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Some electromagnetic properties of charmed mesons*

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We calculate the D^+-D^0 electromagnetic mass difference assuming pole dominance of the form factors. The tadpole, Born-term, and first-intermediate-state contributions, when treated within SU(4) symmetry, imply that $m_{D^+} - m_{D^0} = 12.2$ MeV. Since, in exact SU(4), the calculated charmed-meson production cross section in e^+e^- annihilation saturates the observed total cross section, contrary to experiment, a simple SU(4)-symmetry-breaking scheme has been introduced. Using this scheme it is estimated that in the c.m. energy range 4.2–4.6 GeV, the total charmed-meson production cross section ($D\bar{D}$, $D\bar{D}^* + \bar{D}D^*$, $D^*\bar{D}^*$, $F\bar{F}$, $F\bar{F}^* + \bar{F}F^*$, $F^*\bar{F}^*$) rises from 3.8 to 6.9 nb. Also, the estimate of the D^+-D^0 mass difference is lowered to 5.8 MeV.

Recent experimental data[1,2] have suggested a candidate for the D meson, a charmed pseudoscalar meson with isospin $\frac{1}{2}$, at 1.86 GeV. The mass spectrum of the particles recoiling against the D shows peaks at 2.0 and 2.15 GeV, which in turn may be candidates for the vector meson D^*, or the mixed[3,4] axial-vector meson D_A, both expected in this region.

To explain the suppression of charged D's in the experiment, De Rújula, Georgi, and Glashow argued[5] that the electromagnetic mass splitting of the D's had to be of the order 10 MeV, with the $D^\pm$ heavier. Splitting of this order is indicated by experiment, with the D^0 having a mass[1] of 1865 $\pm$15 MeV and $D^\pm$ having[2] 1876$\pm$15 MeV. Estimates from the charmonium picture[5-9] range from 1 to 15 MeV, in rough agreement with experiment.

If one takes the naive SU(4) approach of simply adding a scalar tadpole[10] which transforms like the third component of isospin, one finds[11,12] $m_{K^0}{}^2 - m_{K^+}{}^2 = m_{D^+}{}^2 - m_{D^0}{}^2$ (or $\Delta m_D = 1.1$ MeV) and no pion mass splitting. This is certainly not enough to explain the properties of the D's. We will show here that a complete analysis including the electromagnetic corrections to the meson propagator does yield a much larger D^+-D^0 splitting. Furthermore, the electromagnetic form factors we use to calculate the self-masses determine the production cross sections for pairs of charmed particles in e^+e^- annihilation. We first use exact SU(4) symmetry, which very likely overestimates the mass difference. We then repeat the calculation using a simple-SU(4)-breaking scheme, which we feel gives a more accurate estimate.

We begin by calculating the Born-term contribution which is represented by the process $D \to D+\gamma$ (virtual)$\to D$. Using the SU(4) generalization of Socolow's[13] mixed propagator, we dominate the electromagnetic form factor by the lowest-lying 16-plet of vector mesons,

$$F(DD,q^2)=\frac{1}{2}\left[\pm\frac{m_\rho{}^2}{m_\rho{}^2-q^2}-\frac{\sin\theta}{3}\,(\sin\theta+\sqrt{2}\cos\theta)\frac{m_\omega{}^2}{m_\omega{}^2-q^2}-\frac{\cos\theta}{3}\,(\cos\theta-\sqrt{2}\sin\theta)\frac{m_\phi{}^2}{m_\phi{}^2-q^2}+\frac{4}{3}\frac{m_\psi{}^2}{m_\psi{}^2-q^2}\right],\quad(1)$$

where θ is the vector mixing angle and the upper (lower) sign refers to the D^+ (D^0). In a previous SU(4) analysis of meson decays,[3] we found $\theta=37.3°$; the other two SU(4) mixing angles were so close to their ideal values that they have been set equal to them here. Since calculations using extended vector-meson dominance have shown[14] that the coupling of higher-mass multiplets is not substantial in SU(3), we have not included them in our expression for the form factor. Equation (1) can be substituted into the expression[15] for the Born-term contribution to the self-mass dm,

