Role of direct emission in strong and electromagnetically induced inclusive reactions

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Direct emission models which have been proposed for hadronically induced inclusive reactions are tested by comparison with electromagnetically induced reactions. It is found that proton emission in both (p,p') and (γ,p) reactions is consistently described by a direct emission model. The available (e,α) data are sufficiently sensitive to the assumed residual excitation energy of the model that a firm conclusion cannot be drawn for the role of direct emission in fragmentation processes. Predictions for particle multiplicities corresponding to two direct emission models are discussed.

[NUCLEAR REACTIONS Test of models for (p,p'), (γ,p), (p,α), and (e,α) reactions at intermediate energies; predictions of charged particle multiplicities in (p,p') reaction.]

I. INTRODUCTION

A considerable amount of theoretical work[1-19] in recent years has gone into trying to understand the emission at backward angles of protons and light fragments carrying a fair fraction of the incident projectile's energy. There seems to be a growing amount of evidence that the reaction mechanism is not one which involves emission of these particles from a thermally equilibrated excited nucleus. The evidence is as follows:

(1) The target dependence of the double differential inclusive cross section $d^2\sigma/d\Omega dE$ for protons emitted with kinetic energy of 100–200 MeV (incident energy of 500–800 MeV) shows[20-28] that the cross section increases roughly as the mass number of the target A_T for targets from ^{4}He to ^{181}Ta. It is difficult to believe that ^{4}He could behave like a thermally equilibrated system with several hundred MeV excitation energy. Since the cross section for heavier targets than ^{4}He just increase roughly as the number of nucleons in the target,[22] then presumably proton emission for heavy targets is also not exclusively from some thermally equilibrated system.

(2) The analyzing power[23,27] for proton emission at 90° has values which are typically in the 10–20% range. This is in sharp contrast with α-particle emission at the same energies and angles, where the analyzing power[29] is very close to zero. Again, this indicates that the protons do not come from a source in which the knowledge of the incident projectile has been lost.

(3) Calculations[30] involving multiple high momentum transfer collisions of the incident proton with nucleons having a sharp cutoff in the momentum space wave function underestimate the cross section by at least an order of magnitude.

In summary, it is very likely that the incident proton and the observed proton have suffered only a very few collisions during the reaction. Shown in Fig. 1 is the ratio of $d^2\sigma/d\Omega dE$ for the (e,α) to (p,α) reactions[31,32] on a nickel target at similar projectile energies and the same α-particle energy. It is clear that although the (e,α) cross section is less forward peaked than (p,α), the ratio of the cross sections is roughly equal to the electromagnetic fine structure constant squared, α_{em}^2. If one believes that the (p,α) and (e,α) reactions have a common mechanism, then this result tends to argue that a significant fraction of the events require only one scattering of the projectile to produce the ejectile. This conclusion also is supported by a multiple scattering calculation[18] recently done in 10–100 MeV proton induced reactions. Hence, we will concentrate our attention on possible mechanisms which involve only one or two collisions of the incident projectile.

The purpose of this paper is to use electromagnetic probes to test the models proposed for proton induced reactions (including the normalization). In Sec. II we will outline a direct emission model for the (p,p') reaction, and apply it to the (γ,p) reaction.[33-45] Section III will extend this model to the (p,α) reaction,[46-51] and then contrast its predictions for (e,α) with experiment.[31,52,53] Other tests of these models will be discussed in Sec. IV, and the conclusions presented in Sec. V.

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II. DIRECT EMISSION MODEL FOR (p,p') AND (γ,p)

The basic assumption of our model is that the nucleons in the final state can be separated into two groups: the participants in the reaction which carry off significant recoil momentum and spectators which do not. The inclusive cross section then becomes a sum over terms with different numbers of recoiling participants, and for the (p,p') reaction takes the general form (adopting the convention of Bjorken and Drell for spinless objects)[54]

$$\frac{d^3\sigma}{d^3q}=\frac{1}{2^3(2\pi)^2 pE_q}\sum_{i=1}^{A_T-1}\frac{a_i(i+1)}{M_{T_i}}\int\frac{d^3p_F}{E_F}\prod_{j=1}^{i}\frac{d^3k_j}{2E_j(2\pi)^3}\,|\mathcal{T}_i(p,q,p_F,k_i)|\delta^4(\text{energy-momentum})\,, \qquad (1)$$

where we use the labels p, q, p_F, and k_i for the momenta of the incident projectile, observed proton, scattered projectile (after the collision), and recoiling participants, respectively. It is assumed that the remaining A_T-i-1 nucleons are at rest before and after collision. If the observed backward going proton is produced in a direct collision with the projectile it must possess a sizable momentum inside the nucleus before it is struck. Recent calculations[55,56] indicate that high momentum components arise from short-range two-body or few-body correlations. Also we have argued in Sec. I that the data suggest that the number of collisions the incident projectile undergoes is small. Taken together, these facts suggest that the important terms in Eq. (1) are those where i is small and, furthermore, that the recoiling participants are emitted into a fairly limited region of phase space. We therefore replace the phase space integral $\int\Pi_i d^3k_i/E_i$ by an integration over a limited invariant mass spectrum and choose a parametrization of $\mathcal{T}_i$ which gives the inclusive cross section the form

$$\frac{d^3\sigma}{d^3q}=\frac{1}{2^4(2\pi)^5 pE_q}\sum_{i=1}^{A_i-1}\frac{a_i(i+1)}{M_{T_i}}\times\int\frac{d^3p_F}{E_F}d^4K\,D_i(K)n(k)|T|^2\times\delta^4(\text{energy-momentum}), \qquad (2)$$

where K is the total four-momentum of the recoiling i-nucleon jet (k is the three-momentum) and

$$D_i(K)=\frac{i}{2\sqrt{\pi}\,\eta_i\mathfrak{M}_i(k)}\exp\left[-\left(\frac{\mathfrak{M}_i(k)-\mathfrak{M}_i^0}{\eta_i}\right)^2\right] \qquad (3)$$

is its invariant mass distribution. The quantity T is the elementary nucleon-nucleon scattering amplitude evaluated at $t=(p-p_F)^2$ and its presence in (2) reflects the direct nature of the reaction. The remaining dependence of the cross section on the kinematic variables is summarized in the structure function n. The recoiling participants attain the momentum k through few-body momentum correlations in the initial state or multiple scattering in the collision. In either case the probability of imparting a large momentum is small which suggests that it is reasonable to take n to be a function only of k. We note that in a plane wave impulse approximation picture $n(k)$ is the single nucleon inclusive momentum distribution but we emphasize that the *form* of Eq. (2) is more general than that.

