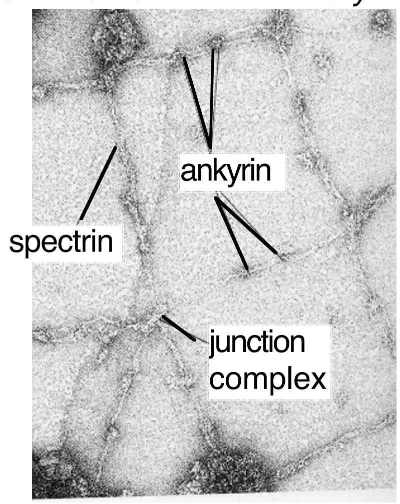
PHYS 4xx net1 - Soft networks and their deformation

Examples of two-dimensional networks in the cell:

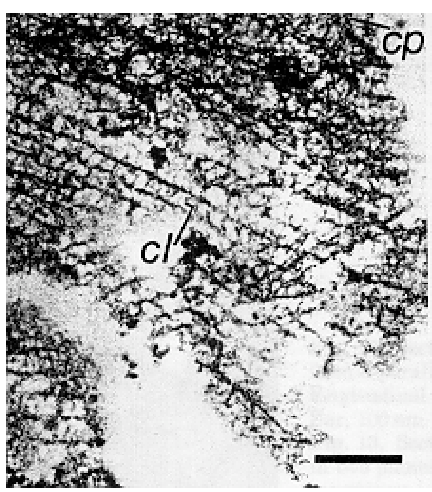
(i) *membrane-associated cytoskeleton of the human erythrocyte*



(from Byers and Branton, 1985)

- a network of spectrin tetramers attached to cytoplasmic side of plasma membrane about midway along their length by the protein ankyrin
- each spectrin tetramer has a 200 nm contour length, but $<r_{ee}>$ ~ 70 nm *in vivo*

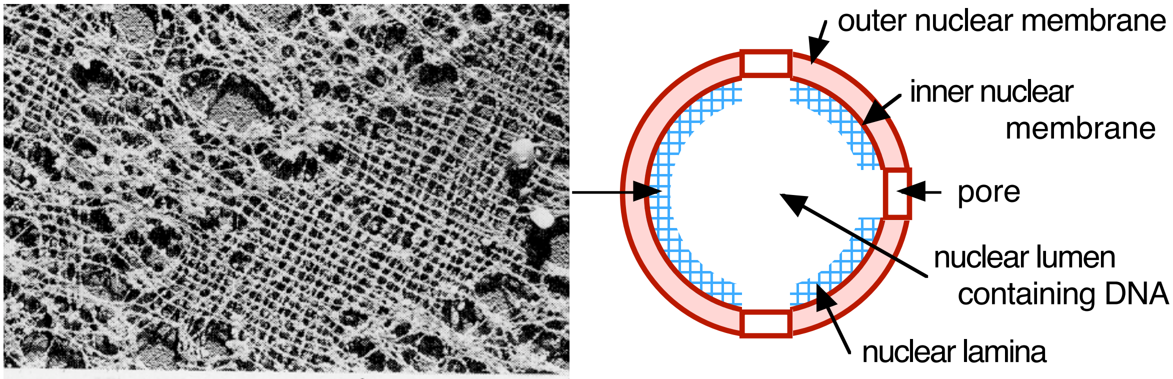
(ii) *auditory outer hair cell*



(from Tolomeo, Steele, and Holley, 1996; bar is 200 nm long)

- lateral cortex lies on cytoplasmic side of the plasma membrane
- principal filaments are about 5-7 nm thick, spaced ~ 60 nm apart and form hoops around the axis of the cylinder; probably actin
- cross-linkers at intervals of ~30 nm (with a range of 10-50 nm) by thinner filaments just 2-3 nm thick, denoted by "cl" above; probably spectrin

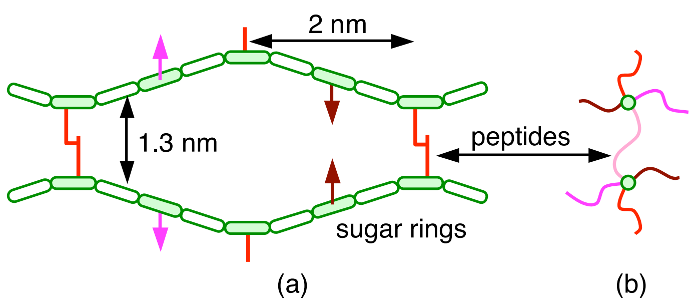
(iii) *nuclear lamina*



(electron micrograph of a region about 2.5 μm in length from the nuclear lamina in a *Xenopus* ooctyte; from Aebi *et al*., 1986)

- four-fold connectivity; network is 10-20 nm thick
- filaments of the protein laminin are 10.5 ± 1.5 nm in diameter and are typically separated by about 50 nm

