

## Probing inclusive production mechanisms with the $(p,2p)$ reaction

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Several models for the inclusive production of protons at backward angles are reviewed. From the model amplitudes, predictions for the  $(p,2p)$  differential cross section in which one of the protons is emitted in the backward direction are calculated. It is found that the magnitudes and angle dependence of the predicted cross sections vary significantly from one model to the next.

[NUCLEAR REACTIONS Inclusive production mechanisms at intermediate energies, effective momentum distributions, predictions for  $(p,2p)$  reaction.]

### I. INTRODUCTION

Since Frankel *et al.*<sup>1</sup> first measured the backward inclusive differential cross section of protons produced in proton-nucleus collisions at intermediate energies, there have been several models<sup>2-6</sup> put forward to explain the data. These models differ considerably in their microscopic detail, but they can each generally fit the data for emitted proton angles in the  $160^\circ$  to  $180^\circ$  range. While recent  $(p,p')$  data at more forward angles has begun to place limits on the range of validity of some of the models, it does not appear likely that further  $(p,p')$  data will be able to distinguish between them. The purpose of this paper is to consider some reactions related to  $(p,p')$ , in particular the  $(p,2p)$  reaction, in an effort to find a means of testing these models.

It will be useful to have some sort of categorization of the models for our purposes. The models usually involve some sort of primary interaction, for example elastic scattering of the incident proton off a cluster of nucleons inside the nucleus. Because the clusters or residual nuclei can carry excitation energy which may lead to their breakup, the few-body intermediate state very likely leads to a many-body final state. It is assumed in all of these models that the protons emitted in the de-excitation of the residual nucleus are of such low energy as not to contribute to the 100–400 MeV protons observed by Frankel *et al.* Hence we will categorize the reactions by the number of particles or clusters there are in the intermediate state following the primary interaction.

We see immediately that two-body intermediate states are unlikely to contribute significantly to the high energy backward events looked at by Frankel since the residual nucleus would have to be carrying off an enormous amount of excitation energy. We will therefore concentrate our attention on three- and four-body intermediate states.

By far the largest number of models fall into the three-body intermediate state category. One of the first models proposed in this group involves the incident proton elastically scattering in the forward direction off a target proton, putting the latter on the mass shell. It is this target proton which is observed in the backward direction. The residual nucleus is then simply a spectator. This model was used by Amado and Woloshyn<sup>3</sup> for describing inclusive proton production at backward angles, and by Boal and Woloshyn<sup>7</sup> for describing the inclusive production of alpha particles at backward angles.<sup>8</sup> We shall call this the direct knockout (DK) model.

A simplification of this model was proposed by Frankel<sup>2</sup>: the quasi-two-body scaling hypothesis (QTBS). In this model, the phase space integral of the transition matrix element is set equal to a product of one function varying rapidly with and depending only on the minimum momentum of the recoiling nucleus, and a second function which varies slowly with the other kinematical variables. This approximation is not unreasonable for  $pp$  scattering and QTBS successfully describes the inclusive proton data. However, the approximation fails for  $p$ -alpha scattering, and indeed QTBS has been shown to be inadequate as a description of the inclusive alpha data. Since QTBS by itself will not allow the prediction of less inclusive processes than  $(p,p')$ , it will not be considered further here.

A model in some respects similar to the DK model is the multiparticle exchange (MPE) model proposed by Weber and Miller.<sup>4</sup> Here, though, the observed proton is the spectator and the primary interaction is between the incident proton and the remaining  $A - 1$  nucleons of the target nucleus (of mass number  $A$ ). The need for detailed knowledge of the  $p + (A - 1)$  collision is avoided because of the inclusive nature of the  $(p,p')$  experiment.

The other three-body intermediate state model which we wish to discuss is the correlated cluster (CC) model of Fujita.<sup>5</sup> In some sense the CC model incorporates both the DK and MPE models in that the incident proton scatters off everything from a single proton in the nucleus to an  $(A-1)$  nucleon cluster. However, the observed proton is now assumed to be the incident proton, scattered through a very large momentum transfer.

Lastly, we take note of a four-body intermediate state model, the cluster recoil (CR) model proposed by Woloshyn.<sup>6</sup> As in the DK model, the primary interaction is  $pp$  scattering. However, whereas the residual nucleus carries off momentum  $\vec{k}$  (where the target proton has momentum  $-\vec{k}$  in the nucleus) in the DK model, in the CR model  $\vec{k}$  is carried off by a "jet" of  $n$  nucleons, the remaining  $A-1-n$  nucleons remaining roughly at rest in the lab. Although one has to model the relative probability of the nuclear proton with momentum  $-\vec{k}$  recoiling against the  $n$ -nucleon jet with momentum  $\vec{k}$  both as a function of  $k$  and  $n$ , the overall normalization is fixed by the  $(p, p')$  data.