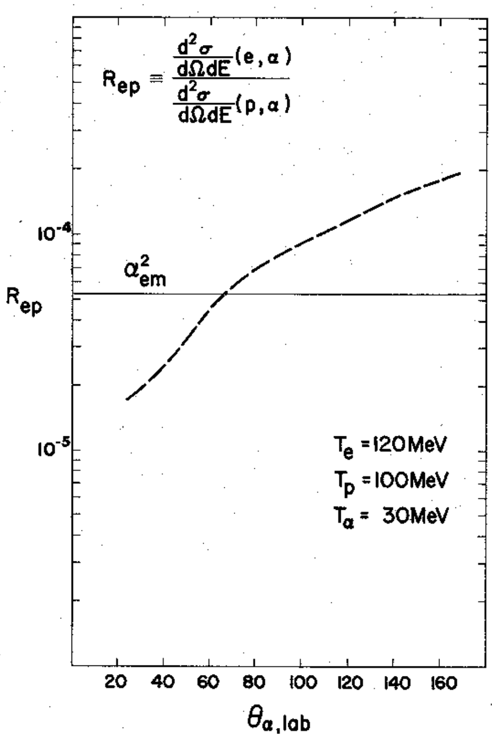


FIG. 1. Ratio of $d^2\sigma/d\Omega\,dE$ for the (e,α) reaction (Ref. 31) to the (p,α) reaction (Ref. 32). Data for both are taken on a nickel target with $T_e=120$ MeV, $T_p=100$ MeV, and $T_\alpha=30$ MeV.

Our particular parametrization of the weighting factor is $a_i=g^{i-1}$ where g is a constant. A second parameter k_0 appears in the expression for the structure function $n(k)$ which is chosen to be

$$n(k)=\frac{C}{\left[2\cosh\left(\frac{k}{2k_0}\right)\right]^2}\equiv CF(k)\,. \qquad (4)$$

The normalization constant C in the above equation was determined by the approximate normalization condition

$$A_T=\frac{C\int F(k)d^3k\sum_{i=1}^{A_T-1}a_i}{2(2\pi)^3M_B}, \tag{5}$$

where we have assumed that the incident projectile can scatter off any of the nucleons in the nucleus. (The average bound nucleon mass M_B is taken to be 931.5 MeV.)

The parametrization of the elementary p-N amplitude was chosen as follows. The elastic and charge exchange NN differential cross sections have the form[57]

$$\frac{d\sigma}{dt}=Ae^{bt}/(64\pi m^2p^2), \tag{6}$$

where t is the four-momentum transfer squared. In the energy range of interest, b has a value of ~10 GeV^{-2} for pp elastic scattering and ~30 GeV^{-2} for pn charge exchange (charge exchange is required if the struck object in the nucleus is to emerge as a proton). However, the projectile is probably striking a two-nucleon system at short separation. Our only hints for the appropriate value of b for such a system come from p-^{2}H elastic scattering[58,59] ($b\sim 25$ GeV^{-2}) or p-^{4}He elastic scattering[60-62] ($b\sim 23$ GeV^{-2}). As a compromise we chose $b=15$ GeV^{-2} for $p<20$ GeV/c, and $b=21$ GeV^{-2} for $p=400$ GeV/c. The constant A was determined by setting the integral of Eq. (6) equal to the total NN cross section, which we chose for convenience as 40 mb throughout the energy range. For the purpose of the inclusive emission calculation, t was defined by the vectors $\vec{p}$ and $\vec{p}_f$, and the appropriate energy was simply defined as the energy of the projectile.

To take into account the distribution of invariant masses the function $D_i(K)$ was introduced into Eq. (2). In the limit $\eta_i\to 0$, $D_i(K)\to i\delta(\mathfrak{M}_i(k)-\mathfrak{M}_i^0)/2\mathfrak{M}_i(k)$ and the d^4k integral becomes the usual d^3k integral on-shell. For the purposes of the calculation, we chose $\mathfrak{M}_i^0$ to be equal to i times the free nucleon mass, and η_i to be i times 15 MeV. For a single nucleon recoiling, we eliminated the distribution and simply put the nucleon mass equal to the free mass plus 10 MeV for the excitation energy of the residual system.

After the energy-momentum delta function is integrated out, there will be three remaining integrals to evaluate. For two of these, we use an analytical approximation described in the Appendix, while the third, which we choose to be over the invariant mass, is done numerically by using Simpson's approximation.

The parameters k_0 and g are determined by fitting the 800 MeV data of Frankel *et al.*[21-23] For ^{6}Li and ^{9}Be targets, the data were well described by $k_0=120$ MeV/c and $g=0.9$, while for Ta the same value of g required $k_0=130$ MeV. A least χ^2 fit was not attempted, but it is likely that both parameters could be changed by up to 10% without seriously affecting the fit. (Of course, the parameters cannot be independently varied by 10%; decreasing k_0 requires increasing g.) The results are shown in Fig. 2. The energy dependence of the model was tested by comparison with $p=6$ and 400 GeV/c data[26,28] taken with a Be target. Shown in Fig. 3 is the comparison at least 6 GeV/c, along with the contribution of each term in Eq. (2) to their sum (data are only available at 162°). The predictions at 400 GeV are shown in Fig. 4, and although the angular dependence is not too far off, there is an obvious deterioration in the quality of the fit.

To test whether this model actually accounts for a significant part of the inclusive (p,p') cross section, i.e., whether the structure function does have a large high momentum tail, we can use the model to make predictions for other reactions whose only unknown is the structure function.

