*Demonstrations:* baseball bat and blocks; shoot the can
*Text*: Fishbane 3-4, 3-5
*Problems* 10, 40, 48, 49, 59 from Ch. 3

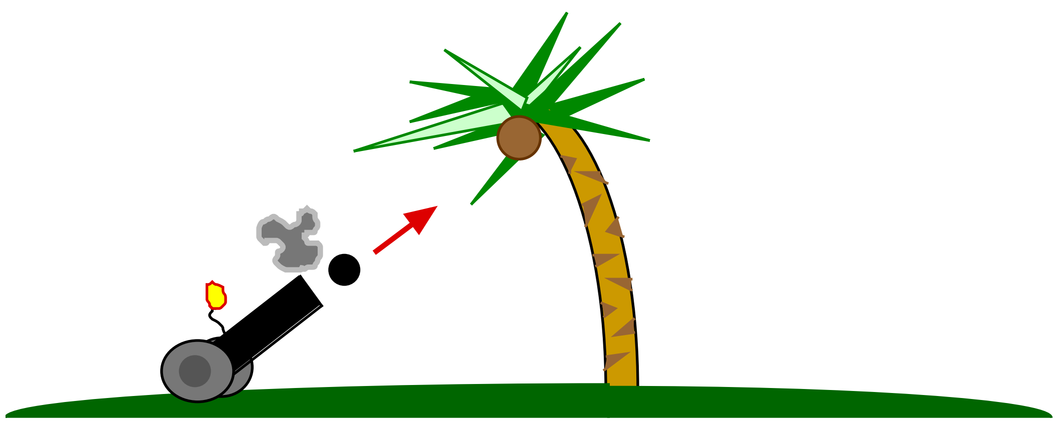
*What's important:*
•projectile motion, range
•circular motion

In Cartesian coordinates, the vector equations of motion separated naturally into three sets of equations - one set (**x**, **v**, **a**) for each direction. Thus, the acceleration due to gravity affects motion in the vertical direction, but not in the horizontal direction.

*Demo*: basball bat and blocks, showing that time taken for block to hit the ground does not depend on the horizontal velocity of the block.

**Example**

The Coconut Shoot problem (*a.k.a.* the Monkey Shoot problem). Suppose that you are trying to knock a coconut from a tree by firing a cannon at it. If you line up the cannon so that it is aimed directly at the coconut, then you know that the cannonball will, in principle, miss the coconut because of the acceleration of the cannonball in the vertical direction due to gravity. What happens if the coconut falls as soon as the cannonball is fired?



After the ball leaves the cannon, it experiences an acceleration in the vertical (y) direction due to gravity, which changes its y-velocity with time as:

$$\text{ball:} \quad \mathbf{v}_y = \mathbf{v}_{oy} - g\mathbf{t}$$

But the coconut experiences the same acceleration, so its velocity in the vertical direction is

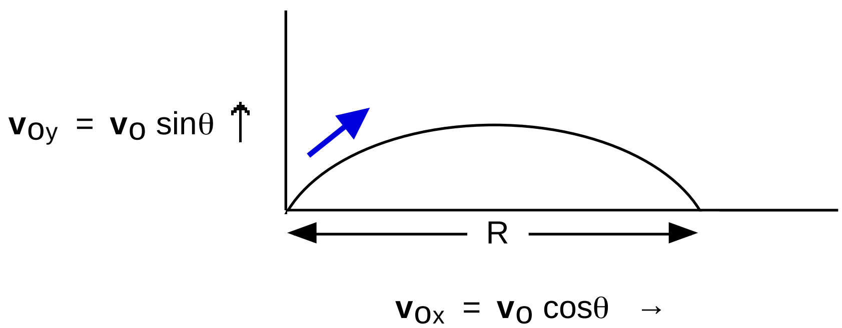
coconut: $\mathbf{v}_y = -\ \mathbf{gt}$

Hence, the deviation of the ball from its original trajectory in the y-direction is $-1/2\ \mathbf{gt}^2$, the same as the coconut.

Both the shoot-the-coconut demo and the bat-and-blocks demo show that the vertical motion of an object, as effected by gravity, is independent of its horizontal motion.

**Example**

A cannonball is fired with an angle $\theta$ with respect to the horizontal. Find the range of the cannonball as a function of $\theta$.




We need to solve for the time of flight **t**. When the cannonball hits the ground at time **t**, the y-coordinate is 0. Hence, we can solve for **t** from the equation of motion in the y-direction

$$\mathbf{y} = \mathbf{v}_o \sin\theta\ \mathbf{t} - 1/2\ g\mathbf{t}^2 = 0$$

$$\mathbf{v}_o \sin\theta = 1/2\ g\mathbf{t}$$

$$\text{or}\ \ \mathbf{t} = 2\mathbf{v}_o \sin\theta / g$$

This gives us the flight time. To get the range, which does not involve an acceleration in the x-direction, we use

$$\mathbf{R} = \mathbf{v_o} \cos\theta \, \mathbf{t} = \mathbf{v_o} \cos\theta \bullet 2\mathbf{v_o} \sin\theta / g$$

or $$\mathbf{R} = \frac{\mathbf{v_o}^2}{g} \underbrace{(2 \sin\theta \cos\theta)}_{\sin 2\theta \text{ by trigonometry}}$$

