SU(4) Multiplet Mixing

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The consequences of mixing between SU(4) multiplets of the same J, P, and C are examined. Within the proposed theoretical framework, the presently available data indicate that there should be substantial mixing between the pseudoscalar multiplets (P and P') while the vector multiples (V and V' containing ψ and ψ') remain largely unmixed. The ratio $F_P/F_{P'} \sim 2$ of the pseudoscalar decay constant leads to the suppression of the decays $\psi \rightarrow \eta_c\gamma$ and $\psi' \rightarrow \eta_c\gamma$.

While the SU(4) approach to the charm interpretation of the J/ψ family of particles has had both quantitative and qualitative success,[1-7] there have been several notable problems. They are the following:

(1) The symmetry-breaking Hamiltonian[1]

$$\mathcal{H} = U_0 + U_8 + aU_{15}, \qquad (1)$$

for the masses demands[2] that the usual SU(3) mixing angle θ be 35.3° if $\psi(3100)$ or its pseudoscalar analog is to be pure $(c\bar{c})$. Thus, the mixing angles obtained by a fit to the pseudoscalar meson masses predict[2,7] gross leakage of $(c\bar{c})$ into η and X^0.

(2) The estimates for the mass of the recently discovered[8] charmed pseudoscalar $D(1865)$ are too high by several hundred MeV for the quadratic mass formula.[1-3,7]

(3) There must be substantial SU(4) breaking of coupling constants[4-6] $g_{VP\gamma}$ in order to suppress the decay $\psi \rightarrow \eta_c(2800)\gamma$.

Because these problems have not yet arisen with the baryons, but only with the pseudoscalar mesons, we must seek a solution within SU(4). Rather than go the route of introducing extra symmetry-breaking terms in the mass matrix[9] or coupling constants,[4-6] we choose to examine the effects of SU(4) multiplet mixing.

There now appear to be two pseudoscalar multiplets,[10] π, K, η, X^0, $\eta_c(2800)$ (which we denote by P) and $K'(1400)$,[11] $E(1420)$, $\eta_c'(3455)$[12] (P', partially complete); and two vector multiplets, ρ, $K^*(892)$, ω, φ, ψ (denoted by V) and $\rho'(1600)$, $\psi'(3700)$ (V', partially complete). Since the mass splitting within a given multiplet is greater than the difference in average masses of the multiplets, there could be significant effects due to intermultiplet mixing.[13]

Now, the mass matrix elements between states of the same multiplet, say multiplet 1, generated by Eq. (1) contain both the symmetry-breaking parameter a and four reduced matrix elements M_1, M_1^0, A_1, and B_1 (see Refs. 1 and 2 for notation). Similarly M_2, M_2^0, A_2, B_2 are introduced for the second multiplet, and T, T^0, A_T, B_T for the cross terms. We set $T = -A_T(1$

TABLE I. Predicted and observed (Ref. 14) masses for the two pseudoscalar multiplets. The parameters have the values $M_1=2.130$, $M_1{}^0=3.260$, $A_1=-0.264$, $B_1=-0.133$, $M_2=4.159$, $M_2{}^0=3.766$, $A_2=-0.302$, $B_2=-0.232$, $T_0=0.945$, $A_T=-0.072$, $B_T=-0.033$ (all in GeV^2) and $a=18.16$. The masses are quoted in MeV.

	Predicted	Observed		Predicted	Observed	Mixing angle
π	138	138.03 ± 0.005	π'	1322	···	0
K	496	495.7 ± 0.08	K'	1418	1400 ± 50	−2.0°
η	549	548.8 ± 0.6	E	1416	1416 ± 3	···
X^0	958	957.6 ± 0.3	X'	1547	···	···
D	1893	1865 ± 15	D'	2596	···	−21.6°
F	1943	···	F'	2653	···	−22.2°
η_c	2564	2750 ± 50	η_c'	3455	3455 ± 5	···

$+a/\sqrt{2})/\sqrt{3}$ so that π-π' (or ρ-ρ') remain unmixed, for simpler coupling-constant definition. To solve for the twelve parameters, we use the nine pseudoscalar (or eight vector) masses, and the eight constraints on the eigenvectors that the $2^{-1/2}(u\bar{u}+d\bar{d})$ and $(s\bar{s})$ content of the $\eta_c(2800)$, $\eta_c'(3455)$ (or ψ,ψ') be equal to 0.0 ± 0.001. These constraints are justified[2] by the small rates of decays such as $\psi,\psi'\to\pi\gamma$, $\psi,\psi'\to K\bar{K}$, and $\eta_c'(3455)\to$ hadrons. The fit is given in Table I for the pseudoscalar masses[14] and the eigenvectors are shown in Table II.

For the vectors, there is no substantial mixing of the two multiplets. The mass of the D^* can be lowered only to about 2150 MeV and has a D^*-$D^{*\prime}$ mixing angle of ~6°. Thus, of the two peaks observed[9] in the recoil mass against $D(1865)$, we would have to assign the one at 2.0 GeV as probably a heavily mixed axial vector.