A good candidate for such a test is the (γ,p) reaction. Although this reaction has been studied previously (see, for example, Refs. 63–66), the

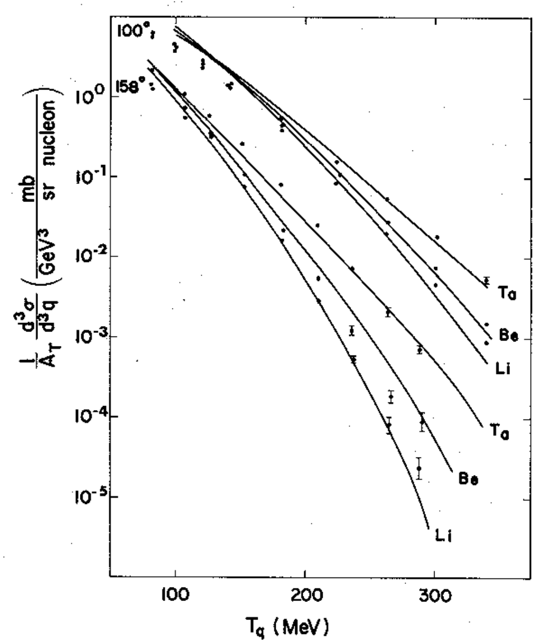


FIG. 2. Comparison of direct knockout model with experiment (Refs. 21–23) at 800 MeV incident proton kinetic energy. The upper set of curves is for protons observed at 100° while the lower set is for 158°. The values of the parameters are $g=0.9$, $k_0=120$ MeV/c for Li and Be; $g=0.9$, $k_0=130$ MeV for Ta.

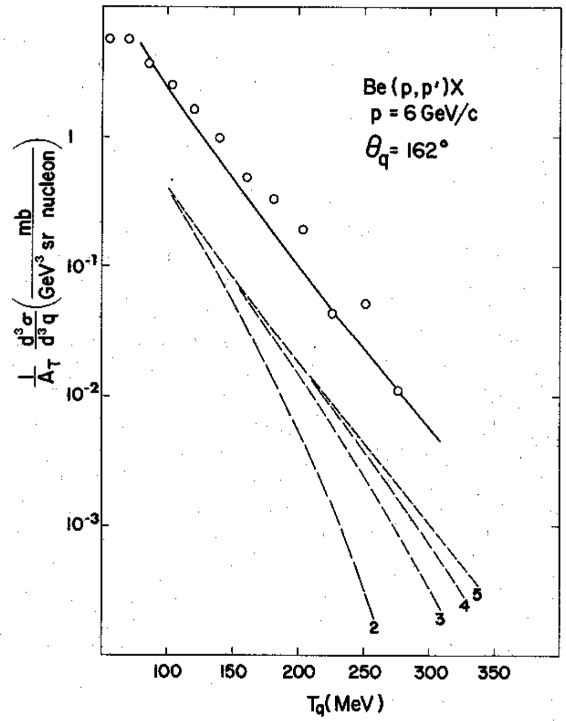


FIG. 3. Comparison of direct knockout model predictions for Be$(p,p')X$ experiment (Ref. 26) at 6 GeV/c incident proton momentum. Contribution of several i-nucleon jets is shown.

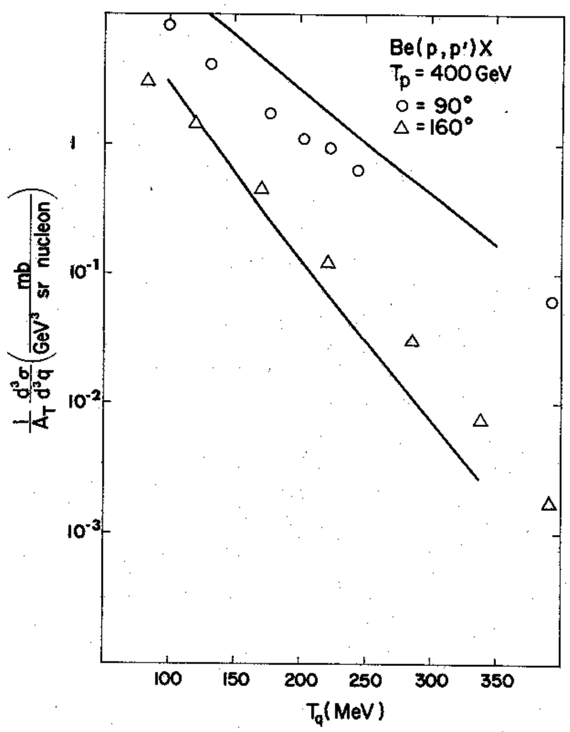


FIG. 4. Comparison of direct knockout model predictions for Be$(p,p')X$ with experiment (Ref. 28) at 400 GeV incident proton energy. The upper curve is for protons observed at 90°, while the lower is for 160°.

models generally involve an arbitrary (or poorly determined) normalization constant (such as appears in the quasideuteron model)[65] which often varies with the incident photon energy, or bremsstrahlung end point energy.[66] The approach taken here will be to assume that the incident photon is captured by one off-shell proton, as in the (p,p') direct knockout model. (This is similar to a model proposed in Ref. 67.) The differential cross section for an incident photon of energy E_γ is then of the form

$$\frac{d^3\sigma}{d^3q}=\frac{1}{2^3(2\pi)^2E_\gamma E_q}\sum_{i=1}^{A_T-1}\frac{a_i(i+1)}{M_{T_i}}\times\int d^4K\,D_i(K)n(k)\overline{\sum}|\mathcal{T}|^2\delta^4(\text{energy-momentum}),\tag{7}$$

where $D_i(K)$ and $n(k)$ are defined in Eqs. (3) and (4), the normalization Eq. (5) changing with the replacement of A_T by the number of protons in the target Z_T. The γ-p vertex has the form

$$\overline{\sum}|\mathcal{T}|^2=e^2\,\overline{\sum}|\epsilon\cdot J|^2,\tag{8}$$

where e is the electromagnetic charge, ϵ the photon polarization vector, and J the proton electromagnetic current. Summing over spins, we have

$$\overline{\sum}|\epsilon\cdot J|^2=4q^2\sin^2\theta_q+2E_\gamma^{\,2}(1+\kappa)^2 + \text{higher order terms.}\tag{9}$$

For comparison with a bremsstrahlung photon spectrum $N(E_\gamma)$ the expression becomes

