Demonstrations:
•sound source for Doppler shift
•big balloon and labels
Text: Mod. Phys. 8.A, 8.B, 8.C
Problems: 1, 3, 6, 7 from Ch. 8

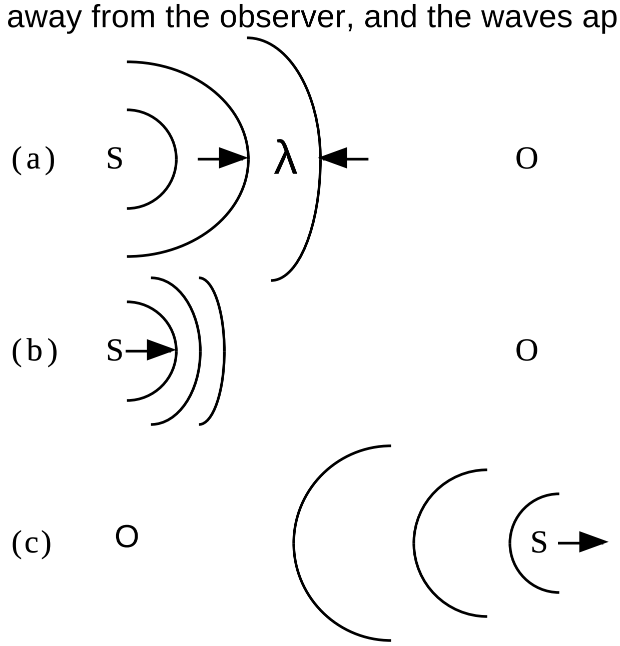
What's important:
•Doppler shift
•Hubble's law
•age of the Universe

Doppler effect

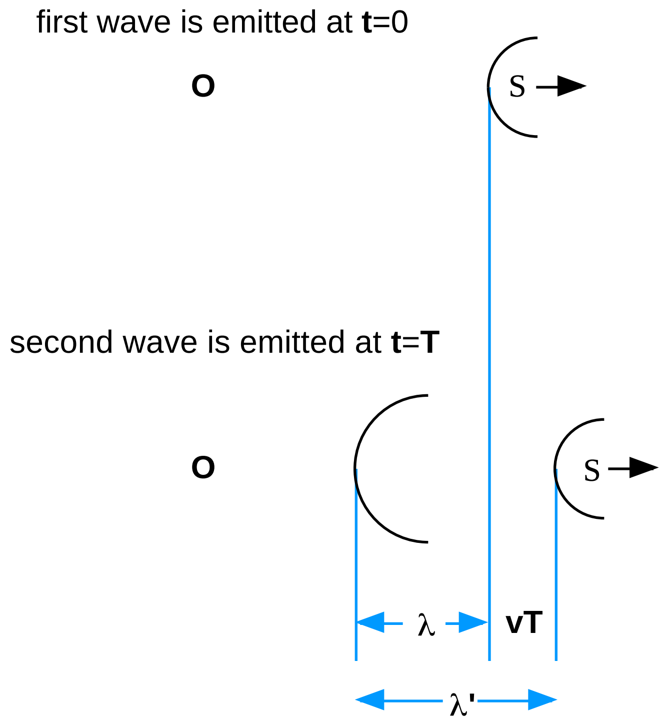
The technique for measuring the velocities of very remote galaxies is based upon the Doppler shift - the shift in the apparent wavelength of a wave due to the relative motion of the emitter of the wave with respect to the observer. Terrestrial example of the Doppler shift: the sound from the siren of an approaching ambulance appears to have a higher pitch (or frequency) than when the ambulance is receding.

Consider a source of waves **S** which is moving with respect to an observer **O**. In the diagram:

(a) shows the natural wavelength λ of the waves emitted by the source
(b) **S** moves towards **O** and the wavelengths appear shorter as seen by **O**
(c) the source moves away from the observer, and the waves appear longer.



We now look at (c) in more detail.



At time **t**=0, wave is emitted by **S**. At time **t**=**T** (the period), another wave is emitted, but **S** has travelled a distance **vT** in time **T**, away from the observer. Hence, the distance between wave crests as seen by observer **O** is

$$\lambda' = \lambda + vT$$
$$(\lambda' - \lambda) / \lambda = vT/\lambda$$

But, the speed **c** of a wave is given by $c = \lambda / T$, so

$$(\lambda' - \lambda) / \lambda = v / c. \qquad (1)$$

The shift in the wavelength from λ at the source to λ' at the observer is referred to as the Doppler shift of the wave. For sources moving towards the observer, the analysis is the same but the sign of **v/c** is reversed and $\lambda' < \lambda$.

Hubble's law

The emission (and absorption) of light under some circumstances involves waves of a very precise frequency that is characteristic of the element or compound from which the light is emitted. In astronomical applications, light emitted at a specific frequency from an element is measured both on Earth and from the distant star or galaxy, and the light from distant stars is observed to be **red-shifted** (shifted to longer wavelengths) than are found in experiments performed on Earth. If the redshift is due to the motion of the stars, then the stars must be moving away from the Earth.

Example

A nebula in the constellation Hydra has a relative redshift of 0.20. Use the Doppler shift expression to find its velocity with respect to Earth.

