

PHYSICS 211 FINAL EXAMINATION

16 August, 2002
Time: 3 hours

Name _____
Signature _____
Student number _____

Calculator and one formula sheet permitted. Circle your answers to questions 1 to 3 on these sheets. Provide detailed solutions to questions 4 to 7 in the examination booklet.

Some useful data:

$$\int_0^{\pi} \sin^3\theta \, d\theta = 4/3$$

1. For each of the following questions, circle the correct answer. (17 marks)

(i) A simple pendulum swings through a vertical plane at the south pole. Which of the following statements is correct?

- (a) The plane rotates clockwise because of the centrifugal force.
- (b) The plane rotates clockwise because of the Coriolis force.
- (c) The plane rotates counter-clockwise because of the Coriolis force.
- (d) The plane rotates counter-clockwise because of the centrifugal force.
- (e) None of [a]-[d] are correct.

(ii) An object is attracted to a fixed location by a central ($1/r^2$) force, and executes an elliptical orbit as a result. If the ratio of the object's maximum speed to its minimum speed is 2, what is the eccentricity of the orbit?

- (a) 1/3 (b) 1/2 (c) 2 (d) 3 (e) none of [a-d]

(iii) An object starts at $t = 0$ with an initial speed v_0 and is subject only to a drag force which is linear in speed. Which statement correctly describes the motion of the object after the passage of a long time, if its motion is strictly horizontal?

- (a) speed constant 0 position constant
- (b) speed infinity position increases without bound
- (c) speed constant 0 position increases without bound
- (d) speed 0 position increases linearly with time
- (e) speed 0 position constant

(iv) An object of mass m is subject to a restoring force $-kx$ and linear damping $-cv$. In terms of c and m , what condition must k satisfy so that the object completes at least one oscillation after release from rest?

- (a) $k = 0$ (b) $k > c^2/4m$ (c) $k = c^2/4m$ (d) $k < c^2/4m$ (e) none of [a-d]

(v) A planet is subject to a $1/r^2$ force from gravity and executes a circular orbit about a star. How does the speed of the planet vary with the radius r ? The speed is proportional to:

- (a) r^1 (b) $r^{1/2}$ (c) $1/r$ (d) $1/r^{1/2}$ (e) $1/r^2$.

2. For each of the following questions, circle the correct answer. (17 marks)

(i) A semi-circular wire ring, a semi-circular laminar disk, and a solid hemisphere all have the same radius R and have uniform density. They each rest on a table which defines $z = 0$. What is the relative magnitude of their centre-of-mass coordinate z_{cm} ?



- (a) ring > disk > hemisphere
 (b) ring < disk < hemisphere
 (c) ring = disk = hemisphere
 (d) disk > ring > hemisphere
 (e) none of [a-d]

(ii) Three cylinders roll without slipping down a smooth incline plane. They each have the same mass but a different mass distribution - one is uniform, one has a hollow core, and one has an extra-dense core. Neglecting air resistance, which arrives at the bottom of the plane first?

- (a) all arrive at the same time
 (b) largest radius arrives first
 (c) uniform density
 (d) hollow core
 (e) dense core

(iii) Which of the following power law forces has a stable circular orbit?

- (a) $1/r^{7/2}$ (b) $1/r^4$ (c) $1/r^{1/2}$ (d) $1/r^6$ (e) none are stable

(iv) In the absence of an external force or torque, what quantity is not conserved in an explosion?

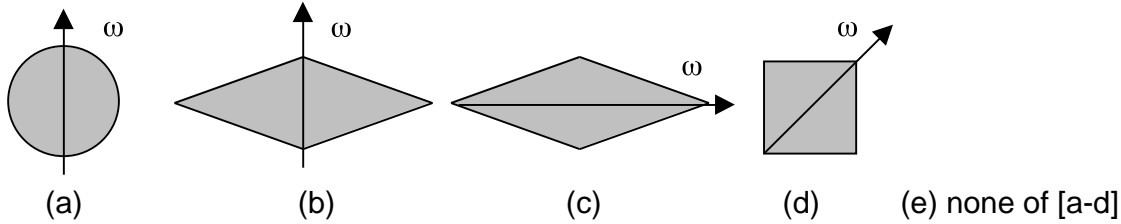
- (a) total momentum (b) total angular momentum (c) total kinetic energy
 (d) kinetic energy of centre-of-mass (e) none of [a-d]

(v) A rocket has an engine which ejects exhaust gas with a constant speed V_{ex} relative to its body. With its engines off, the rocket is stationary with respect to a nearby planet. If the rocket commander wishes to approach the planet with a speed V_{ex} , what will be the rocket's mass after firing its engine to change speed? The initial mass is m_0 .