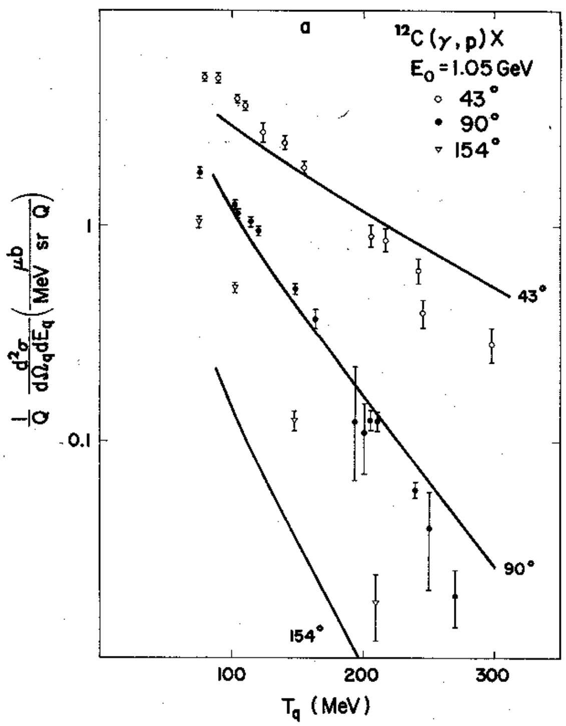
$$\frac{1}{Q}\frac{d^3\sigma}{d^3q}=\frac{1}{Q}\frac{\alpha_{em}}{2^2(2\pi)E_q}\sum_{i=1}^{A_T-1}\frac{a_i(i+1)}{M_{T_i}}\int n(k)D_i(K)\times\overline{\sum}|\epsilon\cdot J|^2\frac{N(E_\gamma)}{E_\gamma}\,\delta(\text{energy})dE_k E_\gamma,\tag{10}$$

where[68]

$$Q=(1/E_0)\int N(E_\gamma)E_\gamma dE_\gamma\tag{11}$$

and E_0 is the bremsstrahlung end point energy. We have used the expression for $N(E_\gamma)$ derived by Schiff.[69]

The predictions of the model for $E_0=1050$ MeV bremsstrahlung photons[38] are shown on Fig. 5. The agreement is remarkably good considering



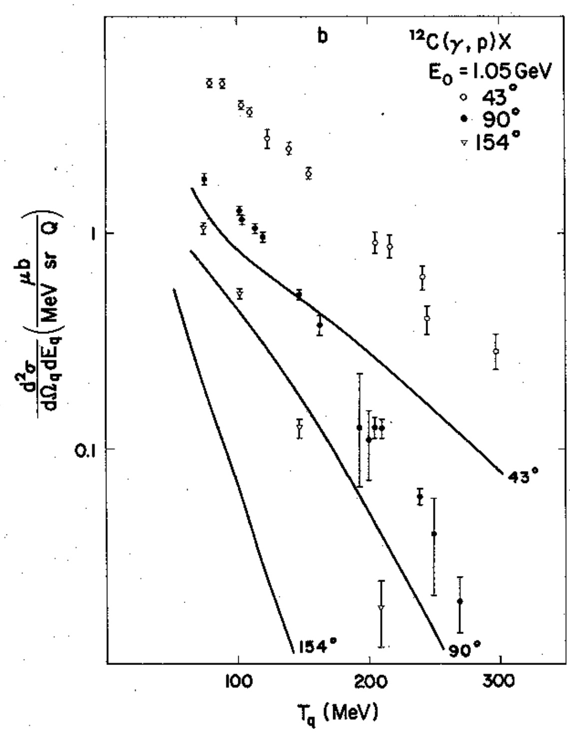


FIG. 5. (a) Comparison of direct knockout model with $^{12}C(\gamma,p)X$ data (Ref. 38) for $E_0=1050$ MeV at 43°, 90°, and 154°. The parameters used are $g=0.9$, $k_0=120$ MeV/c. (b) Comparison of a quasideuteron model calculation (Ref. 66) with $^{12}C(\gamma,p)X$ data (Ref. 38).

the approximations made in fixing the normalization from the (p,p') reactions. The data provide the best test of the calculations since the emitted protons have more than 100 MeV kinetic energy (so one is again looking at high recoil momentum), but the photons required to produce them are not near the kinematic limit of the bremsstrahlung spectrum. The calculations were tried for lower values of the end point energy[39] and it was found that the model began to systematically overpredict the data by $E_0\sim600$ MeV. For example, results for $E_0=335$ MeV on a Li target[35] are shown in Fig. 6. The reason for the overprediction very likely lies in the fact that the residual excitation energy in this model cannot be properly determined from the (p,p') reaction. Hence, as the proton energy comes closer to the kinematic limits, the calculation will be less reliable.

Lastly, shown for comparison in Fig. 5 is a quasideuteron model calculation from Ref. 66. Most of the parameters in the calculation were determined by fitting low E_0 data, and the calculation was then performed for large E_0. Within the context of the calculation of Ref. 66, the quasideuteron constant L would have to be made a function

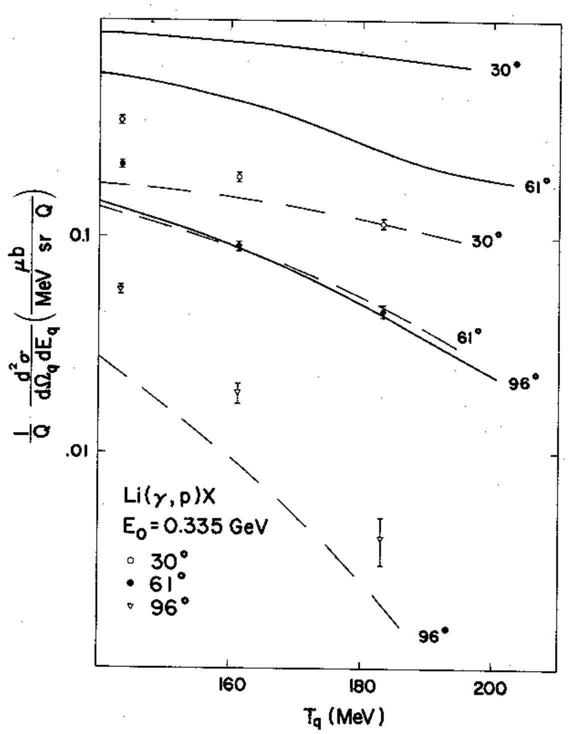


FIG. 6. Comparison of direct knockout model (solid curves) with $Li(\gamma,p)X$ data (Ref. 37) for $E_0=335$ MeV at 30°, 61°, and 96°. Also shown are the quasideuteron model results (Ref. 66) (dashed curves).

of E_0 to obtain satisfactory agreement over the 100–1000 MeV energy range.