$$dm=\frac{i\alpha}{8\pi^3 m}\int d^4q\,\frac{3q^2-4q\cdot p-4m^2}{q^2(q^2-2q\cdot p)}\,[F(PP,q^2)]^2,\quad(2)$$

where p and m are the pseudoscalar meson's (P) momentum and mass, respectively. Substituting (1) into (2), the expression for the mass difference becomes

$$(\Delta m_D)_{\rm Born}=\frac{m_D\alpha c_\rho}{16\pi}\left\{-\frac{\sin\theta}{3}(\sin\theta+\sqrt{2}\cos\theta)\frac{c_\omega}{c_\omega-c_\rho}[f(c_\omega)-f(c_\rho)]\right.$$
$$\left.-\frac{\cos\theta}{3}(\cos\theta-\sqrt{2}\sin\theta)\frac{c_\phi}{c_\phi-c_\rho}[f(c_\phi)-f(c_\rho)]+\frac{4}{3}\frac{c_\psi}{c_\psi-c_\rho}[f(c_\psi)-f(c_\rho)]\right\},\quad(3)$$

where $c_i \equiv m_i^2/m_D^2$, and [13]

$$f(c)=c\ln c-(c-4)^2\int_0^1 dx(x^2-cx+c)^{-1}. \tag{4}$$

This gives $(\Delta m_D)_{\rm Born}=2.91$ MeV, and analogous calculations for the K yield $(\Delta m_K)_{\rm Born}=2.19$ MeV.

We can also estimate the first-intermediate-state contribution, $D\to D^*+\gamma(\text{virtual})\to D$. The self-mass in this case is given by[13]

$$dm=g_{PV}^2\frac{ie^2}{(2\pi)^4 m}\int d^4q\,\frac{[(q\cdot p)^2-q^2m^2][F(PV,q^2)]^2}{q^2(q^2-2q\cdot p+m^2-M^2)}, \tag{5}$$

where M is the mass of the intermediate vector meson, which we take to be 2.0 GeV. The $PV\gamma$ coupling constant g_{PV} was found in Ref. 3 to have the value 2.59 GeV^{-1} [in terms of the quantities used in Eq. (16) of Ref. 3, $g_{PV}=Ng/8\pi^2F_\pi$]. The form factor is again dominated by poles:

$$F(DD^*,q^2)=\frac{1}{2}\left[\pm\frac{m_\rho^2}{m_\rho^2-q^2}-\frac{\sin\theta}{3}(\sin\theta+\sqrt{2}\cos\theta)\frac{m_\omega^2}{m_\omega^2-q^2}-\frac{\cos\theta}{3}(\cos\theta-\sqrt{2}\sin\theta)\frac{m_\phi^2}{m_\phi^2-q^2}+\frac{4}{3}\frac{m_\psi^2}{m_\psi^2-q^2}\right]. \tag{6}$$

Substituting this into (5), we get

$$(\Delta m_D)_{D^*}=\frac{m_D\alpha m_\rho^2 g_{PV}^2}{48\pi}\Big\{\sin\theta(\sin\theta+\sqrt{2}\cos\theta)\frac{c_\omega}{c_\omega-c_\rho}[U(b,c_\omega)-U(b,c_\rho)]$$
$$+\cos\theta(\cos\theta-\sqrt{2}\sin\theta)\frac{c_\phi}{c_\phi-c_\rho}[U(b,c_\phi)-U(b,c_\rho)]+\frac{4c_\psi}{c_\psi-c_\rho}[U(b,c_\psi)-U(b,c_\rho)]\Big\}, \tag{7}$$

where

$$b\equiv m_{D^*}^2/m_D^2-1, \tag{8}$$

$$2cU(b,c)=2c^2+(b-c)[6c-(b-c)^2]\ln[(1+b)/c]+2b^3\ln(1+b^{-1})+[4c-(b-c)^2]^2w(b,c), \tag{9}$$

and

$$w(b,c)=\int_0^1 dx(x^2+bx-cx+c)^{-1}. \tag{10}$$

This gives a contribution to Δm_D of 7.59 MeV, and the analogous result for Δm_K is 0.26 MeV.

