PHYS 211 FINAL EXAMINATION

Tuesday, 5 August, 2003 Name_____
Time: 3 hours Student # _____

Calculator and one formula sheet permitted.

Please show complete solutions to problems 4-8 and explain your reasoning (this requires words, not just equations). State any principles that you use in your solutions.

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I_{disk} (symmetry axis) = $MR^2/2$ I_{ring} (symmetry axis) = MR^2

I_{rod} (through centre) = $ML^2/12$ Binomial theorem $(1 + x)^n = 1 + nx + n(n\text{-}1)x^2/2! +$

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1. For each of the following questions, please circle one selection for your answer. (10 marks)

(i) An object executes a helical path with positive ω, its radial distance from the coordinate origin increasing with time. The change in a unit vector $\mathbf{e}_r$ pointing from the origin to the object is
(a) opposite to $\mathbf{e}_r$ (b) orthogonal to $\mathbf{e}_r$ in a clockwise direction (c) zero
(d) along $\mathbf{e}_r$ (e) orthogonal to $\mathbf{e}_r$ in a counter-clockwise direction

(ii) When subject to a non-dissipative force which depends only on position, the work done on the object
(a) depends only on the initial and final location of the object
(b) always equals the object's kinetic energy
(c) always equals the object's potential energy
(d) equals the force
(e) is never conserved

(iii) A sphere falling slowly through a stationary viscous fluid has a terminal speed. On which quantity does it *not* depend:
(a) the fluid viscosity (b) the sphere's radius (c) gravitational acceleration
(d) initial speed (e) mass

(iv) For an object moving horizontally, subject to linear drag, the velocity:
(a) decreases linearly with time (b) decreases quadratically with time
(c) decreases exponentially with time (d) is constant
(e) approaches a non-zero constant.

(v) The amplitude of a wave is its
(i) maximum displacement from equilibrium (ii) speed
(iii) minimum speed (iv) displacement from equilibrium (e) initial position

2. For each of the following questions, please circle one selection for your answer. (15 marks)

(i) A driven, damped oscillator is described by a set of parameters $\omega_o^2 = k/m$ and $\gamma = c/2m$, where c appears in $F = -cv$. The phase angle of the oscillator is equal to $\pi/2$ at an angular driving frequency of
(a) ω_o (b) $(\omega_o^2 - \gamma^2)^{1/2}$ (c) $(\omega_o^2 - 2\gamma^2)^{1/2}$ (d) $\omega_o - \gamma$ (e) γ

(ii) A conservative force **F** is one for which
(a) **F** = 0 (b) div **F** = 0 (c) **curl F** = 0 (d) **F** is independent of position (e) **F•r** = 0

(iii) Which of the following fictitious forces is the transverse force:
(a) $-2m\ \omega x\mathbf{v'}$ (b) $-m\ (d\omega/dt)\ x\ \mathbf{r'}$ (c) $-m\ \omega x(\omega x\mathbf{r})$ (d) $+2m\ \omega x\mathbf{v'}$ (e) none of [a-d]

(iv) A ball dropped from rest in the northern hemisphere experiences a Coriolis force directed to the
(a) north (b) south (c) east (d) west (e) centre of the earth

(v) The difference between the effective gravitational acceleration **g** and the true gravitational acceleration $\mathbf{g}_o$ (from Newton's law) is a vector $\Delta\mathbf{g} = \mathbf{g} - \mathbf{g}_o$ which
(a) is parallel to the equator, pointing away from the axis of rotation
(b) is parallel to the equator, pointing towards the axis of rotation
(c) points towards the centre of the Earth
(d) points away from the centre of the Earth
(e) points along the axis of rotation

3. For each of the following questions, please circle one selection for your answer. (10 marks)

(i) An object is subject to a central force proportional to $1/r^2$. In a bound state, the cube of the semi-major axis of its orbit is proportional to
(a) T^3 (b) $1/T^3$ (c) $1/T^2$ (d) T^2 (e) none of [a-d]

(ii) The eccentricity e of a parabolic orbit is
(a) zero (b) one (c) less than one (d) greater than one (e) complex

