PHYSICAL REVIEW C VOLUME 20, NUMBER 5 NOVEMBER 1979

Direct knockout model for nuclear fragmentation

David H. Boal
Theoretical Science Institute, Department of Chemistry, Simon Fraser University, Burnaby, British Columbia V5A 1S6, Canada

R. M. Woloshyn
TRIUMF, Vancouver V6T 1W5, Canada
(Received 25 June 1979)

A direct knockout model is developed for intermediate energy inclusive nuclear reactions involving the emission of light fragments in the backward hemisphere. In particular, the model is applied to the $A(p,\alpha)X$ reactions where $A={}^9$Be, ^{27}Al, and Ag. Predictions are made for the $(p,p\alpha)$ reaction with the α particle scattered into the backward hemisphere.

[NUCLEAR REACTIONS Direct knockout model; (p,α) inclusive reactions at intermediate energy; $(p,p\alpha)$ predictions.]

I. INTRODUCTION

Over the past several years, data on the proton induced fragmentation of nuclei at intermediate energy have become available.[1] These data show a clear break in the inclusive differential cross section $d^2\sigma/d\Omega_q dE_q$ of the observed light fragment as a function of T_q, kinetic energy of the emitted fragment. For fragments with between 4 and 12 nucleons, below the transition region of $T_q\sim 30$ MeV the data can be successfully explained by conventional evaporation models. Above this region, the slope of the differential cross section is significantly different from what one would expect from an evaporation model employing physically reasonable level densities and excitation energy distributions. We propose that this "nonevaporative" region can be understood in terms of a mechanism which involves the single scattering[2] of a proton from a transient cluster in the nucleus, the result of the collision being to eject the cluster.

In Sec. II of this paper, we will discuss the question of the validity of the quasi-two-body scaling[3] (QTBS) approach to this problem, and outline the single scattering model used here. Section III contains our results for the Ag$(p,\alpha)X$ reaction, as well as predictions for a possible $(p,p\alpha)$ experiment. Our results are discussed and conclusions presented in Sec. IV. Although data are available for fragments from ^{3}He to ^{12}C with a silver target, we will concentrate our attention exclusively on ^{4}He, for reasons to be outlined below.

II. THE DIRECT KNOCKOUT MODEL AND QTBS

The single scattering model has been quite successful in explaining the high energy inclusive spectra of protons produced in the backward hemisphere.[2,4] It has also been successfully applied to proton emission in high energy heavy ion reactions.[5] Although the single scattering model has theoretical shortcomings,[6] it does provide a useful basis for phenomenological analysis and it is natural to extend it to inclusive reactions in which composite fragments are emitted.

In the single scattering picture an incident proton is scattered elastically from the observed secondary particle (a nucleon or composite fragment), the remainder of the nucleus being a spectator. The closure approximation is usually made so that effectively the final state consists of three particles: the incident proton which is scattered largely in the forward direction, the emitted particle, and the recoiling nucleus approximated by a single state with some average excitation energy. Our kinematic labels for these particles are given in Fig. 1.

The expresssion for the differential cross section is[7]

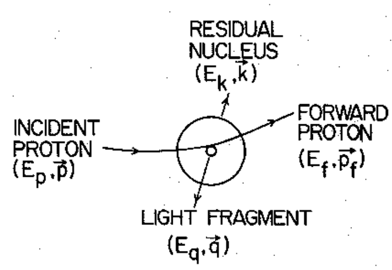

FIG. 1. Direct knockout model notation; the energy and momenta labels of the three particles in the final state are: measured fragment $(E_q, \vec{q})$, forward proton $(E_f, \vec{p}_f)$, and residual nucleus $(\mathcal{E}_k, \vec{k})$. The average excitation energy of the residual nucleus is denoted as $\bar{\mathcal{E}}^*$.