III. DIRECT EMISSION MODEL FOR (p,α) AND (e,α)

In a previous paper,[70] we proposed a direct knockout model for (p,α) in which the interaction of the incident proton was limited to the four nucleons which actually formed the observed alpha particle, and this interaction was approximated by the free p-α amplitude. The fit to the data was good in the 200–500 MeV incident proton kinetic energy range, as had been found in previous applications of such a model to low energy reactions.[71,72] However, some features of the highly localized interaction proposed above, such as the existence of a quasifree peak in the differential cross section or a nonzero analyzing power, do not appear to be observed experimentally.[29]

The model which we used contains one parameter more than those involved in the (p,p') direct knockout model, namely the alpha cluster formation probability. (This may go by several aliases, but is essentially a normalization constant.) There was no *a priori* way of assigning a value to this number, other than that the effective number of alpha particles in the target should not exceed $A_T/4$ by much (given all the other approximations). Indeed, the phenomenologically determined number was only 10 to 20% of $A_T/4$, and so there was no problem with conservation of probability.

A good test of this normalization constant would be to look at an electromagnetic interaction, in the same vein as we looked at (γ,p) to test the model of (p,p'). Our only choice is (e,α), although the data available are at an energy for which the model may be beginning to lose some of its validity. The results which will be presented do not depend significantly on whether coherent recoil is assumed for the residual nucleus (as was done in Ref. 70), or a variable number of nucleons is allowed as in Eq. (2), as long as the calculation is done in a consistent fashion. We extend the model which we have used for (p,p') to both (p,α) and (e,α).

For the (p,α) reaction, Eq. (2) becomes

$$\frac{d^3\sigma}{d^3q}=\frac{1}{2^4(2\pi)^5 pE_q}\sum_{i=1}^{A_T-4}\frac{a_i(i+4)}{M_{T_i}}\times\int\frac{d^3p_f}{E_f}d^4K\,D_i(K)n(k)\,|T_{p\alpha}|^2\times\delta^4(\text{energy-momentum}),\tag{12}$$

where $T_{p\alpha}$ is the taken to be the free proton-^{4}He elastic scattering matrix element, and $n(k)$ is now the probability for finding an i-nucleon recoiling jet with momentum k accompanying an α particle. It is normalized by

$$n_{\text{eff}}=\frac{1}{2(2\pi)^3}\frac{M_\alpha}{M_B^{\,2}}\sum_i a_i\int n(k)\,d^3k\,,\tag{13}$$

where n_{eff} is the effective number of α's in the nucleus, and is treated as an adjustable parameter.

The results of a calculation using $g=0.9$ and $k_0=120$ MeV/c [from the (p,p') analysis] are shown in Fig. 7. The normalization required $n_{\text{eff}}\approx 13$, so that $n_{\text{eff}}=\frac{1}{2}(A/4)$ was used for the (e,α) calculation. Since our main interest is in the magnitude of the (e,α) cross section we did not try to improve upon the fit shown in Fig. 7.

Assuming a direct knockout model with single photon exchange, the differential cross section for the (e,α) reaction is given by

$$\frac{d^2\sigma}{d\Omega\,dE_q}=\frac{1}{2^4(2\pi)^5}\cdot 4m_e^{\,2}\frac{q}{p}\sum_{i=1}^{A_T-4}\frac{a_i(i+4)}{M_{T_i}}\times\int\frac{d^3p_f}{E_f}dE_k D_i(K)n(k)\,|T_{e\alpha}|^2\,\delta(\text{energy})\,,\tag{14}$$

where $\vec{k}=\vec{p}-\vec{q}-\vec{p}_f$ and where we have changed the normalization convention to that appropriate to a spin-$\frac{1}{2}$ object scattering from a spin-0 object. Here, m_e is the electron mass. Now, $|T_{e\alpha}|^2$ is

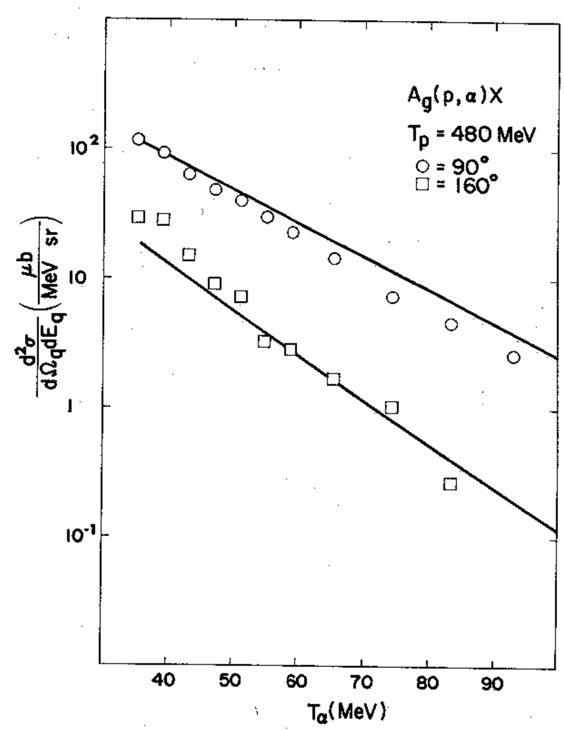


FIG. 7. Comparison of direct knockout model with Ag$(p,\alpha)X$ data (Ref. 73) for $T_p=480$ MeV at 90° and 160°. Normalization required $n_{\text{eff}}=13$.

given by

$$|T_{e\alpha}|^2 = |F_\alpha|^2 \frac{e^4}{m_e^2} q^2 \times \sin^2\theta_q \left[\frac{4E_i^2 E_f^2}{(E_i - E_f)^2} \frac{\sin^2\theta_f}{t^2} \cos^2(\phi_f - \phi_q) - \frac{1}{t}\right], \tag{15}$$

where t is the four-momentum transfer squared between the incoming and outgoing electron, and the alpha electromagnetic form factor is given by

$$F_\alpha(t) = 2(1 + 2.33t)e^{9.14t}, \tag{16}$$

where t is in units of GeV^{-2}.