Solution:

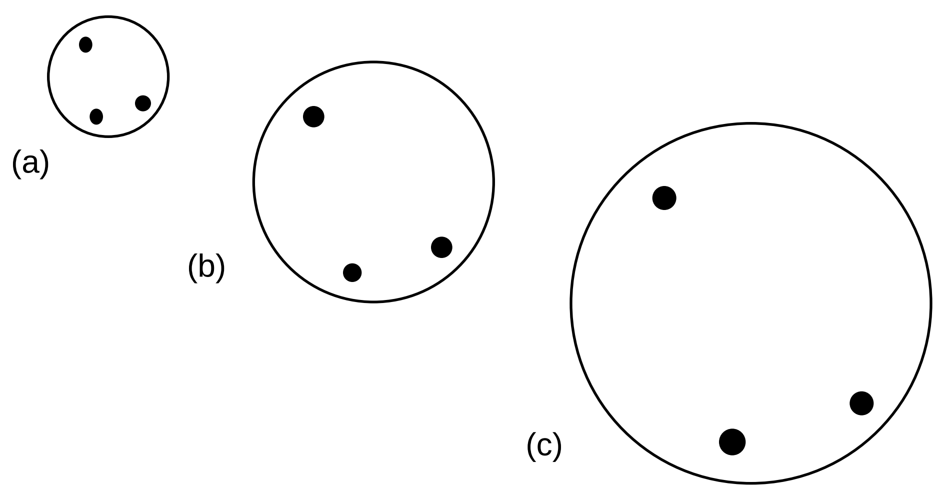
By the words "relative red-shift", we mean $(\lambda' - \lambda)/\lambda = 0.20$.
Hence

$$(\lambda' - \lambda)/\lambda = 0.20 \qquad \mathbf{v} = 0.20\,\mathbf{c} = 6.0 \times 10^4 \text{ km/s.}$$

In 1929, Edwin Hubble found that the velocity of recession **V** of a distant object is related to its distance from the Earth, **R** by

$$\mathbf{V} = \mathbf{HR} \qquad \text{(Hubble's law)} \qquad (2)$$

where **H** is a number called the Hubble parameter. The currently "accepted" value of the Hubble parameter is in the range 40-100 km s^{-1} Mpc^{-1}, where Mpc is shorthand for megaparsec and is equal to 3.26 million light years or 3.09×10^{19} km (1 light-year = 1 ly = 9.46×10^{12} km). Note that the units of **H** are actually [*time*]$^{-1}$.



Hubble's law can be illustrated by the relative motion of positions on the surface of a balloon while the balloon is being inflated [(a) to (c) in the figure]. For all distances between points on the balloon's surface, the distances $\mathbf{R}_i$ in one diagram are related to the distances $\mathbf{R}_i'$ in another diagram by a common scale factor $\mathbf{s}$:

$$\mathbf{R}_i' = \mathbf{s}\mathbf{R}_i. \qquad (3)$$

The change in surface distances between any two diagrams [(a) to (c)] is

$$\Delta\mathbf{R}_i = \mathbf{R}_i' - \mathbf{R}_i = (\mathbf{s} - 1)\mathbf{R}_i$$

so that the velocity of separation between a pair of points is just

$$\mathbf{V}_i = \Delta\mathbf{R}_i / \Delta\mathbf{t} = [(\mathbf{s} - 1)\mathbf{R}_i] / \Delta\mathbf{t}.$$

Finally, since $\mathbf{R}_i$ is not a function of time, then

$$\mathbf{V}_i = \mathbf{R}_i\,[(\mathbf{s} - 1)/\Delta\mathbf{t}] = \mathbf{R}_i \times [\textit{common multiplicative factor}]. \qquad (4)$$

In other words, Hubble's law applies to the motion of particles on the surface of a balloon, or to any system that is expanding according to a scale transformation like Eq. (3), and the Hubble parameter is the common multiplicative factor in Eq. (4). The balloon example shows that Hubble's law applies to all points in the system. There is no "special" position on the balloon's surface.

The Age of the Universe

Hubble's law tells us that the universe as a whole is expanding: at earlier times, the distances between galaxies were much smaller than they are now, and at future times they will be much greater. How long ago were they all in very close proximity?

Suppose that the galaxies started off very close together and each galaxy has always had the same speed, which varies from one galaxy to the next. Then a galaxy covers a distance $\mathbf{R}$ in time $\mathbf{t}$ given by

$$\mathbf{R} = \mathbf{V}\mathbf{t}.$$

What is the velocity of a galaxy? We measure it today to be $\mathbf{V} = \mathbf{H}\mathbf{R}$, which we assume to be constant. Thus, the time it took for the galaxies to separate is

$$\mathbf{t} = \mathbf{R} / \mathbf{V} = \mathbf{R} / (\mathbf{HR}) = \mathbf{H}^{-1}$$

Taking the midvalue of **H** to be 70 km s^{-1} Mpc^{-1} and using 1 Mpc = 3.09 x 10^{19} km,

$$\mathbf{H}^{-1} = 3.09 \times 10^{19}\ [km/Mpc] / 70\ [km\ s^{-1}/Mpc]$$
$$= 4.4 \times 10^{17}\ \text{s} = 14 \text{ billion years.}$$

Hence, if the galaxies have always receded from one another with the same velocities that they have now, then 14 billion years ago the galaxies were very close together. Note that **V** is assumed to be constant in this calculation, not **H**).

Now, correcting for the change in **H** over time, we find that the age of the Universe is approximately (2/3)$\mathbf{H}^{-1}$, rather than just $\mathbf{H}^{-1}$. So, *based on the observed value of the Hubble parameter, we predict that the universe is about 7-14 billion years old*. Can we check this? Not directly, but

(i) The time it has taken light to reach us from the furthest visible galaxies is between 10 and 18 billion years.

(ii) From the age of meteoric material, our solar system is about 4.5 billion years old (not in contradiction with $\mathbf{H}^{-1}$).