Exam #2 Answer Key

Economics 435: Quantitative Methods

Fall 2008

1 Dynamic panel data models

a) First we note that strict exogeneity implies that $cov(u_{i,t}, a_i) = cov(u_{i,t}, y_{i,t}) = cov(u_{i,t}, y_{i,t-1}) = 0$. This implies:

$$\begin{aligned}
0 &= cov(u_{i,t}, y_{i,t}) \qquad \text{by strict exogeneity} \\
&= cov(u_{i,t}, a_i + \beta y_{i,t-1} + u_{i,t}) \qquad \text{by equation (1)} \\
&= cov(u_{i,t}, a_i) + \beta cov(u_{i,t}, y_{i,t-1}) + var(u_{i,t}) \\
&= 0 + \beta 0 + var(u_{i,t}) \\
&> 0 \qquad \text{by equation (2)} \\
&\neq 0
\end{aligned}$$

Since we have just shown that strict exogeneity implies $0 \neq 0$, it cannot possibly hold.

b)

$$\begin{aligned}
\text{plim } \hat{\beta}_1^{IV} &= \text{plim } \frac{\hat{cov}(\Delta y_{i,t}, y_{i,t-2})}{\hat{cov}(\Delta y_{i,t-1}, y_{i,t-2})} \\
&= \frac{cov(\Delta y_{i,t}, y_{i,t-2})}{cov(\Delta y_{i,t-1}, y_{i,t-2})} \\
&= \frac{cov(\beta_1 \Delta y_{i,t-1} + \Delta u_{i,t}, y_{i,t-2})}{cov(\Delta y_{i,t-1}, y_{i,t-2})} \\
&= \beta_1 + \frac{cov(\Delta u_{i,t}, y_{i,t-2})}{cov(\Delta y_{i,t-1}, y_{i,t-2})} \\
&= \beta_1 + \frac{cov(u_{i,t}, y_{i,t-2}) - cov(u_{i,t-1}, y_{i,t-2})}{cov(\Delta y_{i,t-1}, y_{i,t-2})} \\
&= \beta_1 + \frac{0 - 0}{cov(\Delta y_{i,t-1}, y_{i,t-2})} \\
&= \beta_1
\end{aligned}$$

c)

$$\begin{aligned}
\text{plim } \hat{\beta}_1^{FD} &= \frac{cov(\Delta y_{i,t}, \Delta y_{i,t-1})}{var(\Delta y_{i,t-1})} \\
&= \frac{cov(\beta_1 \Delta y_{i,t-1} + \Delta u_{i,t}, \Delta y_{i,t-1})}{var(\Delta y_{i,t-1})}
\end{aligned}$$

$$
\begin{aligned}
&= \beta_1 + \frac{cov(\Delta u_{i,t}, \Delta y_{i,t-1})}{var(\Delta y_{i,t-1})} \\
&= \beta_1 + \frac{cov(u_{i,t}, y_{i,t-1}) - cov(u_{i,t}, y_{i,t-2}) - cov(u_{i,t-1}, y_{i,t-1}) + cov(u_{i,t-1}, y_{i,t-2})}{var(\Delta y_{i,t-1})} \\
&= \beta_1 + \frac{0 - 0 - cov(u_{i,t-1}, y_{i,t-1}) + 0}{var(\Delta y_{i,t-1})} \\
&\neq \beta_1
\end{aligned}
$$

2 Instrumental variables and local average treatment effects

a) All of them are identified.

b)

$$
\begin{aligned}
E(p|z=1) &= 1 * \Pr(complier) + 1 * \Pr(always - taker) + 0 * \Pr(never - taker) \\
&= \Pr(complier) + \Pr(always - taker) \\
E(p|z=0) &= 0 * \Pr(complier) + 1 * \Pr(always - taker) + 0 * \Pr(never - taker) \\
&= \Pr(always - taker)
\end{aligned}
$$

So:

$$
\Pr(complier) = E(p|z=1) - E(p|z=0)
$$

c) First:

$$
\begin{aligned}
E(y|z=1) &= E(y(0) + (y(1) - y(0))p(z)|z=1) \\
&= E(y(0)|z=1) + E((y(1) - y(0))p(1)|z=1) \\
&= E(y(0)) + E((y(1) - y(0))p(1)) \\
E(y|z=0) &= E(y(0) + (y(1) - y(0))p(z)|z=0) \\
&= E(y(0)|z=1) + E((y(1) - y(0))p(0)|z=0) \\
&= E(y(0)) + E((y(1) - y(0))p(0))
\end{aligned}
$$

Then take the difference:

$$
\begin{aligned}
E(y|z=1) - E(y|z=0) &= (E(y(0)) + E((y(1) - y(0))p(1))) - (E(y(0)) + E((y(1) - y(0))p(0))) \\
&= E\left((y(1) - y(0))(p(1) - p(0))\right) \\
&= E\left((y(1) - y(0))(p(1) - p(0))|complier\right) \Pr(complier) \\
&\quad + E\left((y(1) - y(0))(p(1) - p(0))|noncomplier\right) \Pr(noncomplier) \\
&= E\left((y(1) - y(0))(1)|complier\right) \Pr(complier) \\
&\quad + E\left((y(1) - y(0))(0)|noncomplier\right) \Pr(noncomplier) \\
&= E\left(TE|complier\right) \Pr(complier)
\end{aligned}
$$

Divide the right side by $\Pr(complier)$ and the left side by $E(p|z=1) - E(p|z=0)$ (since we just showed that these are the same) and you get the result.

d) Yes.

e) No.

f) We start with:

$$\begin{aligned} cov(y,z) &= E(yz) - E(y)E(z) \\ &= E(yz|z=1)\Pr(z=1) + E(yz|z=0)\Pr(z=0) - \\ &\quad (E(y|z=1)\Pr(z=1) + E(y|z=0)\Pr(z=0))(1\Pr(z=1) + 0\Pr(z=0)) \\ &= E(y|z=1)\Pr(z=1) - E(y|z=1)\Pr(z=1)^2 - E(y|z=0)(\Pr(z=1) - \Pr(z=1)^2) \\ &= (E(y|z=1) - E(y|z=0))\Pr(z=1)\Pr(z=0) \end{aligned}$$

By the same reasoning:

$$cov(p,z) = (E(p|z=1) - E(p|z=0))\Pr(z=1)\Pr(z=0)$$

Substituting in, we get:

$$\begin{aligned} \frac{cov(y,z)}{cov(p,z)} &= \frac{(E(y|z=1) - E(y|z=0))\Pr(z=1)\Pr(z=0)}{(E(p|z=1) - E(p|z=0))\Pr(z=1)\Pr(z=0)} \\ &= \frac{E(y|z=1) - E(y|z=0)}{E(p|z=1) - E(p|z=0)} \end{aligned}$$

g)

$$\begin{aligned} \text{plim } \hat{\beta}^{IV} &= \text{plim } \frac{\hat{cov}(y,z)}{\hat{cov}(p,z)} && \text{by definition} \\ &= \frac{\text{plim } \hat{cov}(y,z)}{\text{plim } \hat{cov}(p,z)} && \text{by the Slutsky theorem} \\ &= \frac{cov(y,z)}{cov(p,z)} && \text{by definition} \\ &= \frac{E(y|z=1) - E(y|z=0)}{E(p|z=1) - E(p|z=0)} && \text{proved in part (f)} \\ &= E(TE|complier) && \text{proved in part (c)} \end{aligned}$$