Rewriting d^3p_f as $p_f^2 dp_f d\cos\theta_f d\phi_f$, the $|\vec{p}_f|$ integral can be used to eliminate the energy conserving delta function. We then assume the forward peaking approximation, so that the ϕ_f integral is set equal to 2π, and the θ_f integral yields

$$\int |T_{e\alpha}|^2 d\Omega_f = |F_\alpha|^2 \frac{e^4}{m_e^2} q^2 \sin^2\theta_q \frac{2\pi}{2E\,E_f(E - E_f)^2} \times \left[(E^2 + E_f^2)\ln\left(\frac{4E^2E_f^2}{m_e^2(E - E_f)^2}\right) - 4E\,E_f\right], \tag{17}$$

where E_f and F_α are evaluated at $\theta_f = 0°$. The results of the calculation are shown in Fig. 8 for $T_e = 120$ MeV.

The predictions clearly overestimate the experimental data and do not reproduce the shape of the angular distribution particularly well. Because the energy of the incident electron is only 120 MeV, the predictions will be sensitive to the excitation energy of the residual spectator system. In the calculation, this was set equal to zero as was done for the (p,p') model. However, the removal of an α-particle presumably leaves the nucleus in a more excited state than does removal of a proton. To see the effect that giving an excitation energy to the residual system would have on the predictions, we also show in the figure the results of a calculation done with an average excitation energy of 20 MeV. [The (p,α) calculation was performed with this excitation energy first, and showed that n_{eff} had to be equal to $A_T/4$.] One can see that the predicted cross section is reduced considerably, although the angular shape is still not reproduced.

IV. CHARGED PARTICLE MULTIPLICITIES

To test direct emission or other models in greater detail, one must look toward less inclusive experiments. An attempt has already been made[19] to calculate the $(p,2p)$ cross sections for some of the models to see if there is a leading particle effect in the forward direction. Particle multiplicities have also been calculated for the light recoil

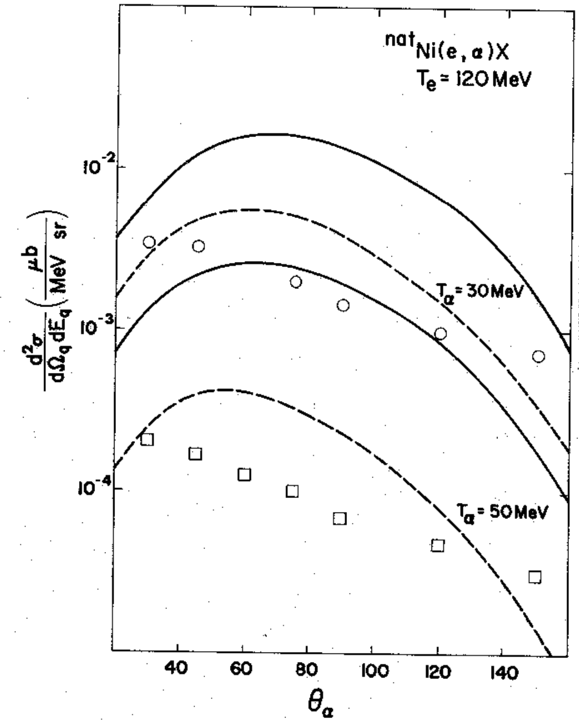


FIG. 8. Comparison of direct knockout model predictions for Ni$(e,\alpha)X$ with experiment (Ref. 31). The upper two curves are for $T_\alpha = 30$ MeV, the lower for $T_\alpha = 50$ MeV. The solid curve corresponds to no excitation energy in the spectator nucleus, while the dashed curve corresponds to 20 MeV.

direct knockout model[8] which was the precursor of the model developed here. We have made an estimate of the charged particle multiplicities (triggered on an energetic proton) of the model outlined in Sec. II, and for the correlated cluster model of Fujita.[7] First, we will deal with the direct knockout model.

In the invariant mass distribution which was shown in Eq. (3) no distinction is made between bound and unbound states, so that it is not possible to obtain a unique prediction for the charged particle multiplicity. We will consider two extreme cases.

(a) All emitted particles are free. For a target of A_T nucleons of which Z_T are protons, the charged multiplicity is

$$\langle n_{\text{ch}}\rangle = 1 + \sum_{i=1}^{A_T-1} \left(\frac{Z_T}{A_T}\right) i \left(\frac{\sigma_i}{\Sigma\sigma_i}\right), \tag{18}$$

where the σ_i represent the partial cross sections and the 1 comes from the incident proton.

(b) All particles emitted with $\mathfrak{M}_i(k)$ above the i free particle mass are free, and all those below are bound into one charged fragment. Then

$$\langle n_{\rm ch}\rangle = 1 + \sum_{i=1}^{A_T-1}\left[\frac{Z_T}{A_T} i\ \sigma_i(>F) + \sigma_i(<F)\right] \Big/ \sum_{i=1}^{A_T-1}[\sigma_i(>F)+\sigma_i(<F)], \tag{19}$$

where $\sigma(\gtrless F)$ refers to the ith partial cross section with $\mathfrak{M}_i$ greater than or less than the i free particle mass. These predictions are shown in Figs. 9 and 10.

For comparison we show the results for charged particle multiplicities predicted by the correlated cluster model of Fujita using Eq. (4.8) of Ref. 7 (curves labeled cc in Fig. 10). This model is also based on the direct emission picture and provides a reasonable description of inclusive (p,p') cross sections in the 0.6 to 0.8 GeV range. The multiplicities in the correlated cluster model are very different from ours and at high incident energies are essentially constant as a function of the observed proton momentum. It is not clear how seriously one can take this latter prediction since this model tends to greatly overestimate the differential cross section at high energy. This is shown in Fig. 11 where a comparison is made of the correlated cluster calculation with Be$(p,p')X$ data at incident proton energies of 0.8 and 5.14 GeV. We conclude that this model is, at best, applicable over a very limited range of energies.