$$\frac{d^2\sigma}{d\Omega_q dE_q} = \frac{qC}{2^5(2\pi)^5 M_A p\,|\vec{p}-\vec{q}|}\iint \frac{k\,dk\,d\phi_k}{\mathcal{E}_k} F(k)\,|T|^2\,, \tag{1}$$

where ϕ_k is the angle between $\vec{p}\times\vec{q}$ and $\vec{k}\times(\vec{p}-\vec{q})$. The quantity T is the proton plus observed particle elastic scattering amplitude and $F(k)$ is some effective structure function. Within the single scattering model $F(k)$ gives the probability that the ejected particle has momentum k in the nucleus. The normalization constant C can be interpreted by use of the relation

$$\frac{Cm_q}{(2\pi)^2 M_A}\int \frac{F(k)k^2 dk}{\mathcal{E}_k} = n_{\rm eff}\,, \tag{2}$$

where m_q is the mass of the observed particle and $n_{\rm eff}$ is the effective number of target particles seen by the incoming proton. We note that for inclusive proton data our analysis indicates that the effective number of target protons equals Z to a good approximation.

Frankel[3] proposed that the expression on the right hand side of (1) could be rewritten as

$$\frac{d^2\sigma}{d\Omega_q dE_q} = \frac{qE_q}{|\vec{p}-\vec{q}|} G(k_{\min}) f(p,q)\,, \tag{3}$$

where f is a slowly varying function of p and q and G is a function only of the nuclear recoil momentum k, when k and p_f are collinear. This is the quasi-two-body scaling (QTBS) hypothesis, and should be distinguished from the single scattering model of Eq. (1). QTBS can be partially derived from (1) when $|T|^2$ is a much more slowly varying function of k than $F(k)$. This is roughly true for backward inclusive proton scattering at intermediate energies, where we would expect a steep falloff in $F(k)$ because of the large internal momenta required for the nuclear protons. In essence, QTBS can be treated as an approximation to the single scattering model where the $\vec{k}\,\|\,\vec{p}_f$ configuration dominates the cross section. QTBS has enjoyed reasonable success in describing the available backward inclusive proton spectra on a wide variety of targets at intermediate energy.

As a first attempt at the fragmentation problem, we try the QTBS approach. We show in Fig. 2 the function $G(k_{\min})$ for 300 MeV protons on Ag, with the α particles being observed at 90° and 160°. The ranges of $k_{\min}$ covered at the two angles do not overlap in this experiment, and it is clear from the figure that the factor of 50 in the normalization of the 90° vs 160° data cannot at all be explained as due to the slowly varying function $f(p,q)$. Hence, QTBS is not an acceptable description of the (p,α) reaction, although it gives us a hint that the single scattering model might be, as the scaling function $G(k_{\min})$ is of the form e^{-k/k_0} for both

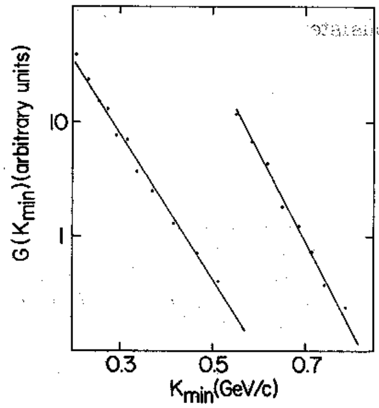

FIG. 2. The function $G(k_{\min})$ for the Ag(p,α) reaction with incident proton energy of 300 MeV.

data sets.

To proceed with the direct knockout model, we need either a model for $F(k)$ or some phenomenological way of isolating it so that it can be extracted from experiment. In a recent paper, Zabolitzky and Ey[8] have looked at the effect of correlations on the ground state momentum distributions of ^{4}He and ^{16}O. They find that the correlations modify the single particle contribution to the high momentum part of the distribution considerably, and obtain a function grossly of the form e^{-k/k_0} with $k_0 \sim 150$ MeV. This falloff is much less steep than what one expects without correlations, for which $k_0 \sim 70$ MeV.