We have outlined above four models for the  $(p, p')$  reaction (DK, MPE, CC, CR) which we feel are significantly different from one another that they should be able to be distinguished experimentally. There are other likely models of which we are aware, and doubtless others of which we are not aware, which we have excluded for brevity. In the following sections, the DK, MPE, and CC models will be reviewed in turn (concentrating most heavily on the DK model) and their  $(p, 2p)$  predictions will be discussed. Unfortunately, the normalization conventions of the various authors are not uniform, nor, of course are their parametrizations of the data which are put into the models. The approach taken in this paper will be to use the same data parametrization (e.g. for  $pp$  scattering) in every model to facilitate comparison, but to leave the normalization conventions intact as much as possible, for easier reference to the original works.

## II. DIRECT KNOCKOUT MODEL

The kinematic labels attached to each participant are shown in Fig. 1. The incident proton has an energy and momentum of  $E_i$  and  $p_i$  in the lab system, and leaves the reaction with  $E_f$  and  $p_f$ . The struck nucleon has momentum  $-\vec{k}$  before the collision, and energy and momentum  $E_q$  and  $q$  after. We will use  $E_k = \bar{\mathcal{E}}^* + (k^2 + M_A^2)^{1/2}$  (where  $M_A$  is the mass of the residual nucleus and  $\bar{\mathcal{E}}^*$  its average excitation energy) to denote the energy of the residual nucleus, and  $\epsilon_k$  to mean  $(k^2 + M_p^2)^{1/2}$ ,

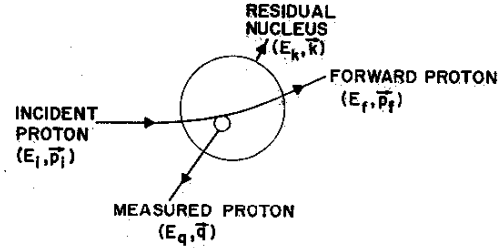


FIG. 1. Energy-momentum labels for direct knockout model.

where  $M_p$  is the proton mass.

Using the phase space normalization conventions of Bjorken and Drell,<sup>9</sup> and using the closure approximation to replace the sum over final nuclear states by a final state with an excitation energy  $\bar{\mathcal{E}}^*$ , Amado and Woloshyn<sup>3</sup> write the  $(p, p')$  differential cross section for a target of  $A$  nucleons in the form

$$\frac{d^2\sigma}{d\Omega_q dE_q} = \frac{M_p^4}{2(2\pi)^3} \frac{q}{p_i} \int \frac{d^3p_f}{E_f \epsilon_k} [n_p(k) |T_{pp}|^2 + n_n(k) |T_{pn}|^2] \times \delta(E_i + M_A - E_f - E_q - E_k), \quad (1)$$

where the probability of finding a proton with momentum  $k$  in a nucleus with  $Z$  protons,  $n_p(k)$ , is normalized by

$$2 \int \frac{M_p}{\epsilon_k} n_p(k) \frac{d^3k}{(2\pi)^3} = Z, \quad (2)$$

and similarly for the neutron momentum distribution  $n_n(k)$ . The collision between the incident proton and the off-mass-shell proton in the nucleus is assumed to be dominated by elastic scattering, the matrix element for which,  $T_{pp}$ , is normalized to

$$\frac{d\sigma}{dt} = \frac{M_p^2}{16\pi p_{lab}^2} |T_{pp}|^2, \quad (3)$$

where  $t$  is the square of the four-momentum transfer. No effort is made to include off-mass-shell effects.

The parametrization for the  $pp$  scattering cross section chosen here is to assume that the integrated cross section is constant (as a function of energy) at 24 mb,<sup>10</sup> and that  $d\sigma/dt$  has the form

$$\frac{d\sigma}{dt} = A e^{bt}. \quad (4)$$

The parameter  $b$  has the rough energy dependence over the range of interest, of

$$b = 14.39T_{lab} - 6.13 \text{ GeV}^{-2}, \quad (5)$$

where  $T_{lab}$  is the lab kinetic energy of the incident proton in GeV. At the limits of applicability of

this formula,  $b$  was restricted to be greater than  $0.7 \text{ GeV}^{-2}$  and less than  $7.0 \text{ GeV}^{-2}$ . Knowing  $b$  for a particular  $T_{1ab}$ , the integrated version of Eq. (4) can be solved for  $A$ . To determine the appropriate  $T_{1ab}$  for the reaction, the momenta of  $p_f$  and  $q$  were used to define the Mandelstam variable  $s$ , for which the corresponding lab energy was then calculated.