V. DISCUSSION AND CONCLUSIONS

We have developed a model for the inclusive (p,p') reaction based on the ideas that the recoiling participants in the reaction are emitted in a limited phase space region and the dominant falloff of the cross section is given by a function of the total recoil momentum. With two energy independent parameters the model gives a good description of large-angle inclusive cross sections for incident proton energies from 0.8 to 400 GeV. This is extended (with no change of parameters) to the (γ,p) reaction (bremsstrahlung photons) and good agreement is obtained for ^{12}C$(\gamma,p)X$ with a photon end point energy E_0 of 1.05 GeV. At $E_0 = 0.335$ GeV, where the observed protons are closer to the kinematic limit, the model systematically overestimates the cross section. This is probably an indication that the strict spectator approximation used here is not appropriate when the recoil energy becomes too small. In any case our description is as good as the quasideuteron model which is the standard description of high energy photodisintegration.

The structure function $n(k)$ represents the probability of producing the final recoil state either

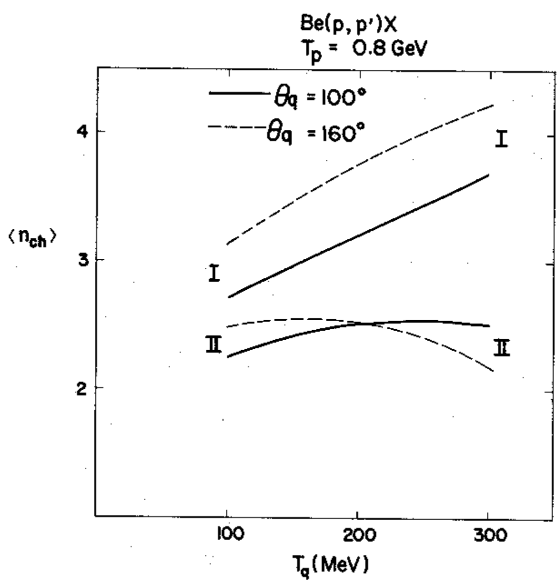


FIG. 9. Predictions of direct knockout model for $\langle n_{\rm ch}\rangle$ at $T_p = 800$ MeV on Be. The angle of the trigger proton is 100° for the solid curve and 160° for the dashed curve. The upper curves are from model I and the lower two from model II.

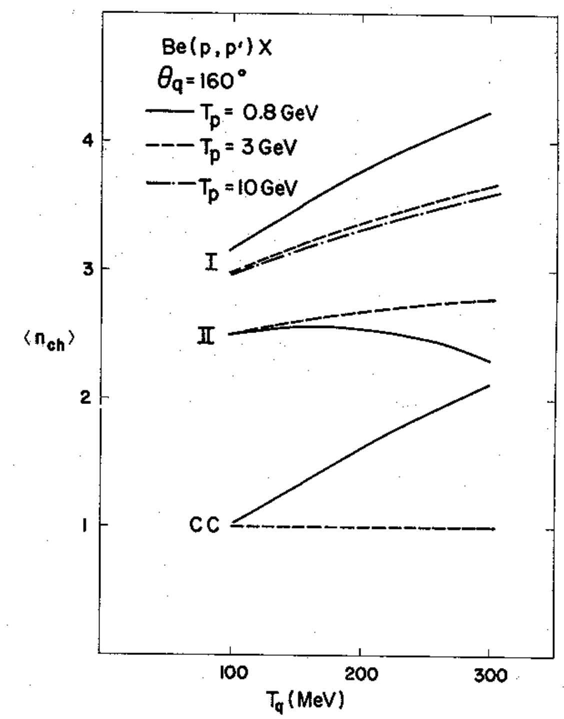


FIG. 10. Predictions of direct knockout model for $\langle n_{\rm ch}\rangle$ at $T_p = 0.8$, 3, and 10 GeV with a proton trigger at 160°. The upper three curves are for model I, the middle two for model II, and the lower two for the correlated cluster model (Ref. 7). Only for model I are the 3 and 10 GeV predictions distinguishable.

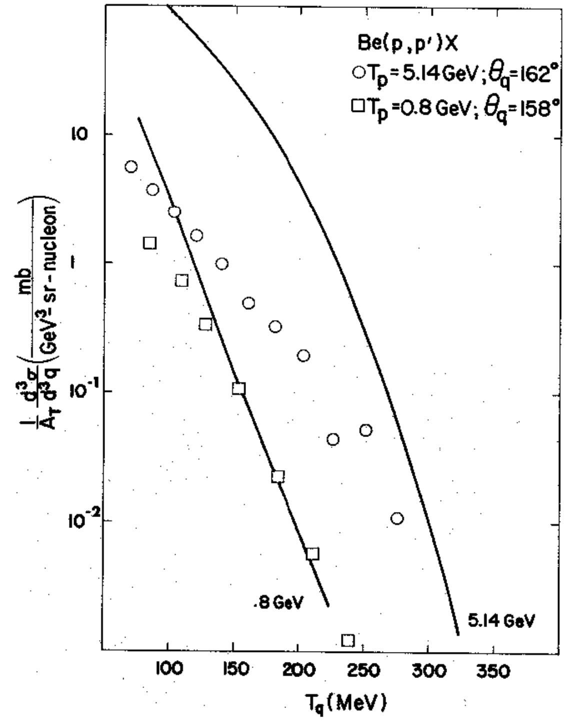


FIG. 11. Predictions of correlated cluster model for $Be(p,p')X$ at $\theta_q = 160°$ and $T_p = 0.8$ and 5.14 GeV.

through multiple collisions or correlations in the initial state wave function. In the plane wave impulse approximation $n(k)$ would be interpreted as the single nucleon momentum distribution. Although the parametrization Eq. (2) is more general than that, the exponential falloff of our phenomenological $n(k)$ [Eq. (4) with $k_0 = 120$ to 130 MeV/c] is nearly the same as the behavior of the high momentum tail of the single nucleon momentum distribution calculated by Zabolitsky and Ey. Furthermore, with the normalization of Eq. (5) the magnitude of $n(k)$ is about a factor of 2 larger than the calculated single nucleon momentum distribution at high momentum (for the Reid soft core N-N potential). In other words the recoil states produced in inclusive reactions are similar to those involved in high momentum ground state correlations which provides good evidence for the direct nature of the reaction mechanism.

