# Fragment emission and the lifetime of pre-equilibrium states

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The production of nuclear fragments in the 10–30 mass number range is viewed as a nonequilibrium process in which individual nucleons in kinetic equilibrium coalesce to form fragments. A simplified set of rate equations for fragment formation is solved numerially which allows for a determination of the lifetime of the fragment formation epoch by comparison with data. The lifetime so calculated is about $4\times10^{-23}$ sec.

The nucleon-nucleon force has similarities to the intermolecular force: attraction at long distances and repulsion at short distances. Such a force could give rise to a van der Waals type equation of state which, in turn, opens up the possibility of a nuclear phase transition,[1] similar to the liquid-gas phase transition observed in the molecular domain. Thermal model analysis[2] of the inclusive cross sections of both proton induced and heavy ion induced reactions has shown that nuclear matter may be excited to the temperature and density regions appropriate for this transition. Analyses of data[3] for the production of heavy fragments have been performed[4,5] using a thermal liquid drop model,[6] which assumes that hot nuclear matter cools through a phase transition,[7] producing nuclear droplets which are the observed heavy fragments. The parameters of the nuclear equation of state which are fitted by this approach have been shown[5] to be self-consistent.

However, a central question regarding any mechanism for fragmentation is the time scale involved. The fact that the (p,p′)/(p,n) ratio is measured[8] to be about 2 (after correcting for the $Z/N$ ratio in a variety of targets) at 100 MeV bombarding energy argues that chemical equilibrium may not be reached in proton-induced reactions.[9] The time scale of the reactions may be so short that the nucleons involved only reach kinetic equilibrium. The production data for heavy fragments also shows that complete chemical equilibrium is not achieved. We show schematically in Fig. 1 the time evolution of a system initially composed only of nucleons. Because the binding energy per nucleon increases with the mass number of the fragment, $A_F$, one would expect that over time the distribution of masses would ultimately be centered about the region with the greatest binding energy per nucleon, the spread about this region being determined by the temperature, among other things. At temperatures greater than 10 MeV (determined by single fireball analysis of inclusive cross section measurements) much of the data of proton induced reactions look similar to the "intermediate time" curve of Fig. 1. Heavy ion data at lower temperatures show a dip in the yield as a function of mass, followed by an increase, although it is not clear whether the data approach the "late time" picture.

Because the reaction time is so short, and the number of participants so small in proton induced reactions, it is possible that a better description of fragment production can be found in a detailed solution of reaction rate equations,[10] rather than in a thermodynamic description which may involve a low temperature phase transition. For the kinetic model to be described below to be valid, the system would have to be sufficiently dilute that many-body interactions could be neglected. The purpose of this paper is to solve a simplified set of rate equations which allow an estimate to be made of the reaction time involved. Our purpose here is not to develop a detailed model, but to show that the kinetic picture appears to be valid for the low density regime and to extract the time scale associated with fragment formation.

We will assume that at the beginning of the fragment formation epoch, the initial hot zone (which has a temperature of 75 MeV for proton induced reactions[11] in the multi-GeV

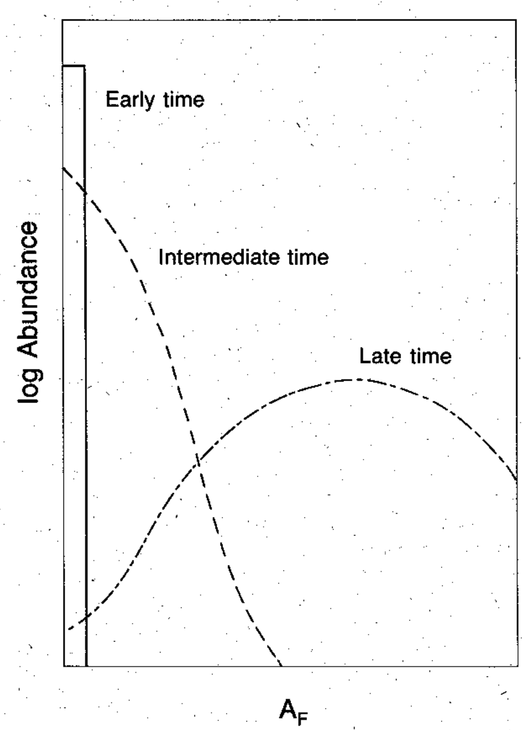


FIG. 1. Schematic representation of the time evolution of the relative abundances in a system composed initially of nucleons.