The parametrization of  $|T_{pn}|^2$  is somewhat more cumbersome. Over the energy range of interest,  $|T_{pn}|^2$  at  $u=0$  (backward scattering) was approximated by (in units of  $\text{mb}/\text{GeV}^2$ )

$$|T_{pn}|_{u=0}^2 = \begin{cases} -1.369 \times 10^4 T_{1ab} + 1.686 \times 10^4, & T_{1ab} < 0.7 \text{ GeV} \\ 0.709 \times 10^4, & T_{1ab} > 0.7 \text{ GeV}. \end{cases} \quad (6)$$

The parameter  $b'$  appearing in

$$\frac{d\sigma}{du} = A' e^{b'u} \quad (7)$$

was fitted with (in  $\text{GeV}^{-2}$ )

$$b' = \begin{cases} -94.34 T_{1ab} + 115.7, & T_{1ab} < 0.876 \text{ GeV} \\ 33.0, & T_{1ab} > 0.876 \text{ GeV}. \end{cases} \quad (8)$$

In general, it was found that scattering from neutrons contributed no more than a few percent of

the predicted cross section, so the parametrization chosen is not terribly crucial to the results presented here. The last ingredient required for the calculation is the form of the momentum distribution. Theoretical understanding of the high momentum components of the momentum distribution is still far from complete, but some recent work by Zabolitzky and Ey,<sup>11</sup> as well as most fits to data, point to an approximately exponential shape in the 300–900  $\text{MeV}/c$  range. Following Amado and Woloshyn, we try

$$n(k) = \frac{C}{[2 \cosh(k/2k_0)]^2}, \quad (9)$$

where  $k_0$  is treated as a parameter and  $C$  is determined by the normalization condition, Eq. (2). The same form is used for both protons and neutrons. Hence, the model contains only one free parameter,  $k_0$ .

Shown in Figs. 2–4 are the predictions of this model with  $k_0 = 88 \text{ MeV}$  for a variety of targets, the bombarding energy being 800  $\text{MeV}$ . In the original paper of Amado and Woloshyn,<sup>3</sup>  $k_0$  was fixed by an analysis of  $(e, e')$  data, and Eq. (1) was not evaluated exactly. Thus, the agreement was not as good as is shown in these figures. It is clear that the agreement is good for Li and Be,

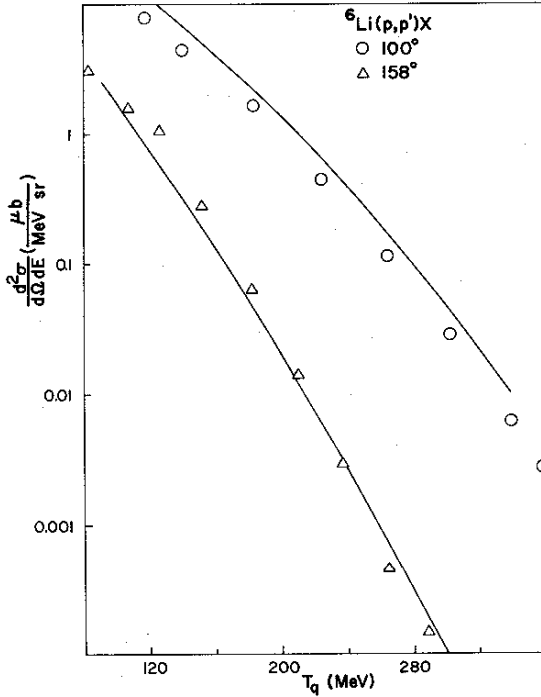


FIG. 2. Comparison of direct knockout model predictions with experiment for the reaction  ${}^6\text{Li}(p, p')X$  at incident proton energy of 800  $\text{MeV}$ , and observed proton angle of  $100^\circ$  and  $158^\circ$ . Data from Ref. 2.

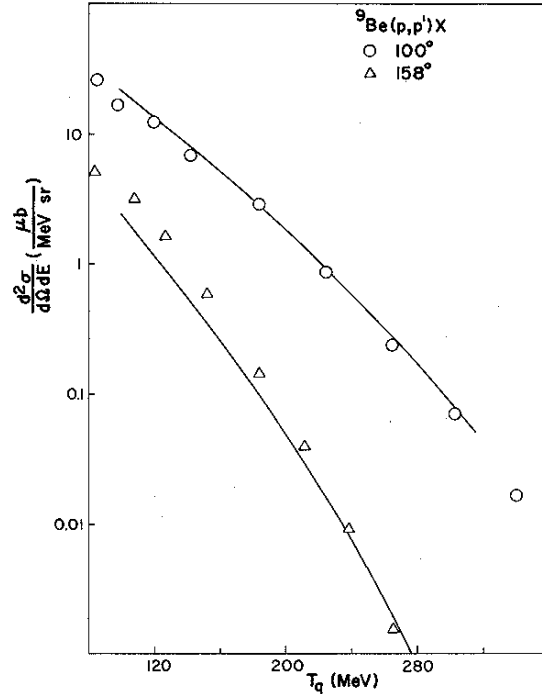


FIG. 3. Comparison of direct knockout model predictions with experiment for the reaction  ${}^9\text{Be}(p, p')X$  at incident proton energy of 800  $\text{MeV}$  and observed proton angle of  $100^\circ$  and  $158^\circ$ . Data from Ref. 2.

