PHYS 211 FINAL EXAMINATION

Friday, 10 August, 2001 Time: 3 hours

Calculator and one formula sheet permitted.

Please show complete solutions to problems and explain your reasoning (this requires words, not just equations). State any principles that you use in your solutions.

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I_{disk} (symmetry axis) = $MR^2/2$ I_{ring} (symmetry axis) = MR^2

I_{rod} (through centre) = $ML^2/12$ Binomial theorem $(1 + x)^n = 1 + nx + n(n\text{-}1)x^2/2! +$

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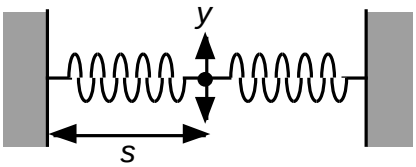
1. A ring of mass M and radius R initially rolls and slides along a horizontal table after receiving an impulse giving it an initial speed v_o. The ring has a non-zero coefficient of kinetic friction μ_k with the table. How long does it take before the ring stops slipping? (13 marks)

2. A mass is attached to two identical Hookean springs (*i.e.*, $V(x) = kx^2/2$) of rest length s and force constant k, as shown:



The other ends of the springs are held fixed, and the mass is permitted to move transversely, in the y-direction. Find the leading-order contribution (in y) to the potential energy $V(y)$ at $x = 0$. Neglect gravity in this problem. (10 marks)

3. The exact expression for the gravitational potential energy between objects of mass M and m separated by a distance r is $V(r) = -GMm/r$. Show that the approximate potential energy difference ΔV at a height h above the surface of the Earth compared to $r = R_e$ is $\Delta V = mgh\,(1 - h/R_e)$, where R_e is the radius of the Earth and g is the local acceleration due to gravity. (11 marks)

4. What is the period, in hours, of a Foucault pendulum at 30° north latitude? (3 marks)

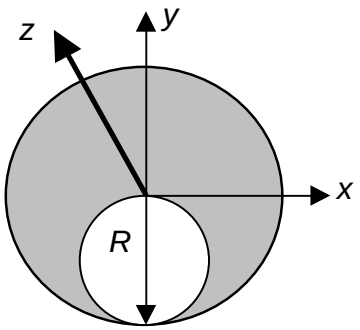
5. A particle of mass m subject to a central force $f(r)$ from the coordinate origin executes a helical orbit $r = c\theta$ in a plane, where r and θ are the usual polar coordinates.
(a) Determine the time-dependence of the polar angle $\theta(t)$. (*Hint: angular momentum L is conserved for a central force*.)
(b) The differential equation for the orbit involves the radial force as

$$-(m / L^2u^2)f(r) = d^2u / d\theta^2 + u.$$

What is $f(r)$ of this orbit? (8+6=14 marks)

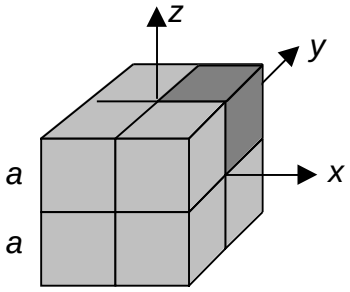
6. Initially at rest, an object explodes into three fragments. One fragment has mass 4*M*, while the other two have mass *M*. The angle between the two lighter fragments as they separate is ϕ, and they break symmetrically with respect to the heavy fragment. Take the heavy fragment to move along the -*x* direction.
(a) Show that the speeds of the light fragments are the same.
(b) Define *R* as the ratio of the kinetic energy of one light fragment to that of the heavy fragment. Determine *R* as a function of ϕ. (2+6=8 marks)

7. A hole is drilled into a thin disk of mass *M* and radius *R*. The hole has radius *R* /2 and is located as shown on the diagram, with the disk lying in the *xy*-plane and the *z* axis towards the viewer.



(a) Find I_{xx}, I_{yy} and I_{zz}, where the origin is at the center of the disk.
(b) Considering each principal axis by itself, does the vector ω appear to precess according to an observer comoving with the axis? (15+3=18 marks)

8. Eight blocks of length *a* to the side are glued together to form a cube as shown.



Each cube has mass *M*, except the shaded cube, which has mass 3*M*. What is the coordinate of the cm position along the *x*-axis? (5 marks)

9. A thin uniform rod of length *L* and mass *M* is constrained to rotate with a constant angular velocity vector ω about an axis passing through the center *O* of the rod and making an angle *ß* with the rod.
(a) Show that the angular momentum **L** about *O* is perpendicular to the rod and has a magnitude $L = (ML^2/12)\,\omega \sin ß$.
(b) Find the magnitude of the torque required to keep the rod oriented and show that it is perpendicular to the rod. (10+8=18 marks)

Answers, where appropriate:

1. $t = v_o / 2\mu g$
2. $V = +ky^4/4s^2$...
3. given
4. 48 hours
5. (a) $\theta(t) = (3L/mc^2)^{1/3}t^{1/3}$ (b) $f(r) = = -(L^2/m)[2c^2/r^5 + 1/r^3]$
6. (a) given (b) $R = 1 / \cos^2(\phi/2)$
7. (a) $I_{zz} = (26/64)MR^2$, $I_{yy} = (15/64)MR^2$, $I_{xx} = (11/64)MR^2$
   (b) precession only occurs about the *y*-axis
8. $x_{cm} = y_{cm} = z_{cm} = a/10$
9. (a) $L = (ML^2/12)\,\omega \sin ß$ (b) $\tau = [ML^2/12]\,\omega^2 \sin ß \cos ß$.