Exam #1 Answer Key

Economics 435: Quantitative Methods

Spring 2006

1 Starter questions (20 points)

a)

$$cov(ax, b + cx + dy) = ac\ var(x) + ad\ cov(x, y)$$

b)

- Yes.
- Yes.
- It is $\hat{\beta}_1 = \frac{0.23+0.77}{2} = 0.5$.

c) Suppose that $E(u|x) = 0$. The law of iterated expectations implies that $E(u) = E(E(u|x)) = E(0) = 0$.

d) Suppose that $E(u|x) = 0$. The law of iterated expectations implies that $E(u) = E(E(u|x)) = E(0) = 0$. Next we note that $E(ux|x) = xE(u|x) = x * 0 = 0$. Applying the law of iterated expectations again, $E(ux) = E(E(ux|x)) = E(0) = 0$. Then $cov(u, x) = E(ux) - E(u)E(x) = 0 - 0E(x) = 0$.

2 Biased sample selection (40 points)

a) β_1 is just the slope of the line connecting (X_0, Y_0) and (X_1, Y_1), so:

$$\hat{\beta}_1 = \frac{\hat{Y}_1 - \hat{Y}_0}{X_1 - X_0}$$

b) Here's how I did it:

$$\begin{aligned}
\text{plim } \hat{P}_Y &= \frac{\text{plim } \frac{1}{n}\sum_{i=1}^{n} I(y_i = Y \cap x_i = X_0 \cap s_i = 1)}{\text{plim } \frac{1}{n}\sum_{i=1}^{n} I(x_i = X_0 \cap s_i = 1)} \\
&= \frac{\Pr(y = Y \cap x = X_0 \cap s = 1)}{\Pr(x = X_0 \cap s = 1)} \\
&= \Pr(y = Y | x = X_0 \cap s = 1)
\end{aligned}$$

c) First, note that Bayes' Law implies:

$$\begin{aligned}
\Pr(y = Y | x = X_0, s = 1) &= \frac{\Pr(y = Y \cap s = 1 | x = X_0)}{\Pr(s = 1 | x = X_0)} \\
&= \frac{\Pr(s = 1 | x = X_0 \cap y = Y)\Pr(y = Y | x = X_0)}{\Pr(s = 1 | x = X_0)}
\end{aligned}$$

Solving for $\Pr(y = Y|x = X_0)$ we get:

$$\Pr(y = Y|x = X_0) = \frac{\Pr(y = Y|x = X_0, s = 1)\Pr(s = 1|x = X_0)}{\Pr(s = 1|x = X_0, y = Y)}$$

d) The condition is:

$$\Pr(s = 1|x = X_0, y = Y) = \Pr(s = 1|x = X_0)$$

In other words, if the probability of selection is a function of the explanatory variables but not the dependent variable, we can still use OLS.

e) The condition is:

$$\Pr(s = 1|x = X_0, y = Y) > 0$$

In other words, if we can never observe cases in which $y = Y$, we cannot tell $\Pr(y = Y)$.

3 Measurement error (40 points)

a)

$$b_1 = \frac{\beta_1 var(x) + cov(e_x, e_y)}{var(x) + var(e_x)}$$

b)

- No.
- Yes.
- Yes.

c)

- Yes.
- Yes.
- No.

d)

- No.
- Yes.
- Yes.

e)

- Uncertain. "No" would be OK too here, because the condition that would have to hold is:

  $$cov(e_x, e_y) = \beta_1 var(e_x)$$

  This is possible, but only by some amazing coincidence.
- Uncertain.

- Uncertain.

f)

- Uncertain. “No” would be OK too here.
- Uncertain.
- Yes.