We now examine the consequences of the pseudoscalar-multiplet mixing on those decays of the J/ψ family which do not depend sensitively on $(c\bar{c})$ leakage. First, we find that the rate for the decay $\psi\to\eta_c(2800)\gamma$ [for our calculation we choose $\eta_c(2565)$] can be lowered from several MeV to less than one keV if the ratio of the coupling constants is $g_{VP'\gamma}/g_{VP\gamma}=2.27\pm 0.05$. Raising the η_c mass or introducing suppression factors[4,15] (of the order m_ψ) in the coupling constants increases the range of values allowed for $g_{VP'\gamma}$. To suppress $\psi'\to n_c\gamma$, the same ratio is required for $g_{V'P'\gamma}/g_{V'P\gamma}$. This is exactly what is found if we use[16] the strong anomalies in the partially conserved axial-vector current to predict the coupling constants: $g_{VP'\gamma}/g_{VP\gamma}=g_{V'P'\gamma}/g_{V'P\gamma}=F_P/F_{P'}$. On the assumption that the decay constants are U(4) invariant, then $F_{P'}\approx 41$ MeV (for $F_P=94$ MeV). Within the anomaly framework, the two photon decay rates of the pseudoscalars are as follows: for π^0, 7.5 eV; for η, 150 eV; for X^0, 18 keV; for π', 36 keV; for E, 60 keV; for X', 52 keV; and for η_c', 2.7 MeV. Again, the $\eta_c\to\gamma\gamma$ rate is suppressed, predicting a small value for the expression $R(\psi\to\eta_c\gamma)R(\eta_c\to\gamma\gamma)$ (R is the branching ratio). Experimentally, the measured upper bound[17] is 2×10^{-4} for ψ decay and 3.7×10^{-4} for ψ'.

We see that the vanishing of $\psi\to\eta_c\gamma$ implies the vanishing of $\psi'\to\eta_c\gamma$ without the use of angular momentum arguments based on a detailed quark model.[18] Several other successes of both approaches should also be noted. The decay $\psi'\to\psi\gamma$

TABLE II. Quark content of the isoscalars.

Particle	P multiplet $(u\bar{u}+d\bar{d})/\sqrt{2}$	 $s\bar{s}$	 $c\bar{c}$	P' multiplet $(u\bar{u}+d\bar{d})\sqrt{2}$	 $s\bar{s}$	 $c\bar{c}$
η	−0.672	0.736	0.004	0.062	−0.016	0.000
x^0	−0.352	−0.379	−0.001	0.779	0.354	0.000
η_c	0.002	0.003	0.915	0.002	−0.002	−0.403
E	0.444	0.340	−0.002	0.617	−0.554	0.001
x'	0.478	0.443	0.001	0.090	0.753	−0.001
η_c'	0.001	−0.001	0.403	0.000	0.000	0.915

is suppressed by SU(4) considerations since the VVV vertex has f-type coupling. The masses of the χ particles can be predicted by SU(4) with knowledge of the masses of the strange members of the $J^P = 0^+$, 1^+, and 2^+ multiplets[19,20]: $\kappa(1250)$, $Q(1350)$, and $K^*(1420)$. On the assumption of ideal mixing in a single multiplet, the χ masses can be found if we know the reduced matrix element A [averaged over the π-K, ρ-$K^*(892)$, and A_2-$K^*(1420)$ mass differences] and the symmetry-breaking parameter a (we use[2] $a = 22$). We find $J^P = 0^+$, 1^+, and 2^+ states at 3.41, 3.44, and 3.47 GeV, respectively.

We can estimate the $\eta_c' \to \psi\gamma$ width by using the $\omega \to \pi\gamma$ rate to fix the coupling constant, assumed to be U(4) invariant. While the result, $\Gamma(\eta_c' \to \psi\gamma) \approx 20$ MeV, is likely too large, it agrees with the experimental result that η_c' has not been seen to decay into hadrons. The $V'P\gamma$ coupling constant can be fixed by assuming that $\eta_c' \to \psi\gamma$ is the dominant decay mode of the η_c' and using the observation[12] $R(\psi' \to \eta_c'\gamma)R(\eta_c' \to \psi\gamma) \approx 1\%$. This gives $g_{V'P\gamma}/g_{VP\gamma} \approx 1/30$. The relation $g_{VPP}/g_{VP\gamma} = g_{V'PP}/g_{V'P\gamma}$ suggested by anomalies implies that the $\rho'(1600) \to \pi\pi$ width should be about $\frac{1}{2}$ MeV. At present, there is no fixed value for this controversial width, except that it is small.[19]

In summary, the mixing of the two pseudoscalar hexadecaplets in SU(4) considerably improves the agreement of their masses and eigenvectors with experiment. It is found that the $\psi, \psi' \to \eta_c\gamma$ rates are easily suppressed in such a scheme. However, we emphasize that the calculations are rough in the sense that no attempt has been made to incorporate U(4) or SU(4) symmetry breaking in the coupling constants.[21] Detailed calculation of the rates involving $(c\bar{c})$ leakage will be given in a later publication.

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