Lecture 8 - Viscosity and drag

What's important:
- drag force depends on velocity
- terminal velocity under gravity

Demo:
- coin and feather, coffee filters, ball bearings in corn syrup

Velocity-dependent forces

Coin and feather demonstration shows two things:
- in a vacuum, the coin and feather drop at the same rate: the force on both objects from gravity is $F = ma$, giving them the same acceleration
- in air, the feather falls more slowly, reaching a constant speed; because the speed is constant, the force it experiences MUST DEPEND ON SPEED: if the force were constant (like mg) then the acceleration would be constant and the feather would continue to accelerate, although at $a < g$.

Thus, the drag forces in fluids are different than the friction force $f_{max} = \mu N$, which is independent of velocity.

The power law for drag force varies with the speed of the object with respect to the medium. Common examples:

Low speeds Stokes' Law (after Irish physicist George Stokes) for the force on a sphere of radius R moving with speed v through a medium with viscosity η:

$$F = 6\pi\eta Rv \qquad \text{(smooth flow, no turbulence)}$$

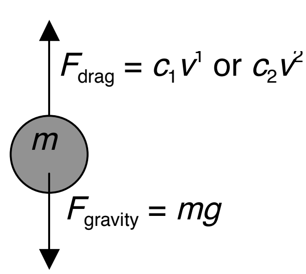
Higher speeds the viscous force may become quadratic in v:

$$F = (\rho/2)AC_D v^2 \qquad \text{(turbulence, does not depend on } \eta\text{)}$$

where ρ = density of medium, A is cross sectional area of object, C_D = drag coefficient:
- $C_D = 0.43$ for BMW roadsters
- $C_D = 0.37$ for Mazda Miata

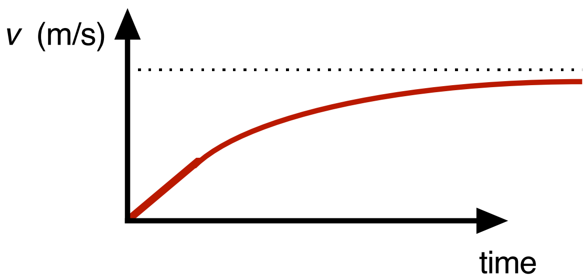
<u>Drag + gravity</u>

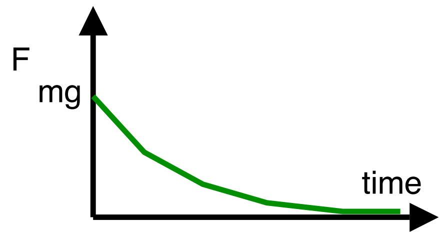
We look at a falling system subject to drag in order to demonstrate the power law behaviour of the drag force. The free-body diagram of an object subject only to gravity ($F = mg$) and drag ($F_{drag} = c_1v^1$ or c_2v^2, where the coefficients are expressed above) is



An object released from rest accelerates downwards because *mg* is greater than F_{drag}, which is initially small because of the small velocity. As the object accelerates, the velocity increases and so does F_{drag}. Ultimately, the drag force may reach *mg*, but it can never exceed *mg* or the object would accelerate upwards. Once the two forces are balanced, the object has reached its terminal velocity:

linear drag:	$c_1 v^1 = mg$	or	$v_{term} = mg / c_1$
quadratic drag:	$c_2 v^2 = mg$	or	$v_{term} = (mg / c_2)^{1/2}$



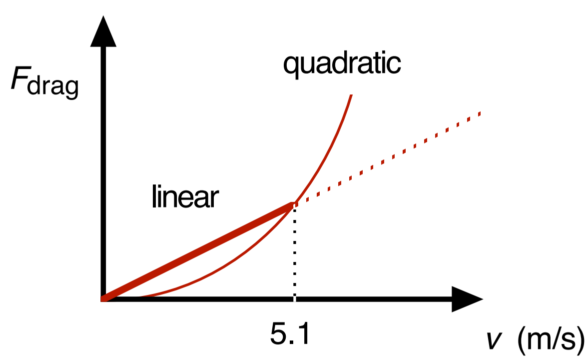


Demo: An example of the quadratic dependence on speed is the motion of falling coffee filters. If the mass of a nested set of coffee filters is increased by a factor of 4 (say 1 filter vs 4) the terminal speed increases by a factor of 2. Thus $v_{term} \sim m^{1/2}$ *(4 filters cover twice the distance in the same time).*

Demo: use linear drag to find viscosity of glycerine
Consider a steel ball-bearing of 0.5 cm radius in glycerine (this corresponds to a 3/8" diameter ball). Assume C_D = 0.5 and use ρ = 1260 kg/m^3. When are the linear and quadratic forces comparable for a ball bearing in glycerine?

$$c_1 v = c_2 v^2$$
$$v = c_1 / c_2 = 5.1 \text{ m/s.}$$

Thus, *for this example*



By observation, *v* is less than 1 m/s, so we assume we are dealing with linear drag (confirm this after the fact).

To find the mass of the ball bearing, use ρ = 7700 kg/m^3 and R = 0.0047 m, so that the mass is

$$m = \rho V = 7700\ (4\pi/3)\ (0.0047)^3$$
$$= 0.0033 \text{ kg}$$

Measurement:

v = 20 cm / 1.1 secs = 0.18 m/s.

Solve for c_1

$c_1 v = mg$ ---> $c_1 = mg / v$

c_1 = 0.0033 x 9.8 / 0.18 = 0.18 kg/s.

Solve for η

$c_1 = 6\pi\eta R$ ---> $\eta = c_1 / 6\pi R$ = 0.18 / (6π • 0.0047) = 2.0 kg/m•s.

Thus, $\eta \sim$ 2 kg / m•s for glycerine.

Fluid viscosities encompass a huge range, as shown in the table

Fluid	η (kg/m•sec at 20 °C)
Air	1.8×10^{-5}
Water	1.0×10^{-3}
Mercury	1.56×10^{-3}
Olive oil	0.084
Glycerine	1.34
Glucose	10^{13}

For cells, the effective viscosity depends on the size of the object – larger objects experience more obstacles and are subject to a higher average viscosity. Typical values would be 10^{-3} to 10^{-2} kg/m•s, which is 1 to 10 times water. The apparent viscosity of the cell as a whole is in the range 10^{+3} kg/m•s.