 

incident energy range) has cooled and expanded such that the temperature has dropped to about $\frac{1}{3}$ its initial value.[12] At this time, which we define as $t=0$, the distribution of number densities $N_i$ of the species present will be assumed to be of the form

$$N_1(t=0)=\rho \ , \quad N_i(t=0)=0, \ \ 2\leq i\leq A_T \ . \tag{1}$$

At this point we will not distinguish between protons and neutrons. At the freeze-out point for pions, the density of nucleons has already decreased[11] to at least $\frac{1}{2}$ nuclear matter density or one nucleon per 12 fm$^3$. Hence we will consider only two body interactions in the rate equations. For early times, the breakup of heavy nuclei will be assumed to be slow compared with the formation rate, although, as one approaches equilibrium these rates will become comparable. Then the rate equations have the form

$$\frac{dN_k(t)}{dt}=+\sum_{i,j}\frac{N_i(t)N_j(t)}{1+\delta_{ij}}\bar{\sigma}_{ij}\delta_{i+j,k}-\sum_i N_i(t)N_k(t)\bar{\sigma}_{ik} \ . \tag{2}$$

Here, $\bar{\sigma}$ is the thermal averaged cross section

$$\bar{\sigma}=4\pi\left[\frac{\mu}{2\pi T}\right]^{3/2}\int v^3\sigma(v)e^{-\mu v^2/2T}dv \ , \tag{3}$$

where $\mu$ is the reduced mass and $v$ is the relative velocity. For simplicity we have assumed that the main contribution to fragment formation is two particle fusion. The omission of more complicated processes will partly compensate for the omission of breakup reactions in these equations.

Before performing a numerical integration of these equations, we can extract a "small time" expansion. For $t$ near zero, assuming that the fusion cross sections do not have a pathological mass number dependence, the rate equations will simplify to

$$\frac{dN_1}{dt}\cong-\frac{\bar{\rho}^2}{2}\bar{\sigma}_{11} \ , \quad \frac{dN_2}{dt}\cong\frac{\bar{\rho}^2}{2}\bar{\sigma}_{11} \ , \quad \frac{dN_3}{dt}\cong N_2\rho\bar{\sigma}_{12} \ , \tag{4}$$

etc. That is, the number density of species $i$ will be building up rapidly as a function of time. One can show that Eq. (4) gives the number densities a general time dependence of the form $N_i\propto t^{A_i-1}$:

$$N_1(t\cong0)=\rho \ , \quad N_2(t\cong0)\cong\frac{\rho}{2}(\rho\bar{\sigma}_{11}t) \ , \quad N_3(t\cong0)\cong\frac{1}{2}\frac{\rho}{2}(\rho\bar{\sigma}_{11}t)(\rho\bar{\sigma}_{12}t) \ . \tag{5}$$

Hence a plot of the log of the yield of fragments against their mass numer would show a straight line with negative slope from which the reaction time $t$ could be obtained. Of course, at large $A$ the cross sections will be increasing, and there will be more reaction channels contributing to a given product, so the expected curve would deviate positively from the straight line decrease. This is indeed what is observed experimentally.

To actually extract an estimate of the reaction time, we will compare with the p+Kr data[4] at 80–350 GeV. For simplicity the integrated reaction cross section in the energy range of interest will be parametrized as[13]

$$\sigma_{AB}=\pi[r_0(A^{1/3}+B^{1/3})+\lambda\!\!\!^{-}]^2 \ , \tag{6}$$

where $\lambda\!\!\!^{-}$ is the reduced de Broglie wavelength, $r_0=1.2$ fm, and $A$ and $B$ are the mass numbers of the fusing nuclei. Of course, not all of the total reaction cross section results in fusion. We estimate the relevant part by using $\sigma_{AB}$ in Eq. (3) but truncating the integral at the Fermi velocity, $v_{\max}$ of nuclear matter at normal density. In other words, a pair of nuclei whose relative velocity is less than $v_{\max}$ will fuse, while those with velocity greater than $v_{\max}$ will not. This is an obvious oversimplication but will suffice for our purposes here. Lastly, we need a temperature and initial density $\rho$, which we will take to be 25 MeV ($\frac{1}{3}$ of the initial proton temperature) and $\frac{1}{2}\rho_0$, respectively. Again, a more sophisticated calculation would allow these to change as the system expands. This temperature justifies in part the use of Eq. (3) which assumes Maxwell-Boltzmann statistics.