To obtain a phenomenological expression for $F(k)$, we look at the expression for the differential cross section at 180°. Then, in Eq. (1), the ϕ_k integration is simply a rotation around the beam axis, and

$$\frac{d^2\sigma}{d\Omega_q dE_q} = \frac{qC}{2^6(2\pi)^4 M_A M_{A'}\, p\,|\vec{p}-\vec{q}|}\int F(k)\,|T|^2 dk^2\,, \tag{4}$$

where we have approximated $\mathcal{E}_k$ by $M_{A'}$ since the recoil and excitation energies are small compared to the mass of the residual nucleus.

Now, as we will show in Sec. III, $|T|^2$ is a rapidly varying function of k, and has the approximate form $\exp-\lambda k^2$ where $\lambda \simeq 13$ GeV^{-2}. If $F(k)$ is not such a rapid function of k, then the integral can be approximated by

$$\lambda^{-1} F(k_{\min})\,|T(k_{\min})|^2\,. \tag{5}$$

Hence, we can extract $F(k)$ directly from the 180° data. Unfortunately, the TRIUMF experiment did not measure the 180° alphas, and so we must content ourselves with extrapolating the 160° data which were obtained. We find that, indeed, $F(k)$ is well represented by $\exp(-k/k_0)$ with $k_0 \sim 75$ MeV. As a consistency check, we observe that such a functional form is more slowly varying than $|T|^2$ over the allowed kinematical range.

Now that we have $F(k)$ explicitly, we can proceed with the predictions of the direct knockout model.

III. PREDICTIONS FOR (p,α) AND $(p,p\alpha)$

The last ingredient for the calculation is the $p+\alpha$ elastic scattering amplitude. This is obtained from a polynomial fit to the parametrized cross section

$$\frac{d\sigma}{dt} = \left(\frac{d\sigma}{dt}\right)_{t=0} e^{bt}, \tag{6}$$

where t is the four-momentum transfer squared. To be consistent with our normalization for the single scattering expression (1), we ignore spin and write

$$\frac{d\sigma}{dt} = \frac{1}{64\pi s(p_{\mathrm{c.m.}})^2}|T|^2, \tag{7}$$

where s is the total center-of-mass energy squared and $p_{\mathrm{c.m.}}$ is the center-of-mass momentum. Our fit to the data in the 250–600 MeV proton kinetic energy range,[9] is

$$\ln(|T|_{t=0}^2) = -15.828 + 7.898\ln T_p - 0.481(\ln T_p)^2, \tag{8}$$

$$b = 4.863\,T_p^{0.259}\,\mathrm{GeV}^{-2}, \tag{9}$$

where T_p is the lab energy of the incident proton in MeV, and $|T|_{t=0}^2$ is in mb – GeV^2. We are then left with two parameters, C and k_0. These are fixed by fitting the backward (p,α) data at 210, 300, and 480 MeV incident proton energy. A comparison of our fits with the data are shown in Figs. 3 and 4. We find that $k_0 \simeq 78$ MeV and

$$N \equiv \frac{C}{2^5(2\pi)^5 M_A M_{A'}} \tag{10}$$

has the value 0.20 GeV^{-4}. From this value of N, the effective number of α particles seen by the incoming proton can be calculated by means of Eq. (2):

$$n_{\mathrm{eff}} \simeq (8\pi k_0)^3 M_\alpha N \tag{11}$$

which gives 5.6 for the effective number of α's.

Similar calculations have been done for (p,α) data[10] taken on ^{9}Be and ^{27}Al with 500 MeV protons (Figs. 5 and 6), the α particle being detected at 120°. The same slope parameter k_0 is found for these reactions as well, and the effective number of alphas is found to be ~0.3 for ^{9}Be and ~1.5 for ^{27}Al. Owing to the error on the absolute normalization of the data alone, there is at least a 25% error associated with these numbers.

Since the two parameters of the model have been determined by these fits, the amplitude for the three-body final state is now completely defined in this model. This means that the differential cross section for either the recoil nucleus or the forward proton in coincidence with the alpha can be predicted.

