

PHYS 4xx Net 5 - Networks in the cell

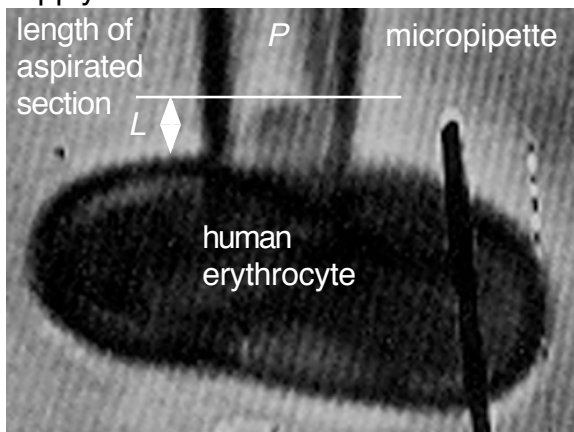
Measuring network elasticity

Two principal techniques:

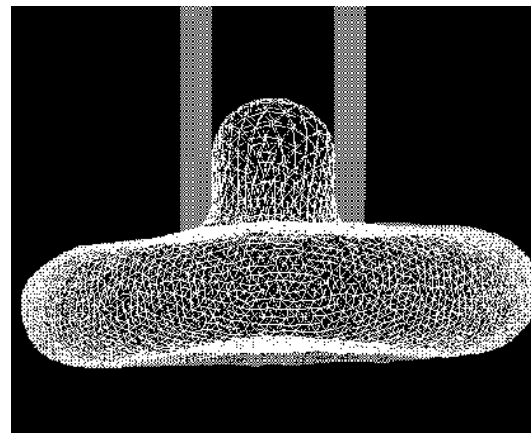
- micromechanical manipulation: a cell or extracted network is subject to a known stress and the response of the cell is observed by optical microscopy or another imaging method
- thermal fluctuations in a system's geometry; these fluctuations are inversely proportional to the stiffness of the system

Human erythrocyte

- apply suction to a flaccid cell



video image of aspiration
(Waugh and Evans)



computer simulation
(Discher, Boal, and Boey)

- length of the aspirated segment L is related to the applied pressure P

$$P = (\mu / R_p) [(2L / R_p) - 1 + \ln(2L / R_p)]$$

$$\mu = 2D \text{ shear modulus}$$

$$R_p = \text{pipette radius} \sim 1/2 \text{ micron}$$
- experiments typically yield $\mu \sim 6-9 \times 10^{-6} \text{ J/m}^2$ for human erythrocytes
 - $\mu \sim 10^{-5} \text{ J/m}^2$ for most mammalian red cells (no nucleus)
 - $\mu \sim 10^{-4} \text{ J/m}^2$ for nucleated cells (Waugh and Evans)
- deforming a cell with optical tweezers gives $\mu = 2.5 \pm 0.4 \times 10^{-6} \text{ J/m}^2$ (Hénon *et al.*)
- thermal fluctuations of red cell shape gives $\mu < 10^{-6} \text{ J/m}^2$ (Peterson, Strey, and Sackmann)

Just how small is this? For a material of thickness t

$$K_V \sim K_A / t.$$

For the red cell cytoskeleton, $K_A \sim 2\mu \sim 10^{-5} \text{ J/m}^2$ and $t \sim 40 \text{ nm}$, so

$$K_V \sim 10^{-5} / 4 \times 10^{-8} = 250 \text{ J/m}^3.$$

Compare this with an ideal gas at STP, where $K_V = P = 10^5 \text{ J/m}^3$: this cytoskeleton is very soft.

Auditory outer hair cell

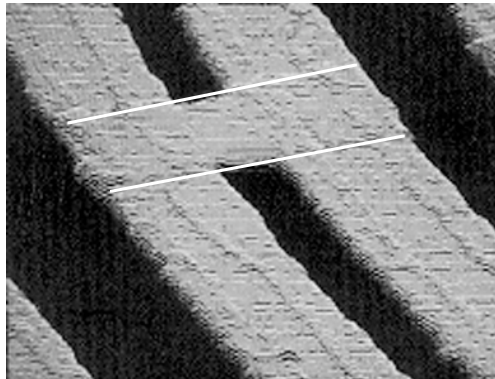
- cell is about $78 \mu\text{m}$ long and $10 \mu\text{m}$ wide



Videomicrograph of an auditory outer hair cell from a guinea pig (Sit *et al.*)