The result of the numerical integration is shown in Fig. 2. To obtain an absolute yield, the calculated number densities must be multiplied by a volume. Here, we neglect this since the data are arbitrarily normalized. The predicted yields match the data well, with deviations occurring at the expected masses. We have not suppressed masses 5 and 8,

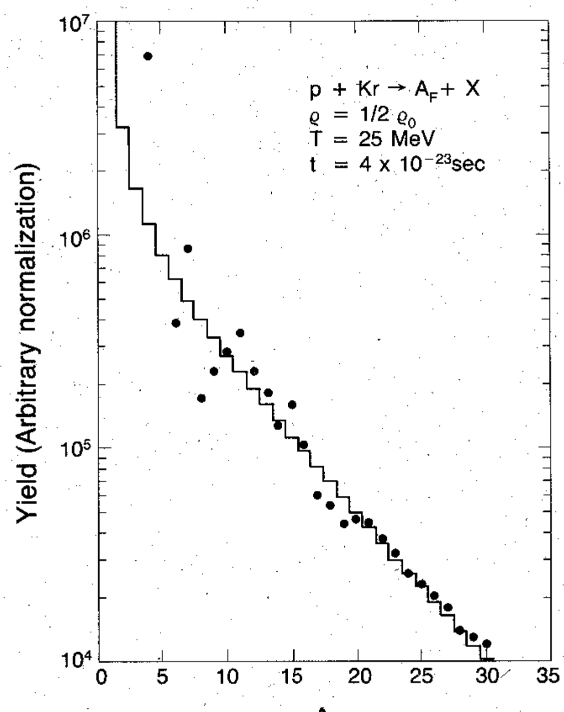


FIG. 2. Solution to the rate equations for fragment formation with $t=4\times10^{-23}$ sec. The data are from the p+Kr reaction at 80–350 GeV (Ref. 4).

so they are necessarily overpredicted. Similarily, these two isobars will enhance mass 4 in their decay, so the underprediction of mass 4 is anticipated. That masses 5 and 8 are suppressed by binding energy considerations during the formation period is probably also responsible for the decrease in isobars immediately above them, i.e., 6 and 9 from the prediction. Lastly, although a specific data set has been chosen, the predicted fall-off of the mass yield with $A$ should be roughly universal, as the formation times should not depend sensitively on the projectile energy or target involved.

The formation time required to produce the observed distribution is $4\times10^{-23}$ sec. This estimate is in accord with what is required to explain the $(p,p')/(p,n)$ ratio, and with the estimated[11] rate of cooling of the hot zone, namely, about $1\times10^{-23}$ sec. One would expect the fragment formation time to be longer than this as the cross sections involved are larger than $(p,p')$, and therefore the heavy fragments go out of equilibrium somewhat later. Of course, there are several effects which would have to be included in a detailed rate calculation before this estimated time could be regarded as firm. As was pointed out above, some of these effects may cancel. Since the data are proportional to the time raised to a power, it is unlikely that the time will be changed by more than a factor of 2 in incorporating extra corrections. Hence, a self-consistent picture emerges of proton induced reactions in which an initially small region of energetic nucleons is produced (in thermal, but no necessarily chemical equilibrium) in around $10^{-23}$ sec, followed by expansion and cooling until fragment formation is complete at around $\frac{1}{2}\times10^{-22}$ sec.

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[1]This has been investigated by many authors. See, for example, H. Jaqaman, A. Z. Mekjian, and L. Zamick, Phys. Rev. C 27, 2728 (1983); for cosmological applications, see D. H. Boal, Michigan State University Report No. MSUCL-419.

[2]For a review, see S. Das Gupta and A. Z. Mekjian, Phys. Rep. 72C, 133 (1981).

[3]J. E. Finn, S. Agarwal, A. Bujak, J. Chuang, L. J. Gutay, A. S. Hirsch, R. W. Minich, N. T. Porile, R. P. Scharenberg, B. C. Stringfellow, and F. Turkot, Phys. Rev. Lett. 49, 1321 (1982).

[4]A. S. Hirsch, A. Bujak, J. E. Finn, L. J. Gutay, R. W. Minich, N. T. Porile, R. P. Scharenberg, B. C. Stringfellow, and F. Turkot (unpublished); R. W. Minich, S. Agarwal, A. Bujak, J. Chuang, J. E. Finn, L. J. Gutay, A. S. Hirsch, N. T. Porile, R. P. Scharenberg, B. C. Stringfellow, and F. Turkot, Phys. Lett. 118B, 458 (1982).

[5]A. L. Goodman, J. I. Kapusta, A. Z. Mekjian, Lawrence Berkeley Laboratory Report No. LBL-16471.

[6]M. E. