Exam #2 Answer Key

Economics 435

Spring 2004

1 A few issues in research design

a) You cannot. The reason is that while you have a panel data set, you are estimating the effect of one variable *at a particular point in time* on another variable at a particular (though different) point in time. So even though your data set has multiple periods, the portion you will be using does not.

b) You cannot. There is only one instrument here, and two endogenous explanatory variables. As a result the IV estimator does not exist.

2 Time series and stationarity

Note that this means that:

$$\begin{aligned} E(x_t) &= \mu_x \\ E(y_t) &= \mu_y \\ var(x_t) &= \sigma_x^2 \\ var(y_t) &= \sigma_y^2 \\ cov(x_t, x_s) &= \sigma_x(|t-s|) \\ cov(y_t, y_s) &= \sigma_y(|t-s|) \end{aligned}$$

a) Applying the usual rules, we get:

$$\begin{aligned} E(ax_t) &= a\mu_x \\ var(ax_t) &= a^2\sigma_x^2 \\ cov(ax_t, ax_s) &= a^2\sigma_x(|t-s|) \end{aligned}$$

This obviously satisfies the definition of covariance stationary.

b) Applying the usual rules, we get:

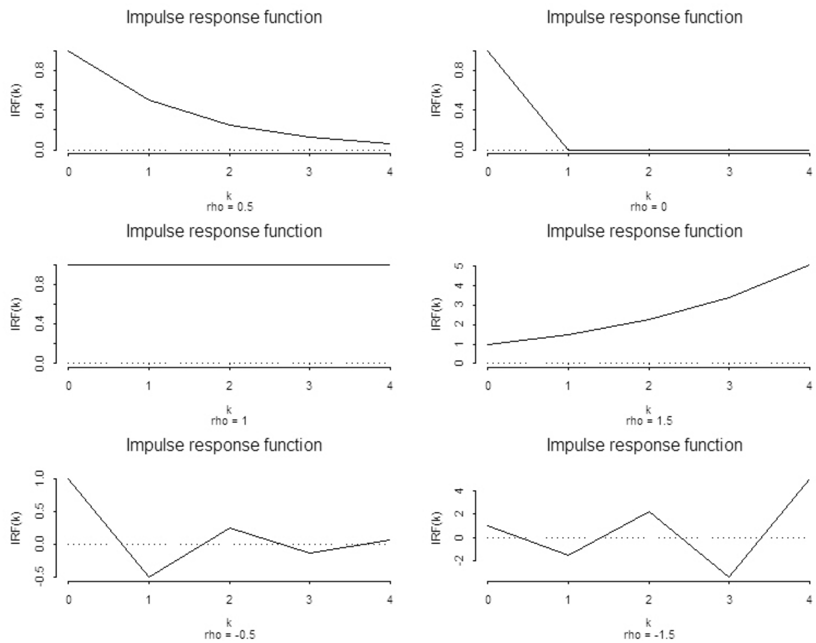
$$\begin{aligned} E(x_t + y_t) &= \mu_x + \mu_y \\ var(x_t + y_t) &= \sigma_x^2 + \sigma_y^2 \\ cov(x_t + y_t, x_s + y_s) &= \sigma_x(|t-s|) + \sigma_y(|t-s|) \end{aligned}$$

This obviously satisfies the definition of covariance stationary.

c) The impulse response function is:

$$irf(k) = \rho^k$$

d) The impulse response functions are in the figure below.



3 Fixed effects with measurement error

a)

$$\begin{aligned}
\text{plim } \hat{\beta}^{OLS} &= \text{plim } \frac{c\hat{o}v(y_{it}, \tilde{x}_{it})}{v\hat{a}r(\tilde{x}_{it})} \\
&= \frac{cov(y_{it}, \tilde{x}_{it})}{var(\tilde{x}_{it})} \\
&= \frac{cov(\alpha_i + \beta x_i + \beta v_{it} + u_{it}, x_i + v_{it} + \epsilon_{it})}{var(x_i + v_{it} + \epsilon_{it})} \\
&= \frac{cov(\alpha_i, x_i) + \beta var(x_i) + \beta var(v_{it})}{var(x_i) + var(v_{it}) + var(\epsilon_{it})} \\
&= \frac{\beta(\sigma_x^2 + \sigma_v^2) + \sigma_x \sigma_a \rho_{\alpha,x}}{(\sigma_x^2 + \sigma_v^2) + \sigma_\epsilon^2}
\end{aligned}$$

b)

$$\begin{aligned}
\text{plim } \hat{\beta}^{FE} &= \text{plim } \frac{c\hat{o}v(\Delta y_{it}, \Delta\tilde{x}_{it})}{v\hat{a}r(\Delta\tilde{x}_{it})} \\
&= \frac{cov(\Delta y_{it}, \Delta\tilde{x}_{it})}{var(\Delta\tilde{x}_{it})} \\
&= \frac{cov(\beta\Delta v_{it} + \Delta u_{it}, \Delta v_{it} + \Delta\epsilon_{it})}{var(\Delta v_{it} + \Delta\epsilon_{it})} \\
&= \frac{\beta var(\Delta v_{it})}{var(\Delta v_{it}) + var(\Delta\epsilon_{it})} \\
&= \beta\frac{\sigma_v^2}{\sigma_v^2 + \sigma_\epsilon^2}
\end{aligned}$$

c) If there is no fixed effect, then the probability limit of the OLS estimator is:

$$\text{plim } \hat{\beta}^{OLS} = \beta\frac{(\sigma_x^2 + \sigma_v^2)}{(\sigma_x^2 + \sigma_v^2) + \sigma_\epsilon^2}$$

Since $\frac{(\sigma_x^2+\sigma_v^2)}{(\sigma_x^2+\sigma_v^2)+\sigma_\epsilon^2}$ is closer to one than $\frac{\sigma_v^2}{\sigma_v^2+\sigma_\epsilon^2}$ is, the OLS estimator has smaller asymptotic bias.

d) If the fixed effect is uncorrelated with x_i, then the probability limit of the OLS estimator is:

$$\text{plim } \hat{\beta}^{OLS} = \beta\frac{(\sigma_x^2 + \sigma_v^2)}{(\sigma_x^2 + \sigma_v^2) + \sigma_\epsilon^2}$$

Since $\frac{(\sigma_x^2+\sigma_v^2)}{(\sigma_x^2+\sigma_v^2)+\sigma_\epsilon^2}$ is closer to one than $\frac{\sigma_v^2}{\sigma_v^2+\sigma_\epsilon^2}$ is, the OLS estimator has smaller asymptotic bias.

e) If there is no measurement error, then the probability limit of the OLS estimator is:

$$\text{plim } \hat{\beta}^{OLS} = \frac{\beta(\sigma_x^2 + \sigma_v^2) + \sigma_x \sigma_a \rho_{\alpha,x}}{(\sigma_x^2 + \sigma_v^2)}$$

and the probability limit of the FE estimator is

$$\text{plim } \hat{\beta}^{FE} = \beta$$

Since the FE estimator is consistent and the OLS estimator is not, the FE estimator obviously has smaller asymptotic bias.

f) The first statement is the correct one.