# Lecture 3 - Velocity-dependent forces - I

*Text*: Fowles and Cassiday, Chap. 2
*Demo: coffee filters, ball bearings in oil...*

Most examples of force in first year physics are position-dependent [$V(x)$ or $\mathbf{F}(x)$]. We now generalize this to velocity-dependent forces: for instance, the magnetic force on a moving charge $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$ changes the direction of the velocity. Common examples in mechanics include drag forces on objects moving through fluids, such as Stoke's Law for the force on a sphere of radius $R$ moving with speed $v$ through a medium with viscosity $\eta$:

$$F = 6\pi\eta R v \qquad \text{(appropriate at small } v\text{)} \qquad (3.1)$$

At higher speeds, the viscous force may become quadratic in $v$:

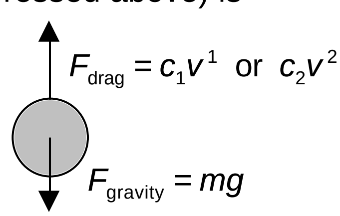
$$F = (\rho/2) A C_D v^2 \qquad \text{(turbulence, does not depend on } \eta\text{)} \qquad (3.2)$$

where $\rho$ = density of medium, $A$ is cross sectional area of object, $C_D$ = drag coefficient ($C_D$ = 0.43 for BMW roadsters, 0.37 for Mazda Miata). This is different from the friction between solid surfaces $f = \mu N$ (presented in first year) which does not depend upon speed. We break up our treatment of drag into two sections:

(i) physical properties
(ii) mathematical description and analysis.

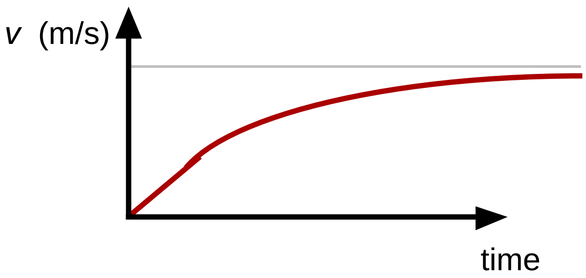
<u>Drag + gravity</u>

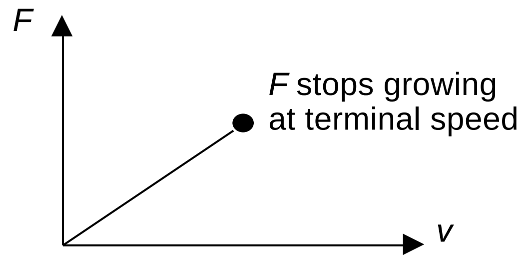
We jump ahead a little to look at a falling system subject to drag in order to demonstrate (in class) the power law behaviour of the drag force. The free-body diagram of an object subject only to gravity ($F = mg$) and drag ($F_{drag} = c_1 v^1$ or $c_2 v^2$, where the coefficients are expressed above) is




An object released from rest accelerates downwards because $mg$ is greater than $F_{drag}$, which is initially small because of the small velocity. As the object accelerates, the velocity increases and so does $F_{drag}$. Ultimately, the drag force may reach $mg$, but it can never exceed $mg$ or the object would accelerate upwards. Once the two forces are balanced, the object has reached its terminal velocity $v_T$:

linear drag: $c_1 v^1 = mg$ or $v_T = mg / c_1$ (3.3)
quadratic drag: $c_2 v^2 = mg$ or $v_T = (mg / c_2)^{1/2}$ (3.4)




The viscosities of fluids encompass a huge range, as shown in the table. In class, we determine the viscosity of corn syrup using linear drag.

| Fluid | $\eta$ (kg/m•sec at 20 $^{o}$C) |
|---|---|
| Air | $1.8\times10^{-5}$ |
| Water | $1.0\times10^{-3}$ |
| Mercury | $1.56\times10^{-3}$ |
| Olive oil | 0.084 |
| Glycerine | 1.34 |
| Glucose | $10^{13}$ |

*Demo*

- •velocity of falling coffee filters follows $m^{1/2}$ ---> quadratic drag
- •balls in corn syrup harder to show with steel balls because radius changes along with mass

*Experiment: use linear drag to find viscosity of corn syrup*

First, we establish what power law regime our experiment obeys. To see where the cross-over between linear and quadratic regimes occurs, we consider a steel ball of radius 0.5 cm (= 3/8" diameter) in a somewhat viscous fluid with $\eta$ = 1 kg/m•s; take the density to be $10^3$ kg/m$^3$ (like water or similar fluids)

Linear regime:

$F = c_1 v$ where $c_1 = 6\pi\eta R$

$c_1 = 6\pi\eta R = 6\pi \bullet 1 \bullet 0.005 = 0.094$ kg/s

Quadratic regime:

$F = c_2 v^2$ where $c_2 = (\rho/2)AC_D$

Assume: $\rho = 10^3$ kg/m$^3$ $C_D = 0.5$ $A = \pi R^2$ with $R = 0.5$ cm

--> $c_2 = (\rho/2)AC_D = (1000 / 2)\ \pi\ (0.005)^2\ 0.5 = 0.020$ kg/m

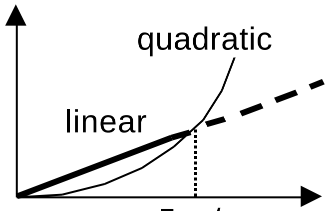
When do these forces become comparable for a ball bearing in glycerine?

$c_1 v = c_2 v^2$

$0.094\ v = 0.020\ v^2$

$v = 0.094 / 0.020 = 4.8$ m/s (this is the cross-over point)

Thus, *for this example*




In the corn syrup demo, the speed of the falling ball is less than 0.1 m/s, meaning that the drag force is probably in the linear regime, and we can analyze the motion using Stoke's law:

$$mg = 6\pi\eta R v_{\text{TERM}} \qquad \text{or} \qquad v_{\text{TERM}} = mg / 6\pi\eta R.$$

To find the mass of the ball bearing, use $\rho$ = 8000 kg/m$^3$ and $R$ = 0.005 m, so

$$m = \rho V = 8000(4\pi/3)(0.005)^3 = 0.0042 \text{ kg} = 4 \text{ grams}$$

In the demo, we obtain $\eta$ ~ 20-25 kg / m•s for corn syrup.