PHYSICS 211 FINAL EXAMINATION

Thursday, 17 December, 1998 Time: 3 hours

Calculators and two formula sheets permitted.

Show complete solutions to all problems.

Some useful constants:

$k = 8.99 \times 10^9$ Nm^2/kg^2 $c = 3.0 \times 10^8$ m/s

$|Q| = 1.6 \times 10^{-19}$ C electron mass = 9.11×10^{-31} kg

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1. (a) Find the three cartesian components of the force corresponding to the potential energy function

$V(r) = cz / r^3$. (8 marks)

(b) By explicitly evaluating all components of its **curl**, verify that this force is conserved. Show every wretched derivative in detail. (12 marks)

2. A pair of billiard balls are at rest on a frictionless table, just touching one another. A third identical ball moving with velocity v_o strikes both simultaneously, resulting in an elastic collision. Find the velocities of all three balls in terms of v_o immediately after the collision. (14 marks)

3. A particle of mass m moves in two dimensions, subject to the potential energy

$V(x,y) = -V_o\,[\cos(\pi x/a) + \cos(\pi y/a)]$,

where V_o and a are positive constants.

(a) What values of (x, y) correspond to positions of stable equilibrium? (4 marks)

(b) What is the angular frequency of oscillation ω if the particle is displaced slightly from a position of equilibrium? (6 marks)

(c) What is the general form of $X(t)$ and $Y(t)$ for small oscillations about an equilibrium position, where $X(t)$ and $Y(t)$ are the displacements from equilibrium? (3 marks)

4. An electron travelling at 1/10 of the speed of light is scattered from a proton held fixed in a target.

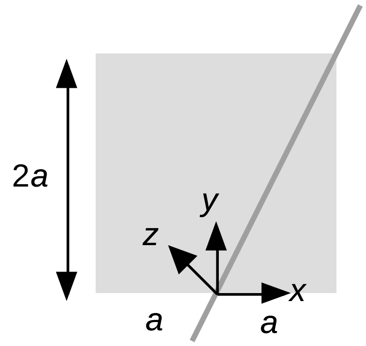
(a) Find the differential cross section $\sigma(\theta)$ for scattering angles (in the lab frame) of 0, $\pi/2$ and π radians. (10 marks)

(b) What are the corresponding values of the impact parameter if $\tan\theta_o = 2Eb / \Gamma$?

(5 marks)

(c) If the electron were replaced by a positron (same mass, opposite charge) how would the cross section change? How would the classical trajectory change? Explain your reasoning. (3 marks)

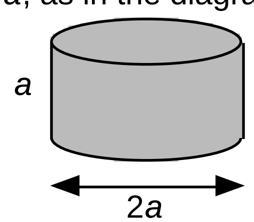
5. (a) Find the three-dimensional inertia tensor I_{ij} of a square lamina of mass M and sides of length $2a$. Use the cartesian coordinate system in the diagram, where the lamina lies in the xy plane and the coordinate origin is at the centre of one edge.



(16 marks)

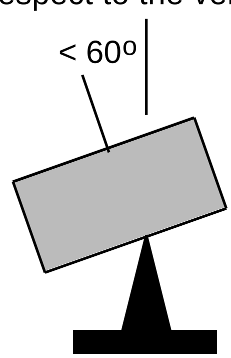
(b) Find the moment of inertia for the same lamina about an axis running from the coordinate origin to a corner on the opposite side, as indicated by the dashed line in the diagram. (7 marks)

6. (a) Find the moments of inertia about the three principal axes of a cylinder with mass M and radius and thickness a, as in the diagram.



Start your calculation using the result that the moment of inertia of a laminar disk about an axis through its centre, perpendicular to its plane, is $mR^2/2$. (8 marks)

(b) Find the minimum value of the spin (in terms of a, g *etc.*) required to keep the cylinder spinning at an angle with respect to the vertical of less than 60^o. (4 marks)



Answers

1. $F_x = 3cxz(x^2+y^2+z^2)^{-5/2}$

$F_y = 3cyz(x^2+y^2+z^2)^{-5/2}$

$F_z = 3cz^2(x^2+y^2+z^2)^{-5/2} - c(x^2+y^2+z^2)^{-3/2}$

2. incident ball = $-v_o/5$; stationary balls = $(2\sqrt{3} / 5)v_o$

3. (a) $x/a = 2n_x$, $n_x = 0, \pm1, \pm2...$ $y/a = 2n_y$, $n_y = 0, \pm1, \pm2...$

(b) $(\pi/a)\bullet(v_o/m)^{1/2}$

(c) $X(t)=A \cos(\omega t+\phi_x)$, $Y(t)=B \cos(\omega t+\phi_y)$

4. no longer part of PHYS 211

5. (a) $I_{xx} = 4Ma^2/3$; $I_{yy} = Ma^2/3$; $I_{zz} = 5Ma^2/3$; $I_{xy} = I_{xy} = I_{zy} = 0$

(b) $I_n = (8/15)Ma^2$

6. (a) $I_1 = I_2 = Ma^2/3$; $I_3 = Ma^2/2$

(b) $S > (4g / 3a)^{1/2}$.