Lecture 19 - MB continued

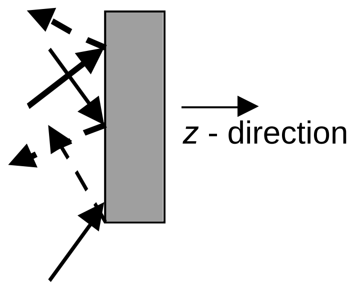
*What's Important:*

- pressure of an ideal gas

*Text*: Reif

**Pressure of an ideal gas**

Our picture of an ideal gas hitting a wall is




With each collision, an amount of momentum $2mv_z$ is transferred to the wall, giving rise to a pressure, or force per unit area. If the collisions occur over a time interval $\Delta t$, then for a given value of $v_z$

[*momentum change*] = $2mv_z$ • [*collisions per unit time*] • $\Delta t$

But the force from the collisions is equal to $\Delta p / \Delta t$, so for a given value of $v_z$

[*force per unit area*] = $2mv_z$ • [*collisions per unit time per unit area*] • $\Delta t / \Delta t$.

That is, the pressure from $v_z$ is

[*pressure*] = $2mv_z$ • [*collisions per unit time per unit area*].

Now, the [*collisions per unit time per unit area*] is also called the flux. If the particles all moved perpendicular to the wall, then the flux would be

[*flux*] = [*number per unit volume*] $v_z$,

which clearly has the units of [*number*] • [*time*]$^{-1}$ • [*area*]$^{-1}$. However, the flux is reduced if the particles hit at an angle $\theta$, becoming

[*flux*] = [*number per unit volume*] $v_z \cos\theta$

Defining $f(v)$ as the probability density for the velocity distribution, we can express the mean pressure as (this is the mean pressure because the calculation is performed at constant volume):

$$\begin{aligned}\bar{p} &= \int_{v_z>0} d^3v\, f(v)\cdot v\cos\theta \cdot 2mv_z \\ &= 2m \int_{v_z>0} d^3v\, f(v)\cdot v_z^2 \\ &= m \int_{all\, v_z} d^3v\, f(v)\cdot v_z^2 \\ &= mn\overline{v_z^2}\end{aligned} \qquad (19.1)$$

where $n$ is the number density and the expectation is

$$\overline{v_z^2} = \frac{1}{n}\int_{all\, v_z} d^3v\, f(v)\cdot v_z^2 \qquad (19.2)$$

Next ,we use translational symmetry to obtain

$$v^2 = \overline{v_x^2 + v_y^2 + v_z^2} = 3\overline{v_z^2}$$

which finally gives an expression for the mean pressure in terms of microscopic variables:

$$\bar{p} = \frac{1}{3}nm\overline{v^2}\,. \qquad (19.3)$$

A related expression for the pressure can be obtained from the mean kinetic energy

$$\bar{K} = mean\ kinetic energy = \frac{1}{2}m\overline{v^2}$$

which yields

$$\bar{p} = \frac{2}{3}n\left(\frac{1}{2}m\overline{v^2}\right) = \frac{2}{3}n\bar{K} \qquad (19.4)$$

In other words, the mean pressure is 2/3 of the mean kinetic energy per unit volume, or the pressure is 2/3 of the energy density.

Finally, from the equipartition theorem

$$\bar{K} = \frac{3}{2}k_BT$$

comes the relation

$$\bar{p} = \frac{2}{3}n\frac{3}{2}k_BT = nk_BT\,, \qquad (19.5)$$

with is just the ideal gas law.