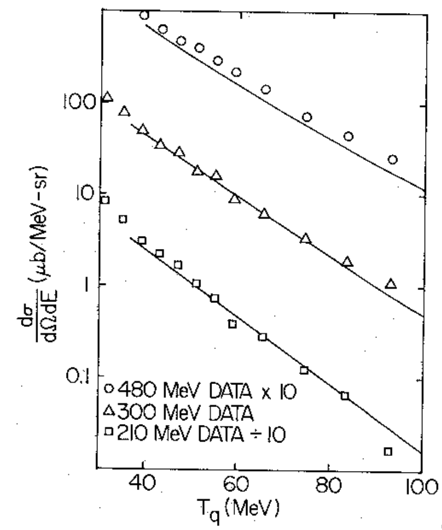

FIG. 3. Comparison of the single scattering model predictions for Ag$(p,\alpha)X$ with experiment for the α observed at 90°.

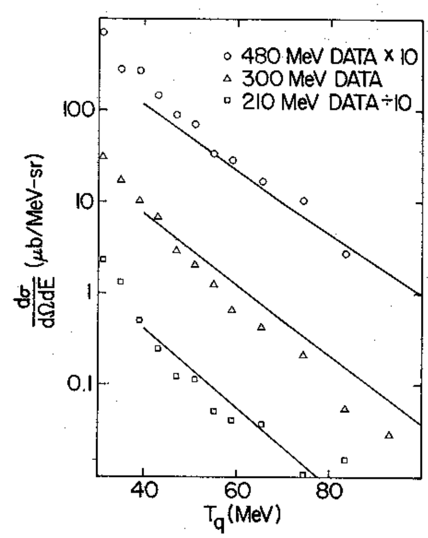

FIG. 4. Comparison of the single scattering model predictions for Ag$(p,\alpha)X$ with experiment for the α observed at 160°.

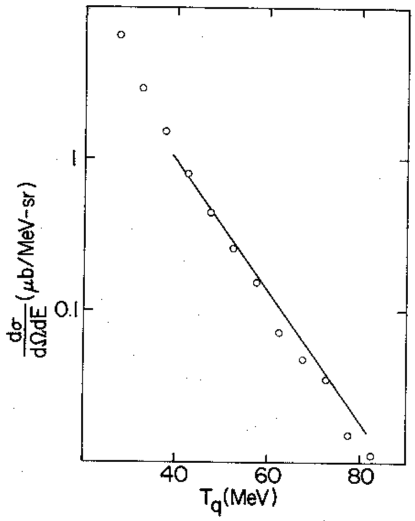

FIG. 5. Comparison of the single scattering model for ^{9}Be(p,α) X with experiment. The incident proton energy is 500 MeV and the alpha is observed at 120°.

In our calculation we have set the average excitation energy of the residual nucleus A' equal to zero. In reality the residual nucleus will carry some excitation energy. Evaporation calculations indicate that the average excitation is in the 10–20 MeV range so that a large number of residual nuclear states contribute to the inclusive cross section. This would undoubtedly complicate the interpretation of alpha plus recoil nucleus coincidence experiments.

On the other hand the binding energy per nucleon is grossly the same for all the states involved so that

$$T_p \approx T_{p_f} + T_q + \overline{\mathcal{E}}^* + T_k . \tag{12}$$

The last two terms will sum to perhaps 20 MeV, so that, for $40 \leq T_q \leq 90$ MeV, T_{p_f} will be fairly large, especially for large T_p. This relationship then says that T_{p_f} will be roughly the same, independent of $\theta_{f,\mathrm{lab}}$, the angle between the forward proton and the beam axis. Hence, a measure of T_{p_f} in the $(p,p\alpha)$ reaction should give a good measure of $\overline{\mathcal{E}}^*$.

