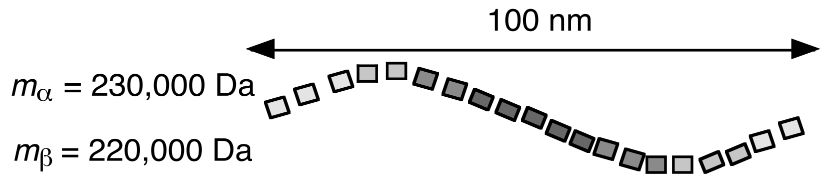
PHYS 4xx Poly 4 - Biopolymers

Some important filaments in the cell

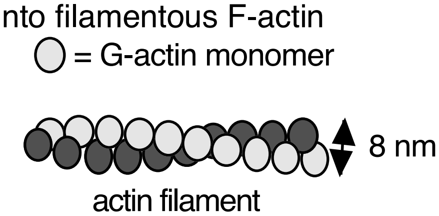
DNA
- monomeric unit is phosphate + sugar + organic base
- phosphate and sugar units alternate along each strand of a double helix
- length along the helix is 0.34 nm per base pair; diameter is 2 nm

Spectrin
- tetramer is two pairs of chains, joined end-to-end, total contour length of 200 nm
- pair has two intertwined and inequivalent (α and β) strings of spectrin (pairs join end-to-end to form a tetramer)

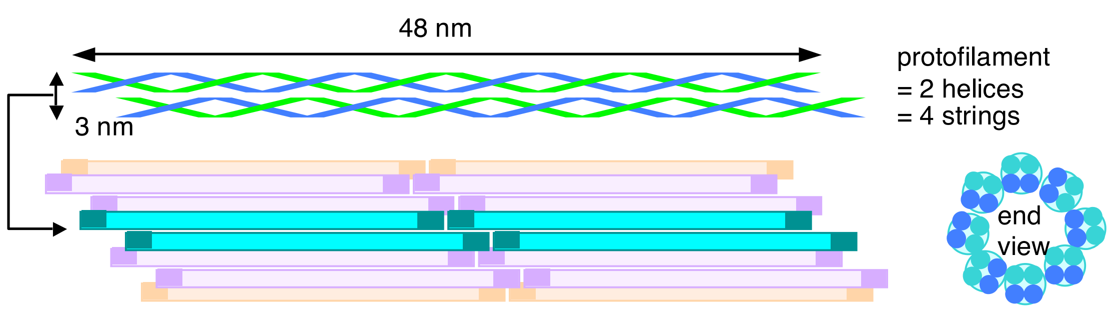


- chain folds back on itself repeatedly, so that each monomer is a series of 19 or 20 relatively rigid barrels 106 amino acid residues in length

Actin
- G-actin (*G* for globular), a single chain of ~375 amino acids; mass ~ 42,000 D
- G-actin units assemble into filamentous F-actin

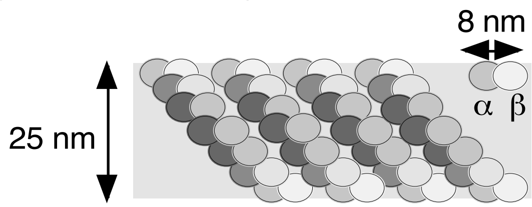


Intermediate filaments
- two protein chains intertwined as a helix
- pairs of helices lie side-by-side to form a linear protofilament ~ 2 - 3 nm in diameter
- filament is a hollow bundle of 8 protofilaments, about 10 nm in diameter
- many protofilaments have lengths of the order 50 nm



<u>Microtubules</u>

- heterodimer of tubulin (α-tubulin and β-tubulin) about 8 nm in length
- dimers assemble α to β successively into a hollow microtubule consisting of 13 linear protofilaments (in almost all cells)



Measurements of persistence length

(mass per unit length λ_p and persistence length ξ_p)

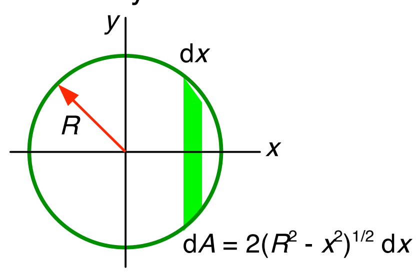
Polymer	*Configuration*	λ_p (D/nm)	ξ_p (nm)
Long alkanes	linear polymer	~110	~0.5
Spectrin	2-strand filament	4,500	10-20
DNA	double helix	1,900	53 ± 2
F-actin	filament	16,000	10-20 x 10^3
Intermediate filaments	32 strand filament	~35,000	0.1-1 x 10^3
Tobacco mosaic virus		~140,000	~1 x 10^6
Microtubules	13 protofilaments	160,000	1-6 x 10^6