- (a) m_0 (b) m_0/e (c) em_0 (d) $m_0/(e-1)$ (e) none of [a-d]

3. For each of the following questions, circle the correct answer. (20 marks)

(i) For which of the following lamina does the angular velocity ω precess about the angular momentum \mathbf{L} ? All of the lamina, as well as ω , lie in the plane of the paper.



(ii) The inertia tensor I_{ij} is symmetric under exchange of i and j . Which of the following statements follows from this property?

- (a) \mathbf{L} must be parallel to ω .
- (b) I_{ij} is a 3x3 matrix in three dimensions.
- (c) \mathbf{L} must be equal to ω .
- (d) I_{ij} can be diagonalized.
- (e) \mathbf{L} is constant in the absence of an external torque.

(iii) A lamina disk rotates with ω not parallel to \mathbf{L} . The symmetry axis of the disk (normal to its plane) makes an angle of 45° with respect to ω . What is the angle between the symmetry axis and \mathbf{L} ?

- (a) 63.4°
- (b) 45°
- (c) 26.6°
- (d) 22.5°
- (e) 0°

(iv) What is the ratio of the maximum instantaneous power transmitted by a wave on a string under tension compared to its average power? The ratio P_{\max}/P_{av} is:

- (a) 2
- (b) 3
- (c) 1
- (d) 1/3
- (e) 4

(v) At one instant, the configuration of a wave on a string under tension τ is described by

$$u(x) = A \sin(\pi x/L)$$

where L is the length of the string. Considering only its magnitude, what is the largest vertical component of the tension experienced by an element of the string?

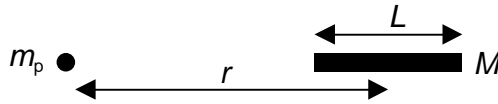
- (a) $\tau A/L$
- (b) 2τ
- (c) τ
- (d) $\tau A/L$
- (e) $2\tau A/L$

4. The interaction between molecules separated by a distance r is frequently described by the Lennard-Jones potential

$$V(r) = 4V_0 \left[\left(\frac{a}{r} \right)^{12} - \left(\frac{a}{r} \right)^6 \right]$$

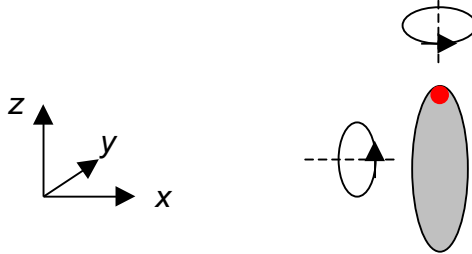
where V_0 and a are parameters. The potential is repulsive at short distances and attractive at large distances. At what value of r is $V(r)$ a minimum? (12 marks)

5. A point mass m_p interacts via gravity with a uniform rod of mass M and length L . The rod lies a distance r from the point mass, and has the orientation:



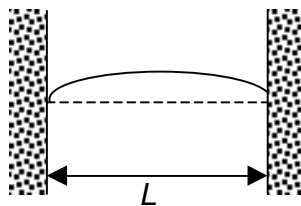
Determine the force due to gravity between these masses. (14 marks)

6. Relative to a fixed set of Cartesian coordinates xyz , a thin disk initially lies in the xz plane. The disk then rotates counterclockwise about the z axis at one revolution per second, keeping its axis of symmetry in the xy plane. At the same time, a red dot painted on the rim of the disk completes one revolution of its own every second, in a counterclockwise direction as seen facing the dot.



- What are the rates of change of the Euler angles ϕ , θ and ψ ?
- Ignoring the motion of the red dot, what are the components of the angular velocity ω' of the body as it rotates about z ? Quote your components in the frame $x'y'z'$ which follows the shape of the body as it moves, but not the motion of the red dot. (12 marks)

7. A string of length L is placed under tension by attaching each end to fixed points on opposite walls. The string is drawn a small distance to the side, and released from rest, as in



The initial displacement $u_0(x)$ is given by

$$u_0(x) = h \sin^2(x/L).$$

The initial displacement $u_0(x)$ of the string can be expressed by the Fourier series

$$u_0(x) = \sum_{n=1}^{\infty} A_n \sin(\omega_n x/c) \cos(\omega_n t), \quad \text{where } \omega_n = nc/L.$$

Determine the leading coefficient A_1 . (8 marks)

Answers

1. c, a, e, b, d

2. a, e, c, c, b

3. b, d, c, a, d

4. $r = 2^{1/6} a$

5. $F = Gm_p M / (r^2 - L^2/4)$

6. (a) $d\phi/dt = 2 \text{ s}^{-1}$; $d\theta/dt = 0$; $d\psi/dt = 2 \text{ s}^{-1}$

(b) $\omega'_x = 0$; $\omega'_y = 2 \text{ s}^{-1}$; $\omega'_z = 0$.

7. $A_1 = 8h/3$.