In light of the above, we focus our attention on the $(p,p\alpha)$ reaction. We will calculate $d^3\sigma/d\Omega_q\, d\Omega_f dE_q$, integrating over $\vec{k}$ and $|\vec{p}_f|$ to get rid of the four dimensional delta function. We find

$$\frac{d^3\sigma}{d\Omega_f\, d\Omega_q\, dE_q} = N\frac{q}{p}\,\frac{M_{A'}p_f^2}{[p_f(E_p+M_A-E_q)-|\vec{p}-\vec{q}|\,E_f\cos\theta_f]} \times F(k)\,|T|^2 , \tag{13}$$

where θ_f is the angle between $\vec{p}_f$ and $\vec{p}-\vec{q}$. As a sample calculation, we have looked at 300 MeV protons producing α particles at 90° and 160°. The differential cross section for the forward protons is shown in Figs. 7 and 8. We have chosen the kinetic energies of the alphas to be 40, 70, and 100 MeV. The angle θ_f is positive for the proton on the opposite side of the beam from the alpha, and the particles are all taken to be coplanar. As before, $\overline{\mathcal{E}}^*$ is set equal to zero.

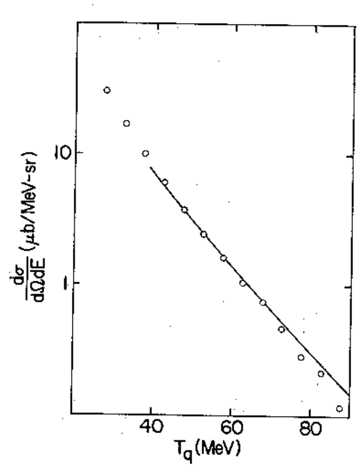

FIG. 6. Comparison of the single scattering model for ^{27}Al(p,α) X with experiment. Same conditions as with Fig. 5.

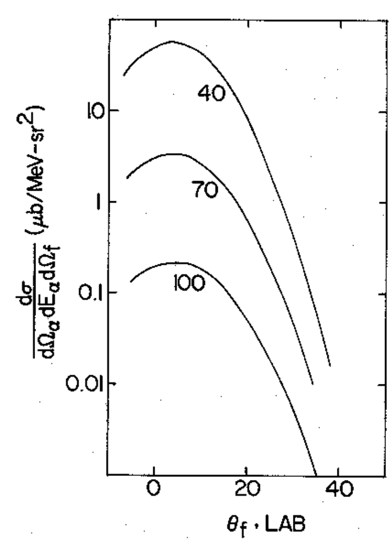

FIG. 7. Predictions of the single scattering model for Ag$(p,p\alpha)$ X with 300 MeV protons and the alpha at 160°.

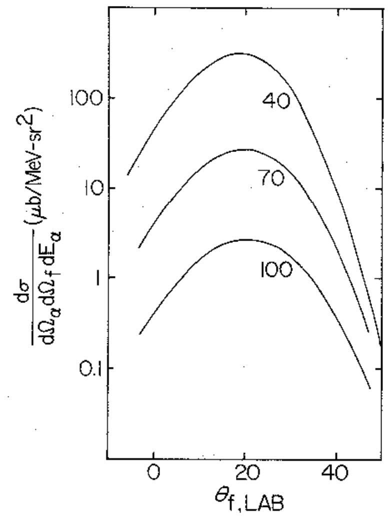

FIG. 8. Predictions of the single scattering model for Ag($p,p\alpha$)X with 300 MeV protons and the alpha at 90°.

IV. DISCUSSION AND CONCLUSION

We have shown that the inclusive (p,α) data at backward α angles are well described in terms of a single scattering mechanism. From the model amplitude cross section for the ($p,p\alpha$) reaction for α's in the backward hemisphere is predicted. While our sample calculation was for Ag($p,p\alpha$)X, we feel confident about our predictions for any target with $20 \leq A \leq 120$. Measurement of the ($p,p\alpha$) reaction should provide a reasonable test of this model, provided multiple scattering effects are not too important for the proton.