The tadpole contribution is then fixed from the observed $\Delta m_K=-3.99$ MeV. The value so obtained (1.71 MeV) is about 50% larger than that found[13] from the proton-neutron mass difference. The total for the D meson is then $\Delta m_D=12.2$ MeV.

We can use these expressions for the form factors, and their analogs for the F mesons, to determine the production cross sections of charmed mesons in e^+e^- annihilation.[16] Assuming unpolarized beams, the cross section for the production of two pseudoscalars ($P\overline{P}$) at c.m. energy squared s is given by

$$\sigma(e^+e^-\to P\overline{P})=\frac{\pi\alpha^2}{3s}\left(1-\frac{4m_P^2}{s}\right)^{3/2}|F(PP,s)|^2. \tag{11}$$

The numerical evaluation of this expression yields the results shown in Fig. 1. We see that in the c.m. energy range 4.2–4.6 GeV $\sigma(e^+e^-\to D\overline{D}$, all charges) is approximately 0.1 nb, while $\sigma(e^+e^-\to F^+F^-)$ is slightly less than half this value. These estimates are in good agreement with experiment, insofar as neither has yet been seen. The suppression of production of F's relative to D's drops asymptotically to a ratio of 1:2 (F:D, all charges).

Similarly, the cross section for the production of a vector-pseudoscalar pair is given by

$$\sigma(e^+e^-\to P\overline{V})=\sigma(e^+e^-\to\overline{P}V)=\frac{\pi\alpha^2}{6}\left[1-\frac{(m_P+m_V)^2}{s}\right]^{3/2}\times\left[1-\frac{(m_P-m_V)^2}{s}\right]^{3/2}\times g_{PV}^2|F(PV,s)|^2. \tag{12}$$

Experiment demands that production of $D\overline{D}^*+\overline{D}D^*$ be considerably larger than $D\overline{D}$, and this indeed is what we find (see Fig. 1). Again, in the energy range 4.2 to 4.6 GeV, $\sigma(e^+e^-\to D\overline{D}^*+\overline{D}D^*$, all charges) rises from 11.0 nb to 14.4 nb, while $\sigma(e^+e^-\to F\overline{F}^*+\overline{F}F^*)$ rises from 2.9 nb to 5.3 nb because of threshold effects. Of course, at higher energies the cross sections will fall off as $1/s^2$.

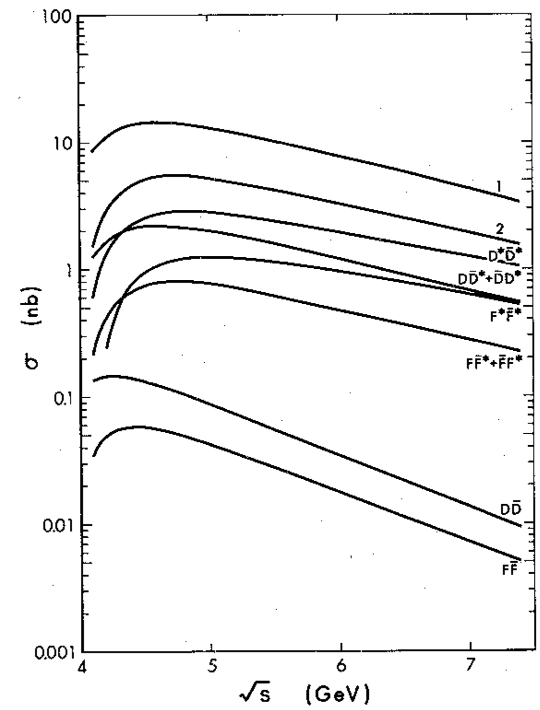

FIG. 1. Predicted cross sections for the production of specific charmed mesons in e^+e^- annihilation is shown as a function of c.m. energy. The curves labeled 1 and 2 are the predictions for $D\bar{D}^*+\bar{D}D^*$ and $F\bar{F}^*+\bar{F}F^*$, respectively, without the mass suppression in the coupling constants.