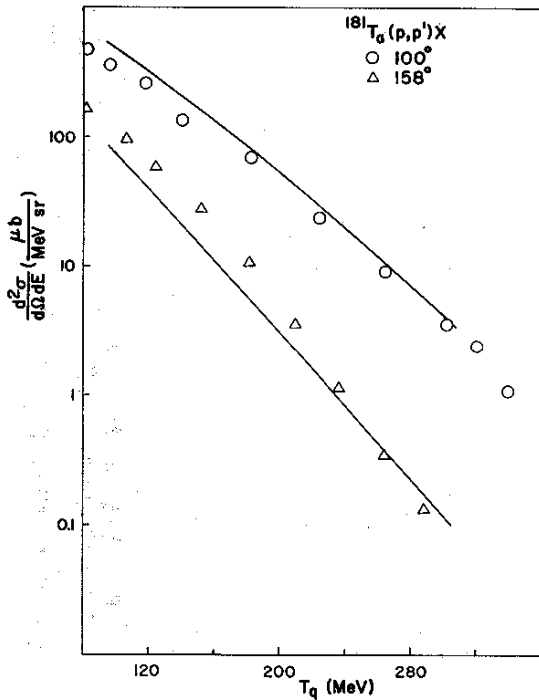


FIG. 4. Comparison of direct knockout model predictions with experiment for the reaction  $^{181}\text{Ta}(p, p')X$  at incident proton energy of 800 MeV and observed proton angle of  $100^\circ$  and  $158^\circ$ . Data from Ref. 2.

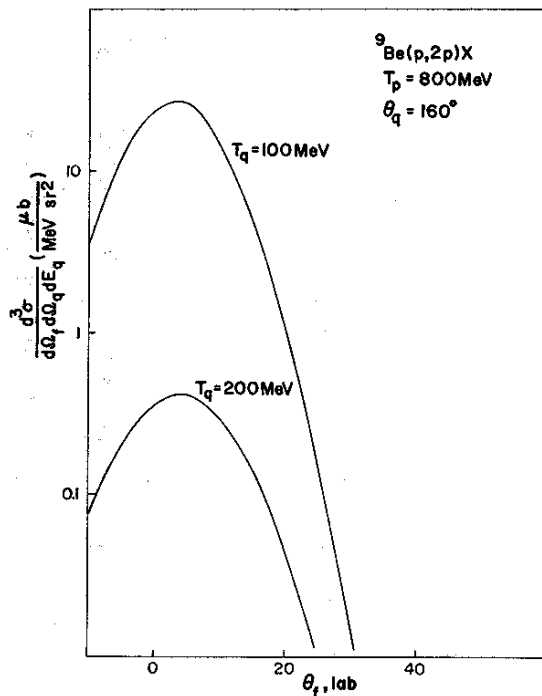


FIG. 5. DK predictions for  $^9\text{Be}(p, 2p)X$  with  $\theta_q = 160^\circ$  and 800 MeV incident proton kinetic energy.

but has begun to deteriorate slightly for Ta. Had the simple exponential been used instead of Eq. (9) for  $n(k)$ , the predicted results would have been roughly 20% lower.  $\mathcal{E}^*$  was set equal to zero for these calculations.

In this model, multiple scattering effects have been neglected, and it is important to find out just how significant they are. A recent experiment at TRIUMF<sup>12</sup> tried to address this question by using  $^4\text{He}$  as a target. While the data have not yet been finalized for publication, the preliminary results are in good agreement with the DK predictions.

To test this model further, one would have to go to a less inclusive process than  $(p, p')$ . Since the DK model assumes that the incident proton is scattered forward without large momentum transfer, one can easily calculate the DK predictions for the  $(p, 2p)$  reaction. A comparison of light vs heavy target could then be used to investigate the importance of multiple scattering.

The differential cross section for two proton coincidence reactions is given by

$$\frac{d^3\sigma}{d\Omega_f d\Omega_q dE_q} = \frac{M_p^4}{2(2\pi)^3} \frac{q}{p_i \epsilon_k} \frac{E_B}{\left[ p_f(E_k + E_f) - |\vec{p}_i - \vec{q}| E_f \cos \theta_f \right]} \times n(k) |T_{pp}|^2, \quad (10)$$

where  $\theta_f$  refers to the angle between  $\vec{p}_f$  and  $\vec{p}_i - \vec{q}$ . For this and other models,  $^9\text{Be}$  has been chosen as the target nucleus. Shown in Fig. 5 is the DK prediction for the backward proton being observed at  $160^\circ$ . As expected, the peak in the distribution of the forward proton is near  $0^\circ$  ( $\theta_{f, \text{lab}}$  is the angle between  $p_f$  and the beam axis, a positive  $\theta_{f, \text{lab}}$  being on the opposite side of the beam from  $\theta_q$ ). For  $\theta_q = 90^\circ$ , shown in Fig. 6, the peak shifts out to about  $15^\circ$ . In both of these calculations,  $\vec{p}_f$  has been chosen to lie in the plane formed by  $\vec{p}_i$  and  $\vec{q}$ . The cross section decreases as  $\vec{p}_f$  moves out of the plane. The incident proton kinetic energy in both these results is 800 MeV.