The two parameters of the model, C and k_0, are extracted by a fit to the (p,α) data (this is not a least χ^2 fit, as the computing costs would have been fairly high). Presently, we do not have any *a priori* way of determining these parameters, but some comments on their values are worth making. The value of k_0 that was obtained is within 10% if the value found for the (p,p') reaction. This is not unreasonable if the structure function $F(k) = e^{-k/k_0}$ approximately represents the probability of having a nucleon of momentum k. Then, the probability of finding N nucleons in a cluster with momentum k, would be roughly proportional to

$$(e^{-k/Nk_0})^N = e^{-k/k_0}\,. \tag{14}$$

While this argument is quite crude, it indicates that k_0 should not vary dramatically from one emitted cluster to the next.

The constant C is more of an enigma. It should really be a product of three effects:

(1) the probability of cluster formation, as discussed above,

(2) the probability that the cluster will not be rescattered after it is struck by the proton,

(3) the probability that the cluster is emitted with sufficiently small excitation energy that it will not break up after leaving the nucleus. It is unlikely that this will be important for alpha particles, but may be significant for higher mass fragments.

It is of interest to look at the A dependence of the effective number of alpha particles. We expect that rescattering effects which decrease the cross section should become more important as A increases. The fact that our n_{eff} increases monotonically with A could then be interpreted as indicating that the probability of cluster formation is a rapidly increasing function of A. We note that this behavior of n_{eff} is different from that obtained by Dollhopf *et al.*[11] in the ($\alpha,2\alpha$) reaction. There, n_{eff} was found to increase with A in the range $6 \leq A \leq 23$ and tended to be constant for higher mass targets.

It is likely that rescattering effects are more important in ($\alpha,2\alpha$) and lead to this discrepancy.

ACKNOWLEDGMENTS

We wish to express our gratitude to R.E.L. Green and R.G. Korteling of the fragment emission experiment at Simon Fraser University, and to G. A. Moss and G. Roy of the $p+\alpha$ elastic scattering experiment at the University of Alberta, for making their data available to us prior to publication. Our thanks also go to G. Igo for providing us with tabulations of the UCLA $p+\alpha$ elastic scattering data. We are indebted to R. E. L. Green, K. P. Jackson, and R. G. Korteling for measuring the ^{9}Be and ^{27}Al (p,α) cross sections for use with this model. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada.

[1] R. E. L. Green and R. G. Korteling, Phys. Rev. C 18, 311 (1978); J. R. Wu, C. C. Chang, and H. D. Holmgren, *ibid.* 19, 698 (1979); J. P. Alard *et al.*, Nuovo Cimento A30, 320 (1975); V. I. Bogatin *et al.*, Nucl. Phys. A 260, 446 (1976).

[2] R. D. Amado and R. M. Woloshyn, Phys. Rev. Lett. 36, 1435 (1976).

[3] S. Frankel, Phys. Rev. Lett. 38, 1338 (1977); Phys.

Rev. C 17, 694 (1978).
[4]S. Frankel, W. Fratt, R. M. Woloshyn, and D. Yang, Phys. Rev. C 18, 1379 (1978).
[5]R. L. Hatch and S. E. Koonin, Phys. Lett. 81B, 1 (1979).
[6]R. D. Amado and R. M. Woloshyn, Phys. Lett. 69B, 400 (1977).
[7]See D. F. Jackson, Nucl. Phys. A173, 225 (1971) for a detailed derivation.
[8]J. G. Zabolitzky and W. Ey, Phys. Lett. 76B, 527 (1978).
[9]G. A. Moss, G. Roy *et al.*, private communication; H. Courant *et al.*, Phys. Rev. C 19, 104 (1979); V. Comparat *et al.*, *ibid.* 12, 251 (1975); E. Aslanides *et al.*, Phys. Lett. 68B, 221 (1977); J. Fain *et al.*, Nucl. Phys. A262, 413 (1976).
[10]R. E. L. Green, K. P. Jackson, and R. G. Korteling, private communication. The data for ^{9}Be and ^{27}Al targets were collected specifically to check the normalization constant in the model. The experimental conditions were similar to those described in Green and Korteling, Ref. 1.
[11]W. E. Dollhopf, C. F. Pendrisat, P. Kitching, and W. C. Olsen, Nucl. Phys. A316, 350 (1979).