Asymptotically, the ratio of FF^*:DD^* will be 1:2. We also note that

$$\sigma(e^+e^-\to DD)/\sigma(e^+e^-\to DD^*)\approx 0.3/s \quad (s \text{ in GeV}^2)\,.$$

These estimates for PV production are probably too high, in that the total cross section for e^+e^- annihilation averages[17] 27 ± 3 nb in the c.m. energy region 3.9–4.6 GeV, falling to 18 ± 2 nb at 4.8 GeV. If our estimates are correct, then almost all of the total cross section would be due to the production of charmed mesons. More likely, there is SU(4) breaking in the coupling constant, as appears to be required to explain the suppression of $\psi\to\eta_c(2.8)\gamma$ and $\psi\to e^+e^-$.

One way of breaking SU(4), based on simple dimensional arguments, would be to write the $PV\gamma$ coupling constant as $g_{PV}=f_{PV}/m_V$, where f_{PV} is a dimensionless SU(4)-invariant constant and m_V is the mass of the *produced* vector meson. [We have chosen to introduce SU(4) breaking only in the coupling constant and not in the form factor. This is suggested by the strong-anomaly framework.[3]] Since the phase space for the $\psi\to\eta_c\gamma$ rate cannot be determined until the η_c mass is accurately known, our only experimental guide is the suppression observed in $\psi\to e^+e^-$, where SU(4) results are down by a factor of 2 in the amplitude. Because this is a relatively small decrease, we choose a low power of the vector mass to break SU(4).[18] We normalize f_{PV} to the ω mass, so that $f_{PV}=2.03$. This suppresses the e^+e^- cross sections by about a factor of 6 as is evident from Fig. 1. The average values for total cross sections in the c.m. energy range 4.2 to 4.6 GeV are then 2.0 nb for $(D\bar{D}^*+\bar{D}D^*)$ and 0.6 nb for $(F\bar{F}^*+\bar{F}F^*)$. The ratio $\sigma(e^+e^-\to DD^*)$:$\sigma(e^+e^-\to DD)$ declines to about 13:1.

To estimate $D^*\bar{D}^*$ and $F^*\bar{F}^*$ production, we have employed a U(4) Yang-Mills–type VVV coupling[3,19]

$$\mathcal{L}^{VVV}=-gf_{ijk}\partial^\mu\phi_i^\nu\phi_\mu^j\phi_\nu^k\,, \tag{13}$$

where the 16 vector mesons ϕ_i^μ form a U(4) Yang-Mills field. The production cross section is then given by

$$\sigma(e^+e^-\to V\bar{V})=\frac{\pi\alpha^2}{12m_V{}^4s}\,(s^2+20sm_V{}^2+12m_V{}^4)\times\left(1-\frac{4m_V{}^2}{s}\right)^{3/2}|F(VV,s)|^2\,, \tag{14}$$

where the vector-dominated form factor $F(D^*D^*,q^2)$ is given by (1). The resulting vector-meson production cross sections are shown in Fig. 1. In the c.m. energy region 4.2–4.6 GeV, $\sigma(e^+e^-\to D^*\bar{D}^*)$ ranges from 1.3 nb to 2.7 nb, while $\sigma(e^+e^-\to F^*\bar{F}^*)$ ranges from 0.2 nb to 1.1 nb. According to (14), $\sigma(e^+e^-\to D^*\bar{D}^*)$ and $\sigma(e^+e^-\to F^*\bar{F}^*)\propto 1/s$ asymptotically, and we find that these channels contribute ~0.5 to R at high energies.