### III. MULTIPARTICLE EXCHANGE MODEL

The particle labels used for this model will be the same as those used in the previous section. In order to be able to make the  $(p, 2p)$  predictions for this model, we will have to go beyond the model of Weber and Miller<sup>4</sup> and actually construct a matrix element for the  $p$ -residual nucleus collision. Thus, the  $(p, 2p)$  predictions generated here are going to have significantly more model dependence than that proposed by Weber and Mil-

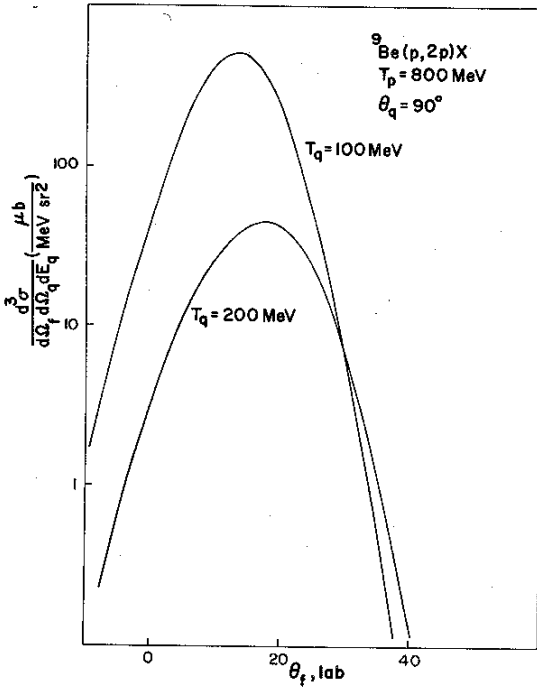


FIG. 6. DK predictions for  ${}^9\text{Be}(p, 2p)X$  with  $\theta_q = 90^\circ$  and 800 MeV incident proton kinetic energy.

ler.

The normalization for the cross section chosen here will be that of Bjorken and Drell<sup>9</sup> for spinless particles. This is the same normalization taken in our previous work on  $(p, \alpha)$  reactions. Denoting the  $p$ -residual nucleus matrix element by  $T_{pR}$ , the differential cross section for inclusive proton production is given by

$$\frac{d^2\sigma}{d\Omega_q dE_q} = \frac{C}{2^5 (2\pi)^5 M_A M_{A'} p_i} q M_{A'} F(q) \times \int \frac{d^3 p_f}{E_f E_h} |T_{pR}|^2 \delta(E_i + M_A - E_q - E_f - E_h). \quad (11)$$

The function  $F(q)$  is proportional to  $n(k)$  of the direct knockout model. The constant  $C$  will be determined by fitting the inclusive data.<sup>2</sup> Now if the kinematic limits for the  $p_f$  integration were close to those of the free  $p$ - $R$  reaction, then the integral would simply be proportional to the  $p$ - $R$  cross section. That means that for fixed  $E_i$  and  $|\vec{q}|$ , the inclusive differential cross section should vary only slowly with  $\theta_q$ . Experiment shows a fairly rapid variation as one goes to forward values of  $\theta_q$ , so the particular version of the MPE model under investigation here is probably limited to large  $\theta_q$ . To get an idea of the effect of shifting the integration limits as a function of  $\vec{q}$ , the fol-

lowing parametrization is chosen.

(i)  $F(q)$  is simply set equal to  $e^{-q/k_0}$ . As in the DK model, a value of  $k_0$  of 88 MeV adequately describes the large  $\theta_q$  data.

(ii) The total  $p$ - $R$  cross section is assumed to have the same form as given in Weber and Miller, i.e.,

$$\sigma = 86(A-1)^{2/3} \text{ mb}, \quad (12)$$

independent of energy in this region.

(iii) The differential cross section for  $p$ - $R$  collisions is assumed to have the same  $t$  dependence as Eq. (4), and  $b$  is chosen by a rough fit to the slope parameters of elastic  $p$ - $R$  scattering at intermediate energy<sup>13</sup>

$$b = 5.5A^{1.06} \text{ GeV}^{-2}. \quad (13)$$

Lastly, with this normalization, the  $p$ - $R$  differential cross section is given by

$$\frac{d\sigma}{dt} = \frac{|T_{pR}|^2}{64\pi M_R^2 p_{1ab}^2}. \quad (14)$$

While this is admittedly not the best parametrization available, it will do for the purpose of this paper.