Analysis:

- persistence length $\xi_p = \beta\kappa_f = \kappa_f / k_B T$ (1)
- $\kappa_f = Y\mathcal{I}$ (2)

  where Y = Young's modulus, units of [*energy* • *length*$^{-3}$]

  $\mathcal{I}$ = the moment of inertia of the cross section, units of [*length*^{4}]
- calculate $\mathcal{I}$ of a uniform solid cylinder:



$$\mathcal{I}_y = \int_{-R}^{R} x^2 \, dA = 4\int_o^R x^2 (R^2 - x^2)^{1/2} \, dx$$

Integrating: $\mathcal{I} = 4R^4 \int (x/R)^2\,[1 - (x/R)^2]^{1/2}\,d(x/R)$

$= 4R^4 \int \cos^2\theta\,[1 - \cos^2\theta]^{1/2}\,d\cos\theta$ where $x/R = \cos\theta$

$= 4R^4 \int \cos^2\theta \sin^2\theta\, d\theta$ where $0 \le \theta \le \pi/2$

In detail: $\int \cos^2\theta \sin^2\theta\, d\theta = \int (\sin 2\theta\ /2)^2\, d\theta$

$= (1/8) \int \sin^2\alpha\ \ d\alpha$ where $0 \le \alpha \le \pi$

$= \pi/16$

Thus: $\mathcal{I} = \pi R^4/4$ (solid cylinder) (3)

- for a hollow core of radius R_i, (3) is reduced by $\mathcal{I} = \pi R_i^4/4$ of the core:

  $\mathcal{I}_y = \pi(R^4 - R_i^4)/4$ (hollow cylinder). (4)

ξ_p and Young's modulus

- view the polymers as flexible rods; according to (1) and (2), ξ_p is

  $\xi_p = Y\mathcal{I} / k_B T$ (5)

- moment of inertia of the cross section for hollow rods of inner radius R_i and outer radius R is from (4)

  $\mathcal{I} = \pi(R^4 - R_i^4)/4$.

- assume $R >> R_i$:

  $\xi_p \cong \pi Y R^4 / 4 k_B T$, (6)

  good for tobacco mosaic virus ($R / R_i \sim 4.5$)

  factor-of-two error for microtubules ($R \sim 14$ nm and $R_i \sim 11.5$ nm)

- replace R by the mass per unit length λ_p using $\lambda_p = \rho_m \pi R^2$ for a cylinder, where ρ_m is the mass per unit volume:

  $\xi_p \cong (Y / 4\pi k_B T \rho_m^2)\ \lambda_p^2$ (7)

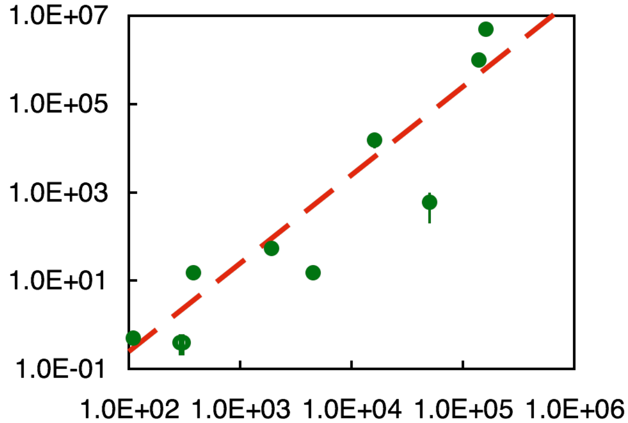
- compared to filament radii, Y and ρ_m are relatively constant among filaments

- straight line through data is $\xi_p = 2.5 \times 10^{-5}\ \lambda_p^2$, where ξ_p is in nm and λ_p is in D/nm

- equating the fitted numerical factor

  $2.5 \times 10^{-5}\ \text{nm}^3/\text{D}^2 = Y / 4\pi k_B T \rho_m^2$

----> $Y = 0.5 \times 10^9$ J/m^3 for $k_B\,T = 4 \times 10^{-21}$ J and $\rho_m = 10^3$ kg/m^3



Some comparative values:

material	Y (J/m^3)
diamond	1.2×10^{12}
steel	2×10^{11}
dry cellulose	8×10^{10}
bone (tension)	1.6×10^{10}
wood (along grain)	1.4×10^{10}
collagen	$1\text{-}2 \times 10^{9}$
rubber	7×10^{6}