(iii) The total energy E of a parabolic orbit is
(a) zero (b) one (c) less than zero (d) greater than zero (e) complex

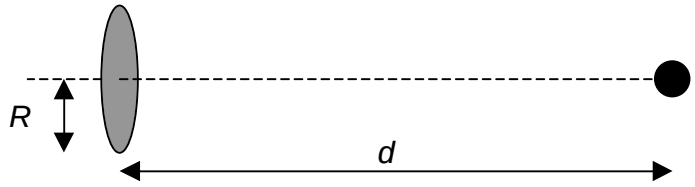
(iv) For orbital stability, the condition on the effective potential $U = V(r) + m\ell^2/2r^2$ is
(a) $dU/dr > 0$ (b) $dU/dr < 0$ (c) U = constant (d) $d^2U/dr^2 < 0$ (e) $d^2U/dr^2 > 0$

(v) For a collection of N point objects, the centre-of-mass position is
(a) $\Sigma_i m_i \mathbf{r}_i^2$ (b) $\Sigma_i m_i \mathbf{r}_i$ (c) $\Sigma_i m_i \mathbf{r}_i / \Sigma_i m_i$ (d) $\Sigma_i \mathbf{r}_i$ (e) none of [a-d]

4. Subject to a force $F(x)$, an object moves along the $x$axis with a speed described by $v = bx^2$.
(a) Establish that, in general, the acceleration is given by $a = v\,(dv/dx)$.
(b) As a function of x, find $F(x)$ from which the observed speed arises. (9 marks)

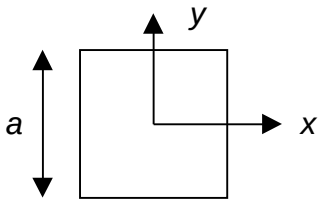
5. A thin solid disk rotates about its axis completely submerged in a viscous fluid, from which it experiences a drag force given by $\phi = cv$, where ϕ is the force per unit exposed area of the disk, v is the local speed of the disk's surface with respect to the fluid, and c is a constant which is determined by the viscosity of the fluid. The axis of the disk does not move with respect to the fluid. The disk has an angular speed ω and a radius R. Find the torque experienced by the disk. (12 marks)

6. Find the gravitational force experienced by a mass m lying on the symmetry axis a distance d from a thin, flat disk of mass M and radius R. [*Hint*: *it may help to use* $r\,dr = dr^2/2$ *when integrating*]

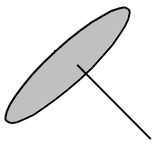


(19 marks)

7. A planar sheet has the shape of a square of length a to the side. Find the components I_{xx}, I_{yy} and I_{xy} of the inertia tensor for a coordinate system whose origin lies at the center of the square and whose axes are parallel to the sides of the square.
(9 marks)



8. A mini-gyroscope is made by welding a Canadian dime to a straight pin as shown, with the schooner *Bluenose* facing away from the pivot point. The mass and radius of the coin are 5 g and 0.5 cm, respectively, and the pin itself has negligible mass and a length of 2 cm.



The dime is made to spin about its symmetry axis. This mini-gyro completes one circuit about the vertical axis in 2 seconds, and at each revolution, the *Bluenose* reverses it orientation (from right-side-up to upside-down to right-side-up ...).
(a) What are the rates of change of the Euler angles?
(b) Is the mini-gyroscope spinning fast enough to maintain a vertical orientation? (16 marks)

The angle between the pin and the vertical axis (θ) *is constant.*

Answers:

1. e, a, d, c, a.
2. a, c, b, c, a.
3. d, b, a, e, c.
4. $F = 2mb^2x^3$.
5. $\tau_{total} = \pi c \omega R^4$.
6. $F = 2GmM[1 - d/(R^2+d^2)^{1/2}]$
7. $I_{xx} = I_{yy} = Ma^2/12$; $I_{xy} = 0$.
8. (a) $d\phi/dt = \pi\ s^{-1}$, $d\theta/dt = 0$, $d\psi/dt = \pi/2\ s^{-1}$; (b) $S = 3\pi/2$ is insufficient to keep the gyro vertical.