The value of g found in the fit may appear to be somewhat surprising at first, but is probably not unreasonable. Among the approximations which we have used, the $2+i$ body-phase space of Eq. (1) was approximated by a 3-body phase space in Eq. (2). Since this phase space grows as a power of i, the "real" value of g would be less if this approximation had not been made. Other evidence that a large value of g is not unwarranted comes from the relative differential cross sections for heavy fragment production. One finds[47–51] that after a rapid drop in cross section in going from inclusive proton to inclusive α production, the decrease for heavy fragment emission is slow for increasing fragment mass. If heavy fragment emission has its origins in some multiple scattering mechanism, then the probability for multiple scattering does not decrease rapidly with increasing number of collisions (most of these "extra" collisions are of low momenta compared to the initial momentum given the struck nucleon, and will have a larger cross section if the momentum is low enough).

A similar model was then developed for proton induced α emission. No attempt was made to get a best fit to all of the (p,α) data, insofar as we were largely interested in obtaining the magnitude of the cross section. As in the proton emission case, the model was then extended to an electromagnetically induced reaction, namely (e,α). Assuming the same parameters as found for the (p,p') reaction, and adding no excitation energy to the residual spectator system, it was found that the predicted cross section was up to an order of magnitude larger than the data, and was much more sideways peaked than the data.

However, because the energy of the incident electron is only 120 MeV for these data, and therefore the assumed invariant mass distributions are broad compared to the available phase space, the calculations are sensitive to the excitation energy assumed for the residual system. It was found that when an average exciation energy of 20 MeV was put into the residual spectator system, the magnitude of the predictions was reduced by a factor of 3–10 although the sideways peaking was not significantly changed.

To sort this problem out, it would be most useful to have data on electron induced fragmentation at electron energies of at least 400 MeV, so that a 50 MeV emitted fragment, for example, is not near the kinematic limits of the experiment. Until such data are available, it will be difficult to draw any firm conclusions about the mechanism for fragment emission. At the present time, it would appear that models which assume α emission from a thermally equilibrated source have difficulty explaining the rather unusual constraints placed by experiment on the nature of the source.[73] However, if the lack of sideways peaking in the (e,α) data persists to higher energies, then it is likely that the projectile-fragment interaction will have to be treated in more detail, as has been attempted in the pre-equilibrium models.[74–76]

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APPENDIX

Because Eq. (2) involves a triple integral after removal of the energy and momentum conserving delta functions, it was decided to develop an approximation for two of the integrals and use Simpson's approximation to numerically evaluate the third. We begin by rewriting Eq. (2) as

$$\frac{d^3\sigma}{d^3q}=\frac{1}{2^4(2\pi)^5 pE_q}\sum_{i=1}^{A_T-1}\frac{a_i(i+1)}{M_{T_i}}\int D_i(\mathfrak{M}_k)\mathfrak{M}_k\,d\mathfrak{M}_k\,I(\mathfrak{M}_k)\,, \tag{A1}$$

where

$$I(\mathfrak{M}_k)=\int\frac{n(k)|T|^2}{E_f E_k}\,d^3k\,d^3p_f\,\delta^4(\text{energy-momentum})\,. \tag{A2}$$

The k integral is used to integrate out the momentum conserving δ function, and the $|p_f|$ integral the energy conserving δ function (the angles are defined with respect to the beam)

$$I(\mathfrak{M}_k)=\int\frac{n(k)|T|^2}{E_f E_k PSP}\,d\cos\theta_f\,d\phi_k\,, \tag{A3}$$

where

$$PSP=\left|p_f\left(\frac{1}{E_f}+\frac{1}{E_k}\right)+\frac{1}{E_k}(q\cos\theta_q-p_i)\cos\theta_f +\frac{1}{E_k}q\sin\theta_f\cos\phi_f\sin\theta_q\right|\,. \tag{A4}$$

From the form of $n(k)$ and $|T|^2$ adopted in the model [Eqs. (4) and (6)] at large k the numerator of the integrand has the form

$$AC\,e^{bt-(k/k_0)}\equiv AC\,e^{-\xi} \tag{A5}$$

which has a maximum at

$$\sin\theta_f\simeq\frac{|\vec{p}-\vec{q}|\sin\theta_q}{2bp_f k_0 k+|\vec{p}-\vec{q}|\cos\theta_q}\equiv\sin\theta_{\max}\,. \tag{A6}$$

This expression can be solved iteratively, but in practice, the angle is sufficiently close to 0° that one can substitute the values of p_f and k at 0° and obtain a reasonably close answer (p_f is a very slowly varying function of θ_f at small θ_f). Assuming that PSP and E_k are slowly varying functions of θ and ϕ_f (which is true for large E_k), then

$$I(\mathfrak{M}_k)\simeq\frac{AC}{PSPE_fE_k}\int_{\xi_{\min}}^{\xi_{\max}}e^{-\xi}\left[-2bpp_f\sin(\gamma+\theta_{\max})+\frac{p_f|\vec{p}-\vec{q}|}{kk_0}\sin(\alpha-\gamma-\theta_{\max})\right]^{-1}\sin\gamma\,d\xi\,d\phi_f\,, \tag{A7}$$

where α is the angle between $p-q$ and the beam and $\gamma=\theta_f-\theta_{\max}$. Since α and $\theta_{\max}$ are usually small, the integral can be approximated by

$$I(\mathfrak{M}_k)\approx\frac{AC}{PSPE_fE_k}e^{-\xi}\left(2bpp_f+\frac{p_f|\vec{p}-\vec{q}|}{k_0k}\right)^{-1}, \tag{A8}$$

where all quantities are to be evaluated at $\theta_{\max}$. In tests of this approximation, we found that it was generally accurate to about 20% for $b>15$ GeV^{-2}.

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