We have used the DD^* production cross section obtained with the mass-suppressed $PV\gamma$ coupling constant to obtain the estimate for $\sigma(e^+e^-\to$ charmed PP, PV, and $VV)$ shown in Fig. 2. We also show in Fig. 2 the trend of the total hadronic cross section.[20] We re-emphasize that without this mass suppression, the experimental value for the total hadronic cross section would be almost saturated by charmed-particle production. The only comparison with experiment available is the product of the total cross section with the branching ratio of D to $K n\pi$ final states,[1] which indicates that the charm production cross section has an average value greater than 0.9 nb for $3.9\le\sqrt{s}\le4.6$ GeV.

Because of the mass-suppressed $PV\gamma$ coupling constant, the D^* contribution to the D mass difference decreases to 1.16 MeV. This results in a D mass splitting of 5.8 MeV. Because the mass difference is closely connected with production cross sections in our approach, we can infer that this lower value for Δm_D is more reliable than the value 12.2 MeV obtained without SU(4) breaking.

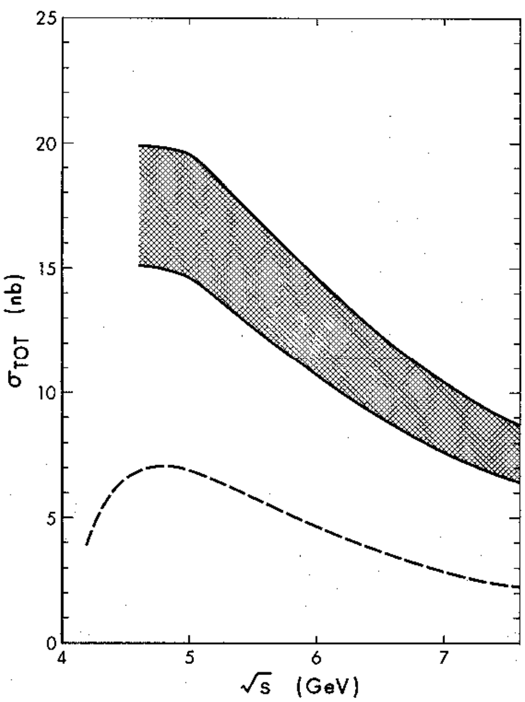

FIG. 2. Predicted $\sigma(e^+e^- \rightarrow$ charmed PP, PV, and VV) as a function of c.m. energy (dashed curve). The shaded area indicates the trend of the total hadronic cross section (within a standard deviation) beyond the resonance region.

If the D^* contribution were completely omitted, the mass difference would be lowered only 1.2 MeV further. Accurate measurements of charmed-meson production would allow us to refine our calculation of the D mass difference.

It seems likely that the structure in $e^+e^- \rightarrow$ hadrons near 4.1 and 4.4 GeV is due to the formation of resonances which decay predominantly into charmed particles. Therefore, our results for charmed-particle production should be modified somewhat for c.m. energies near the resonance structure. However, taking the 4.4-GeV resonance as an example, we find that at 4.6 GeV its contribution to charmed-particle production is negligible compared to the contribution from $\psi(3.1)$. This is because the coupling of the $\psi(4.4)$ to the photon [from $\psi(4.4) \rightarrow e^+e^-$] and to the pseudoscalar multiplet [from a strong-anomaly analysis[3] of $\psi(4.4) \rightarrow$ hadrons] is considerably smaller than that of $\psi(3.1)$.

It is amusing to note that if our estimate of $\sigma(e^+e^- \rightarrow$ charmed PP, PV, and VV) is appropriately scaled, it corresponds remarkably well with the energy dependence of the μe-event cross section of Perl *et al.*[21] Aside from the kinematical cuts, which we have not folded in, this scale factor corresponds to the product of the branching ratios of the charmed mesons into μ, e and particles which escape detection. However, recent estimates[22] of these branching ratios give a scale factor which is several orders of magnitude below what we require to reproduce Perl's results.

Finally, we expect $\Delta m_{D^*} \approx \Delta m_D$, since the tadpole term and the form factors for the Born and D intermediate-state contributions to γD^* scattering are similar to the corresponding quantities for γD scattering.[23]

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