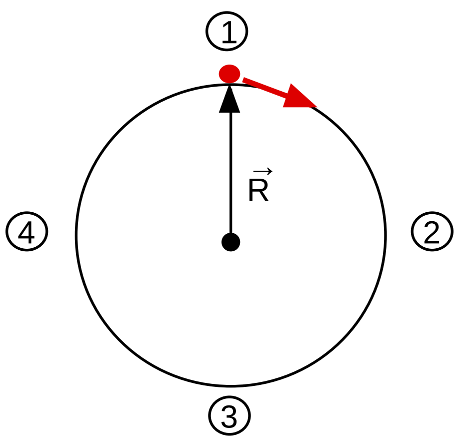
Finally,

$$\mathbf{R} = \frac{\mathbf{v_o}^2}{g} \sin 2\theta$$

Note that the maximum in **R** is at $\theta = 45°$, $\mathbf{R} = \mathbf{v_o}^2 / g$

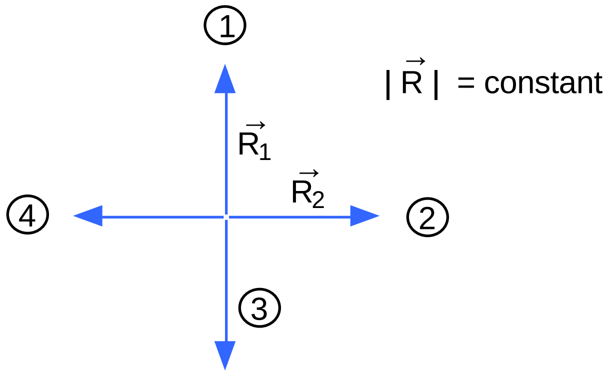
**Circular Motion**

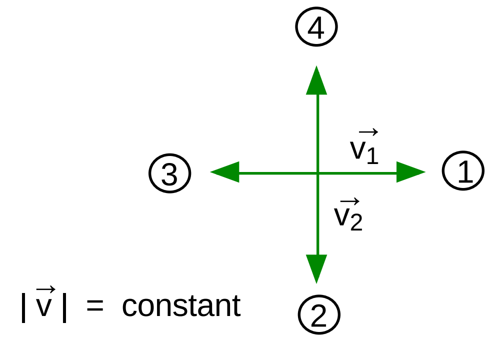
As a simple, but sometimes confusing, application of vectors, we consider circular motion. In the diagram, an object moves in a circular path with radius **R**, in a clockwise direction (as indicate by the red arrow):



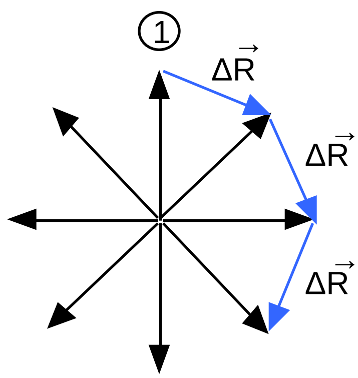

$|\vec{R}|$ = radius (fixed)

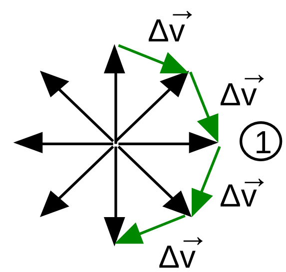
Let's look at the displacement and velocity vectors at four different positions, labelled 1 ... 4 in the diagram.




We can fill in the various intermediate positions to see how the **change in position** and **change in velocity** themselves change in time.




During the period **T** for one complete revolution,

Total distance covered $= \sum |\Delta \mathbf{R}|$
$= 2\pi \mathbf{R}$

speed $= |\vec{v}|$ = distance / time
$= 2\pi R / \mathbf{T}$

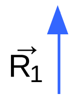
Total change in velocity $= \sum |\Delta \mathbf{v}|$
$= 2\pi v$

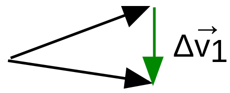
acceleration $= 2\pi \mathbf{v} / \mathbf{T}$

$$= 2\pi \mathbf{v} \left( \frac{\mathbf{v}}{2\pi \mathbf{R}} \right) = \frac{\mathbf{v}^2}{\mathbf{R}}$$

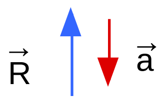
At first sight, this may be a surprising result: the speed is constant, but there is an acceleration $\mathbf{a} = \mathbf{v}^2 / \mathbf{R}$. Of course, even though the speed is not changing, the velocity does change because it is changing direction.

Finally, note that $\Delta\vec{\mathbf{v}}$ is in the opposite direction to $\vec{\mathbf{R}}$.




Further, since $\vec{\mathbf{a}}$ is parallel to $\Delta\vec{\mathbf{v}}$, then $\vec{\mathbf{a}}$ must be in the opposite direction to $\vec{\mathbf{R}}$ as well:




We call $\vec{\mathbf{a}}$ the centripetal (or centre - seeking) acceleration.