PHYSICS 211 FINAL EXAMINATION

Saturday, 5 August, 2000 Time: 3 hours

Calculators and one formula sheet permitted.

Show complete solutions to all problems.

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1. The potential of a quark of mass m in an elementary particle is thought to grow linearly with separation x as in

$$V(x) = k\,|x|,$$

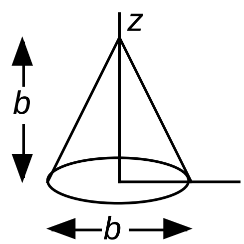
where k has units of energy per unit length. The motion is periodic, but not sinusoidal. What is the period T in terms of m, k and the amplitude of the motion A? (Prove, then use, the integral $\int_0^1 (1-z)^{-1/2}\,dz = 2$ in an energy representation). (10 marks)

2. Stokes' Law for the drag force F experienced by a sphere of radius R moving at a velocity v in a fluid of viscosity η is:

$$F = 6\pi\eta R v.$$

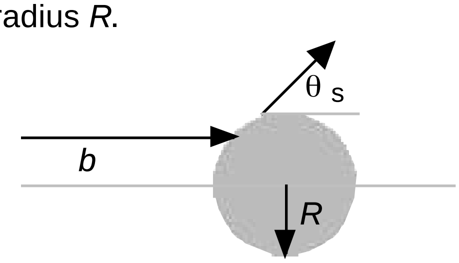
(i) Find the drag force on a white blood cell with R = 10 μm if v = 1 μm/s and $\eta = 10^{-3}$ kg/ m•s.
(ii) How far will the cell travel if it is subject to only this force (starting from v = 1 μm/s). Take the density of the cell to be 10^3 kg/m^3. (15 marks)

3. Find the centre-of-mass position on the z-axis for a solid cone of base and height b.



(17 marks)

4. Find the impact parameter b in terms of the scattering angle θ_s for the scattering of a point particle from a hard sphere of radius R.

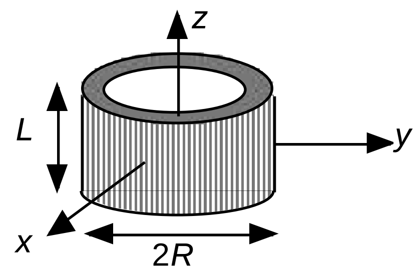


Assume that the kinetic energy of the particle is unchanged by the collision. (10 marks)

5. The mass m of a rocket decreases with time as it burns fuel. Suppose the gas leaves the rocket at a constant speed V_{ex}, and the fuel is used up at a constant rate $\mu = -dm/dt$.
(i) Starting from time t = 0, find the distance D travelled by the rocket before it stops burning fuel. Take the initial mass and velocity to be m_o and 0, respectively; assume that the entire rocket mass is fuel.

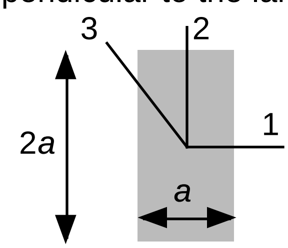
(ii) If the fuel is burned in time T, what is D in terms of V_{ex} and T?
You do not need to rederive the rocket equation. You may use $\int_0^1 \ln(1/x)\, dx = 1$. (10 marks)

6. Find the elements of the inertia tensor for a thin hollow cylinder of radius R, length L and mass M. Take the coordinate system to be at the center of the cylinder as in the diagram.



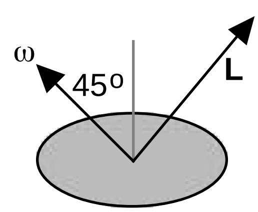
Organize your answer according to (i) I_{zz}, (ii) I_{xx} and I_{yy}, (iii) I_{ij} for $i \neq j$. (16 marks)

7. A rectangular lamina of mass M has dimensions a by $2a$ and symmetry axes 1,2,3, as in the diagram (where the 3-axis is perpendicular to the lamina):



(i) Calculate the moments of inertia $I_1 \ldots I_3$ and rank order them from largest to smallest (you may use the thin rod formula without proof).
(ii) If you spin the lamina about a symmetry axis, the resulting ω_i (i = 1,2,3) may rotate with respect to a space-fixed frame. Treating each axis in turn, explain in words and diagrams whether its ω_i will rotate. (12 marks)

8. A thin disk rotates freely under zero torque. Its angular momentum **L** is fixed in a non-rotating frame, but its ω rotates at an angle of 45^o with respect to the symmetry



axis of the disk. If the period corresponding to ω is 2 seconds:
(i) With what period does ω rotate around the symmetry axis of the disk?
(ii) With what period does ω rotate about the angular momentum vector?
(10 marks; you may use results from class without proof).

Answers:

1. $T = 4(2mA/k)^{1/2}$
2. (i) 0.19 pN; (ii) 0.22Å
3. $z_{cm} = b/4$
4. $b = R\cos(\theta_s/2)$
5. (i) $D = V_{ex}m_o/\mu$; (ii) $D = V_{ex}T$
6. $I_{zz} = MR^2$ $I_{xx} = I_{yy} = MR^2/2 + ML^2/12$ $I_{ij} = 0$ for $i \neq j$
7. (i) $I_1 = Ma^2/3$ $I_2 = Ma^2/12$ $I_3 = (5/12)Ma^2$

(ii) ω precesses about axis 1

8. (i) 2.8 s; (ii) 1.26 s.