The results for  ${}^9\text{Be}(p, p')X$  at incident proton energy of 800 MeV are shown in Fig. 7. The con-

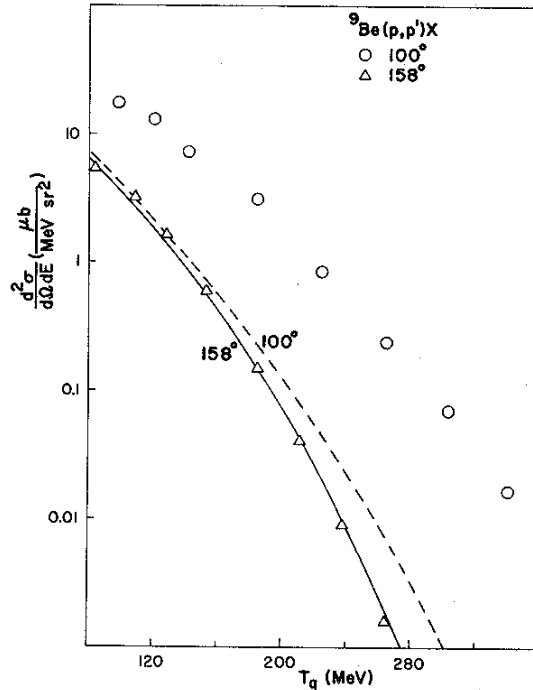


FIG. 7. Comparison of MPE model with experiment for the reaction  ${}^9\text{Be}(p, p')X$  at incident proton energy of 800 MeV and observed proton angle of  $100^\circ$  and  $158^\circ$ . Data from Ref. 2.

stant  $C$  is fixed by the  $158^\circ$  data, and one can see that the cross section predicted for  $100^\circ$  is indeed not too much different.

The  $(p, 2p)$  predictions are given by Eq. (11) with the  $|\vec{p}_f|$  integration performed to take care of energy conservation. The results for a Be target are shown in Figs. 8 and 9. In contrast to the DK model, the peak in the distributions of the MPE model does not shift significantly with  $\theta_q$ .

#### IV. CORRELATED CLUSTER MODEL

The last model which we wish to treat in detail is the correlated cluster model of Fujita.<sup>5</sup> In this model,  $p_f$  is replaced by a cluster of  $N$  nucleons of energy  $E_{cc}$ . The differential cross section then consists of  $A$  terms:

$$\frac{d^3\sigma}{d^3q} = \sum_N G_N \int \frac{1}{E_q E_{cc}} |T_{p-cc}|^2 w_N(k) \frac{d^3p_f}{(2\pi)^3} \frac{1}{(8\pi)^2 F} \times \delta(E_i + NM_p - E_q - E_{cc} - \bar{\epsilon}^*), \quad (15)$$

where

$$F = NM_p p_i, \quad (16)$$

$$G_N = \frac{A}{N} \frac{1}{A^{N-1}} \left(\frac{V_c}{V}\right)^{N-1} \xi_N, \quad (17)$$

$$w_N(k) = \left(\frac{4\pi}{NV}\right)^{3/2} e^{-k^2/N\nu}. \quad (18)$$

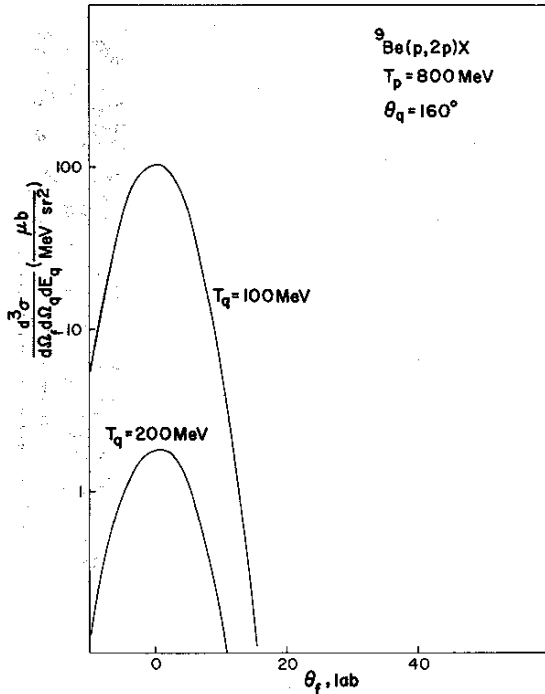


FIG. 8. MPE predictions for  ${}^9\text{Be}(p, 2p)X$  with  $\theta_q = 160^\circ$  and 800 MeV incident proton energy.