- assuming the cortex is isotropic, $\mu = 1.5 \pm 0.3 \times 10^{-2} \text{ J/m}^2$ (Sit *et al.*), which is 1000 times the red cell modulus
- $Y \sim 1 \times 10^7 \text{ J/m}^3$ for principal filaments, $Y = 3 \times 10^6 \text{ J/m}^3$ for cross-links (Tolomeo, Steele, and Holley); 10^{-2} of what we quoted for polymers)
- $\mu \sim 2-4 \times 10^{-3} \text{ J/m}^2$ for inhomogeneous networks in fibroblasts (Bausch *et al.*)

Bacteria



STM image of a bacterial sheath over a $0.3 \mu\text{m}$ gap in a Ga-As substrate (Xu *et al.*)

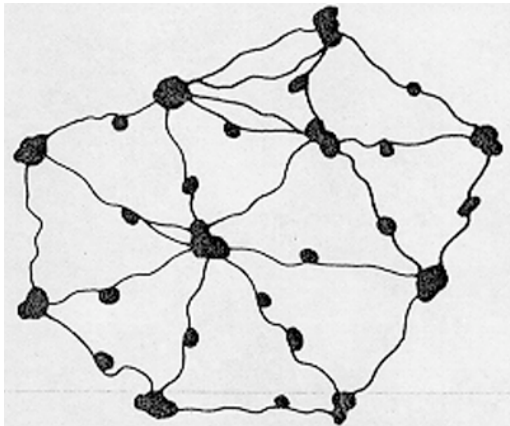
- sheath of the bacterium *Methanospirillum hungatei* has $Y \sim 2-4 \times 10^{10} \text{ J/m}^3$
- bulk Young's modulus of Gram-positive bacterium *Bacillus subtilis* (Thwaites and Surana)
 - $Y = 1.3 \pm 0.3 \times 10^{10} \text{ J/m}^2$ for a dry cell wall
 - $Y \sim 3 \times 10^7 \text{ J/m}^3$ for a wet cell wall

Interpretation of measurements

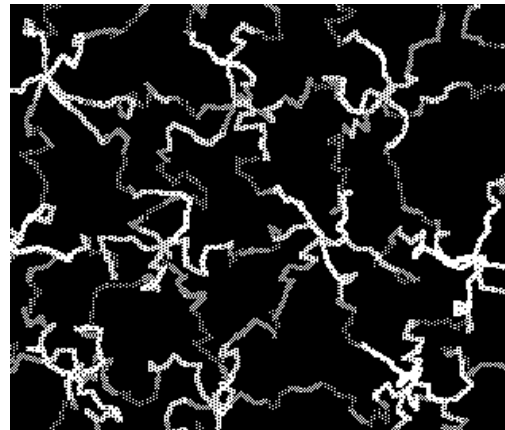
- entropic springs have an effective spring constant $k_{\text{sp}} = 3k_B T / \langle r_{\text{ee}}^2 \rangle$
- area per vertex A_v of an equilateral triangle $A_v \sim \sqrt{3} \langle r_{\text{ee}}^2 \rangle / 2$

$$\text{----> } k_{sp} \sim 3\sqrt{3} k_B T / 2A_v$$

- 2D density of chains $\rho = 3/A_v$ (three chains per vertex)
 $\text{----> } k_{sp} = (\sqrt{3}/2) \rho k_B T$
- six-fold spring network has $\mu = \sqrt{3} k_{sp}/4$ at low temperature
 $\text{----> } \mu = (3/8) \rho k_B T$
- factor of 3/8 isn't too meaningful, so we use
 $\mu \sim \rho k_B T$



spread erythrocyte cytoskeleton
(Liu, Derick, and Palek, 1987)



simulated cytoskeleton
(Boal, 1994)

Applications

- human erythrocyte has a spectrin tetramer density of $\rho \sim 800 \mu\text{m}^{-2}$
 $\text{----> } \mu \sim \rho k_B T \sim 800 \times 10^{12} (\text{m}^{-2}) \cdot 4 \times 10^{-21} (\text{J}) \sim 3 \times 10^{-6} \text{ J/m}^2$
 (within a factor of two or so of experiment)
- micropipette aspiration finds $K_A/\mu \sim 2$ (Discher, Mohandas and Evans, 1994)
 6-fold spring networks obey $K_A/\mu = 2$
- cortical lattice of outer hair cells has two chains per 25 x 65 nm rectangular plaquette,
- corresponding density is $\rho \sim 1.2 \times 10^{15} \text{ m}^{-2}$
 $\text{----> } \mu \sim \rho k_B T \sim 1.2 \times 10^{15} (\text{m}^{-2}) \cdot 4 \times 10^{-21} (\text{J}) \sim 5 \times 10^{-6} \text{ J/m}^2$
- measured μ is three orders of magnitude larger (Sit *et al.*, 1997)!
 ----> shear resistance is probably energetic, not entropic; reflects the stiffness of the network filaments