(iv) *peptidoglycan*

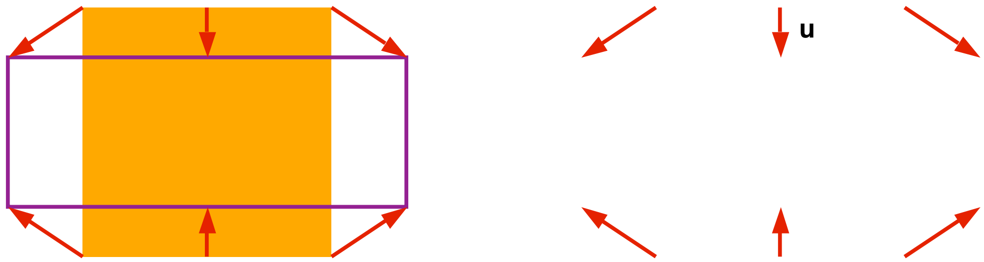


face view side view along glycan
(after Koch and Woeste, 1992)

- bacterial cell wall is made from peptidoglycan network
- stiff chains of sugar molecules (cylinders) are cross-linked both in- and out-of-plane by flexible chains of amino acids (out-of-plane links are indicated by arrows)

Strain tensor

To describe the deformation of an object, we introduce a set of vectors **u**, such that a point moves from its original position **x** to a new position **x** + **u**.



- **u** varies in magnitude and direction across the object (**u** = constant corresponds to translation)
- **u** may have non-zero partial derivatives in any Cartesian direction
- the strain tensor u_{ij}, is related to the rate of change of **u** with position **x** by

$$u_{ij} = 1/2\,[\partial u_i/\partial x_j + \partial u_j/\partial x_i + \Sigma_k\,(\partial u_k/\partial x_i)(\partial u_k/\partial x_j)],\ (i,j,k \text{ are Cartesian indices}) \quad (1)$$

- u_{ij} has $2^2 = 4$ components in 2D and $3^2 = 9$ components in 3D
- •u_{ij} is unitless and is symmetric in indices *i* and *j*.

For small deformations, the last term in Eq. (1) may be neglected:

$$u_{ij} \cong 1/2\,[\partial u_i/\partial x_j + \partial u_j/\partial x_i]. \qquad \text{(small deformations)} \qquad (2)$$

<u>Example:</u> Uniform scaling as in the diagram above:

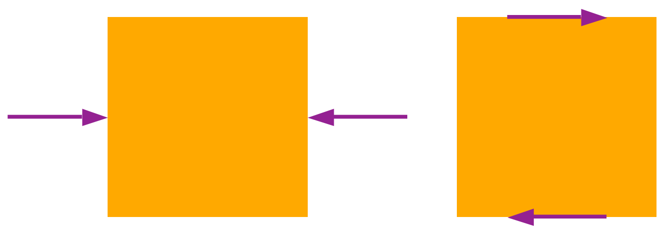
all x go to $1.1x$ ----> $u_{xx} = (1.1 - 1)x / x = 0.1$ everywhere

all y go to $0.9y$ ----> $u_{yy} = (0.9 - 1)y / y = -0.1$ everywhere

change in x does not depend on y ----> $u_{xy} = u_{yx} = 0$.

Sress tensor

- stress tensor σ_{ij} = force per unit area, taking into account the direction of force **F**



- component in the *i*-direction of the net force, F_i, is given by

$$F_i = \Sigma_j\ \sigma_{ij}\ a_j. \qquad (3)$$

- surface area vector **a** is perpendicular to the surface
- σ_{ij} has units of energy density and is symmetric in indices *i* and *j*.
- generally, the diagonal elements of the stress tensor correspond to compression and the off-diagonal elements to shear.

Example: An object under hydrostatic pressure P.

- **F** on a surface is in the opposite direction to the vector **a** describing the surface
  $F_i = -P\, a_i = -P \sum_j \delta_{ij}\, a_j$. (4)
- comparing Eqs. (3) and (4):
  $\sigma_{ij} = -P\, \delta_{ij}$. (5)

Elastic moduli

- for ideal springs in one dimension, the restoring force f is proportional to the displacement from equilibrium x: $f = -k_{sp}x$, where k_{sp} = spring constant.
- corresponding relationship for continuous materials reads [*stress*] ∝ [*strain*], or
  $\sigma_{ij} = \sum_{k,l} C_{ijkl} u_{kl}$ (6)

- material-specific constants C_{ijkl} are the elastic stiffness constants or elastic moduli; units of energy per unit volume for 3D materials, or energy per unit area for 2D

- the elastic moduli of two- or three-dimensional materials form a tensor, as opposed to the single k_{sp} of an isolated spring

Just as the potential energy of a Hooke's law spring is quadratic in the square of the displacement, the change in the free energy density $\Delta\mathcal{F}$ of a continuous object under deformation is quadratic in the strain tensor u_{ij}:

$$\Delta\mathcal{F} = 1/2 \sum_{i,j,k,l} C_{ijkl} u_{ij} u_{kl}. \quad (7)$$

- symmetry considerations greatly reduce the number of independent components of C_{ijkl} from $3^4 = 81$ terms in three dimensions, or $2^4 = 16$ in two dimensions.