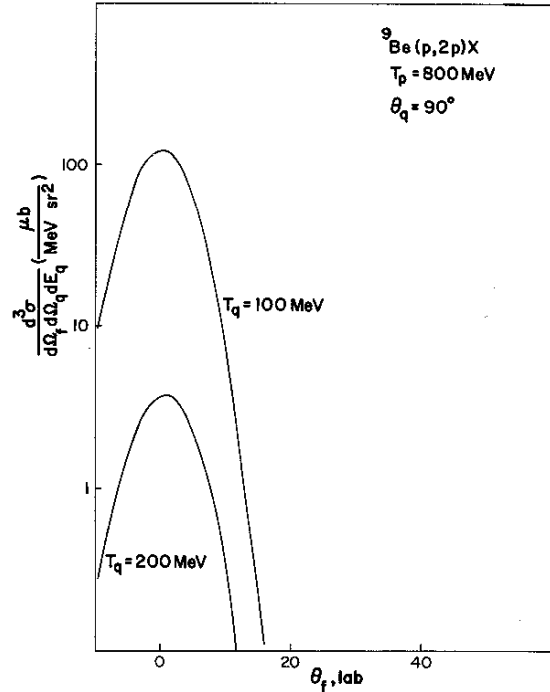


FIG. 9. MPE predictions for  ${}^9\text{Be}(p, 2p)X$  with  $\theta_q = 90^\circ$  and 800 MeV incident proton energy.

The reader is referred to the original paper of Fujita for a detailed discussion of the values of the parameters involved. The values of these parameters which Fujita found to give an acceptable fit to the inclusive data were

$$\nu = 32\,500 \text{ MeV}^2, \quad (19)$$

$$V_c/V = 0.07, \quad (20)$$

$$\xi_N \approx 1. \quad (21)$$

The  $p$ -cluster cross section was replaced by the  $pp$  cross section by Fujita,<sup>5</sup> and the amplitude normalized as in Eq. (14). The same criteria for selecting the appropriate  $s$  and  $t$  and the same  $pp$  cross section parametrization as were used in the DK calculations are used in the results discussed below.

We are interested mainly in the  $N=1$  term of Eq. (15), as the recoiling clusters in Fujita's model do not break up, by and large. Evaluation of Eq. (15) then shows that, depending on  $E_q$ , scattering off a single nucleon contributes only  $\sim 10^{-4}$  to  $10^{-5}$  of the predicted inclusive cross section. It comes as no surprise then that when the  $|\vec{p}_f|$  integration only is performed in Eq. (15) to give the  $(p, 2p)$  prediction, the results are substantially lower than those found in the DK or MPE calculations. The  ${}^9\text{Be}(p, 2p)X$  cross sections for one proton at  $160^\circ$  are shown in Fig. 10.

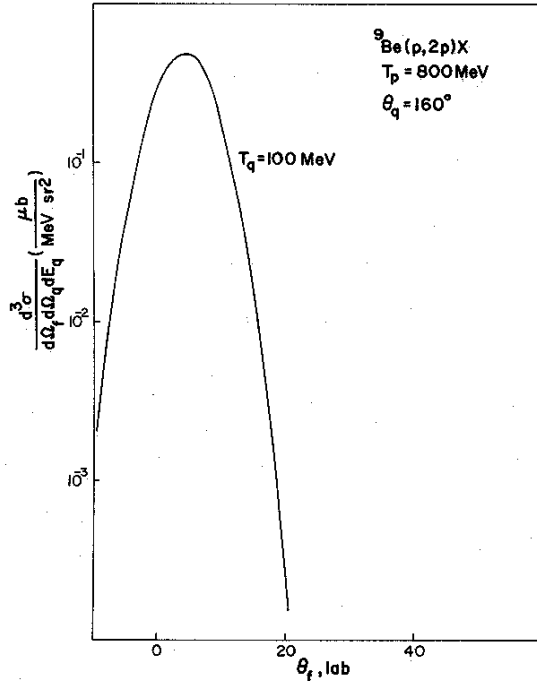


FIG. 10. CC predictions for  ${}^9\text{Be}(p,2p)X$  with  $\theta_q = 160^\circ$  and 800 MeV incident proton energy.

### V. DISCUSSION

In the above, we have calculated some  $(p,2p)$  predictions for a variety of models. Each model has its own characteristic spectrum which should allow experiment to rule at least one of them out. For example, although the peak in the cross section for the DK and MPE models is grossly the same, the peak should shift as  $\theta_q$  is varied in the DK model, but remain roughly centered at the beam axis in the MPE model. The reason that the peak is approximately stationary in the MPE model is that the amplitude assumed here for  $p$ - $R$  scattering falls off very rapidly with momentum transfer of the incident proton to the residual nucleus. If the amplitude did not in fact fall as fast with momentum transfer as has been assumed here, then the forward protons would not be so sharply peaked, although the position of the peak would still change only slowly with  $\theta_q$ . In contrasting both of these models to the CC model, the predicted cross section of the CC model is several orders of magnitude lower than the other two.

