Exam #2 Answer Key

Economics 435: Quantitative Methods

Fall 2010

1 Short answer questions

a)

1. If x and y are independent, then $cov(x, y) = 0$. TRUE.
2. If x and y are independent, then $E(y|x) = E(y)$. TRUE.
3. If $cov(x, y) = 0$ then x and y are independent. FALSE.
4. If $cov(x, y) = 0$ then $E(y|x) = E(y)$. FALSE.
5. If $E(y|x) = E(y)$ then $cov(x, y) = 0$. TRUE.
6. If $E(y|x) = E(y)$, then x and y are independent. FALSE.

b)

1. NO, substantial correlation between x_1 and x_2 does not lead to OLS being inconsistent.
2. YES, nonzero correlation between x_1 and u leads to OLS being inconsistent.
3. NO, heteroskedasticity of u does not lead to OLS being inconsistent.
4. NO, homoskedasticity of u does not lead to OLS being inconsistent.
5. YES, classical measurement error in x_1 leads to OLS being inconsistent.
6. NO, classical measurement error in y does not lead to OLS being inconsistent.
7. NO, a sampling probability that depends on x_1 does not lead to OLS being inconsistent.
8. YES, a sampling probability that depends on y leads to OLS being inconsistent.

c)

$$\begin{aligned} cov(ab, c) &= E(abc) - E(ab)E(c) \\ &= E(a)E(bc) - E(a)E(b)E(c) \qquad \text{by independence} \\ &= E(a)\left[E(bc) - E(b)E(c)\right] \\ &= E(a)cov(b, c) \end{aligned}$$

2 Plausibly exogenous instruments

a)

$$\text{plim }(\hat{\beta}_1^{OLS} - \beta_1) = \frac{\rho_{x,u}\sigma_u}{\sigma_x}$$

b)

$$\text{plim }(\hat{\beta}_1^{IV} - \beta_1) = \frac{\rho_{z,u}\sigma_u}{\rho_{z,x}\sigma_x}$$

c)

$$\frac{\rho_{z,u}}{\rho_{z,x}} < \rho_{x,u}$$

d) "Our IV regression will be inconsistent if our instrument fails to be exogenous, but it will be less inconsistent than the OLS regression as long as the instrument's correlation with unobservables is less than <u>0.2</u> times the explanatory variable's correlation with unobservables"

3 Treatment effects in panel data

a)

$$\begin{aligned} y_{it}^p(1) &= a_i + \beta + u_{it} \\ y_{it}^p(0) &= a_i + u_{it} \\ TE_{it} &= \beta \\ ATE &= \beta \end{aligned}$$

b)

1. The treatment is unrelated to unobserved outcome-relevant factors. NO.
2. Changes in treatment status are unrelated to changes in unobserved outcome-relevant factors. YES.
3. The treatment effect is constant across individuals and time. YES.

c)

$$\begin{aligned} TE_{it} &= \beta + u_{it}(1) - u_{it}(0) \\ ATE &= \beta \end{aligned}$$

d)

$$
\begin{aligned}
\hat{\beta} &\to^{p} \frac{cov(\Delta x_{it}, \Delta y_{it})}{var(\Delta x_{it})} \\
&= \frac{cov(\Delta x_{it}, \beta \Delta x_{it} + \Delta u_{it})}{var(\Delta x_{it})} \\
&= \beta + \frac{cov(\Delta x_{it}, \Delta u_{it})}{var(\Delta x_{it})} \\
&= \beta + \frac{0}{var(\Delta x_{it})} \qquad \text{by the argument below} \\
&= \beta \\
cov(\Delta x_{it}, \Delta u_{it}) &= cov(\Delta x_{it}, u_{it} - u_{it-1}) \\
&= cov(\Delta x_{it}, u_{it}(0) - u_{it-1}(0) + (u_{it}(1) - u_{it}(0))x_{it} - (u_{it-1}(1) - u_{it-1}(0))x_{it-1}) \\
&= cov(\Delta x_{it}, u_{it}(0) - u_{it-1}(0)) + cov(\Delta x_{it}, (u_{it}(1) - u_{it}(0))x_{it}) \\
&\quad - cov(\Delta x_{it}, (u_{it-1}(1) - u_{it-1}(0))x_{it-1})) \\
&= 0 + E(u_{it}(1) - u_{it}(0)) cov(\Delta x_{it}, x_{it}) + E((u_{it-1}(1) - u_{it-1}(0)) cov(\Delta x_{it}, x_{it-1}) \\
&= 0 + 0\, cov(\Delta x_{it}, x_{it}) + 0\, cov(\Delta x_{it}, x_{it-1}) \\
&= 0
\end{aligned}
$$