1) \Pr(x=1) + E(0 \times y|x=0) \Pr(x=0) \\ &= E(y|x=1) \Pr(x=1) + E(0|x=0) \Pr(x=0) \\ &= E(y|x=1) \Pr(x=1) \end{aligned}$$

Next note that:

$$\begin{aligned} E(x) &= 1 \times \Pr(x=1) + 0 \times \Pr(x=0) \\ &= \Pr(x=1) \end{aligned}$$

Then we can divide the first result by the second:

$$\begin{aligned}\frac{E(xy)}{E(x)} &= \frac{E(y|x=1)\Pr(x=1)}{\Pr(x=1)} \\ &= E(y|x=1)\end{aligned}$$

2 The consequences of skipping class

a) Since r_i depends on midterm score, it is likely to be correlated with other factors that affect exam scores such as ability, interest in the subject, motivation, time available to study, etc. As a result, the OLS regression will suffer from severe omitted variables bias.

b) The treatment effect in question is:

$$TE_i = f_i(1) - f_i(0)$$

c)

$$\begin{aligned}\text{plim } \hat{ATE}_c &= \text{plim} \left(\frac{\frac{1}{n} \sum_{i=1}^n f_i r_i c_i}{\frac{1}{n} \sum_{i=1}^n r_i c_i} - \frac{\frac{1}{n} \sum_{i=1}^n f_i (1-r_i) c_i}{\frac{1}{n} \sum_{i=1}^n (1-r_i) c_i} \right) \\ &= \frac{\text{plim} \frac{1}{n} \sum_{i=1}^n f_i r_i c_i}{\text{plim} \frac{1}{n} \sum_{i=1}^n r_i c_i} - \frac{\text{plim} \frac{1}{n} \sum_{i=1}^n f_i (1-r_i) c_i}{\text{plim} \frac{1}{n} \sum_{i=1}^n (1-r_i) c_i} \quad \text{by Slutsky's theorem} \\ &= \frac{E(frc)}{E(rc)} - \frac{E(f(1-r)c)}{E((1-r)c)} \quad \text{by the Law of Large Numbers} \\ &= E(f|rc=1) - E(f|(1-r)c=1) \quad \text{by part (c) of question 1} \\ &= E(f|r=1, c=1) - E(f|r=0, c=1) \\ &= E(f(1)|r=1, c=1) - E(f(0)|r=0, c=1) \\ &= E(f(1)|c=1) - E(f(0)|c=1) \quad \text{by (1) and (2)} \\ &= E(f(1) - f(0)|c=1) \\ &= E(TE|c=1)\end{aligned}$$

d) Setting a small ϵ makes assumptions (1) and (2) more reasonable, but it also means that fewer observations are being used to estimate ATE_c . So there is a tradeoff between bias (through possible violation of (1) and (2)) and variance (by using a smaller sample).

e) The average treatment effect is:

$$\begin{aligned}E(TE_i) &= E(f_i(1) - f_i(0)) \\ &= E(E(f_i(1) - f_i(0)|m_i, r_i)) \\ &= E(E(f_i(1)|m_i, r_i) - E(f_i(0)|m_i, r_i)) \\ &= E(\beta_1 + \beta_2 m_i - (\beta_0 + \beta_2 m_i)) \\ &= E(\beta_1 - \beta_0) \\ &= \beta_1 - \beta_0\end{aligned}$$

f) First, we need to find $E(f_i|m_i, r_i)$.

$$\begin{aligned}E(f_i|m_i, r_i) &= E(r_i f_i(1) + (1-r_i) f_i(0)|m_i, r_i) \\ &= r_i E(f_i(1)|m_i, r_i) + (1-r_i) E(f_i(0)|m_i, r_i) \\ &= r_i(\beta_1 + \beta_2 m_i) + (1-r_i)(\beta_0 + \beta_2 m_i) \\ &= \beta_0 + (\beta_1 - \beta_0)r_i + \beta_2 m_i\end{aligned}$$

Let $u_i = f_i - E(f_i|m_i, r_i)$. By construction:

$$\begin{aligned}f_i &= \beta_0 + (\beta_1 - \beta_0)r_i + \beta_2 m_i + u_i \\E(u_i|r_i, m_i) &= 0\end{aligned}$$

The first of these results corresponds to assumption MLR1, and the second to assumption MLR4. We have a random sample (assumption MLR2) and the needed variation in the explanatory variables (assumption MLR3). Therefore:

$$\text{plim } \hat{\beta}_r = (\beta_1 - \beta_0) = E(TE_i)$$