It should be pointed out that the particular differential cross section used for comparison in these calculations,  $d^3\sigma/d\Omega_f d\Omega_q dE_q$ , involves only those forward protons which result from three-body kinematics as in Fig. 1. Hence,  $T_f$  is

uniquely determined by energy and momentum conservation at a given  $\Omega_f$ ,  $\Omega_q$ ,  $T_q$ , and  $\bar{\epsilon}^*$  and is roughly  $T_f = T_q - \bar{\epsilon}^*$ . These calculations then are restricted to those forward protons with  $T_f$  greater than about  $\frac{1}{2}T_i$ . If the coincidence trigger were defined to be any forward proton at fixed  $\Omega_f$  in coincidence with a high energy proton in the backward hemisphere at fixed  $\Omega_q$  and  $E_q$  (including the low energy protons resulting from evaporation of the residual nucleus) then the predicted angular distributions would look substantially different.

This point is particularly important for any discussion of the CR model. Because the CR model was not broken down into elementary amplitudes, it was not used in the above to predict the  $(p,2p)$  differential cross sections. However, as described in the Introduction, one can evaluate the contribution to the  $(p,p')$  predictions of the term in which the "jet" comprises all of the residual nucleus. This would then give a rough measure of the scale factor which could be applied to the DK model results to approximate what would be the CR results at the  $T_f$  corresponding to coherent recoil of the residual nucleus. The  $n=A-1$  contribution to the  ${}^9\text{Be}(p,p')X$  cross section at incident proton energy of 800 MeV is shown in Fig. 11. As with the CC model, it is observed

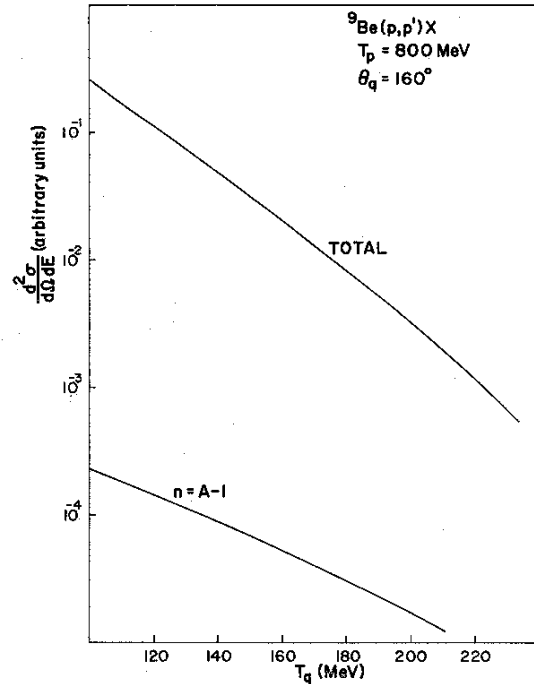


FIG. 11. Comparison of  $n=A-1$  contribution in CR model with total predicted cross section for  ${}^9\text{Be}(p,p')X$  at  $\theta_q = 160^\circ$ ,  $T_p = 800$  MeV.

that the predicted cross section should be much smaller than the DK predictions. However, if one were to measure  $d^4\sigma/d\Omega_f dE_f d\Omega_q dE_q$  as a function of  $E_f$  at fixed  $\Omega_f$ ,  $\Omega_q$ , and  $E_q$ , then rather than observing events at a narrow band of energies centered around  $T_i - T_q - \bar{\mathcal{E}}^*$  (the spreading caused by the  $\bar{\mathcal{E}}^*$  distribution) as predicted by the DK model, the CR model would predict that the maximum in the cross section would be shifted to lower  $E_f$  (corresponding to lighter recoiling jets), the numerical value of the shift depending on the details of the excitation energy and momentum distributions involved.

In all of the above calculations, the average excitation energy of the residual nucleus has been set equal to zero. A more realistic calculation would have a nonzero value, but would introduce extra model dependence into the results, a situation we have tried to avoid. Green and Korteling,<sup>14</sup> using heavy fragment yields as constraints on an evaporation calculation, have estimated that at the beginning of the evaporation cascade, the distribution of excitation energies has an average value in the 20–40 MeV range for an incident proton in the 200–500 MeV range. While this is not necessarily the same distribution that one would expect here, it provides us with a rough value for  $\bar{\mathcal{E}}^*$ . This would manifest itself in the DK model by a downward shift of an amount  $\bar{\mathcal{E}}^*$  in  $T_f$  from the  $\bar{\mathcal{E}}^* = 0$  value of roughly  $T_i - T_q$ .

In performing these calculations, protons with energy 800 MeV and a  $^9\text{Be}$  target were chosen to illustrate the results.  $^9\text{Be}$  was chosen as it is a

readily available and easily manipulated target. The proton energy was chosen to be 800 MeV because an energetic backward moving proton is required for a trigger. This should not imply that the correlation measurement could be done only at this energy. The requirement that the backward moving proton be energetic was made so that one would be measuring (within the context of these models) the high momentum part of the single particle effective momentum distribution, where the distribution seems to be adequately described by an exponential falloff. While there is no reason why the experiment could not be run at lower energy, certainly once the incident proton energy fell much below, say, 200 MeV, it is unlikely that the simple exponential would adequately describe the distribution in the momentum range required for the calculations and they would have to be done with a more appropriate distribution.

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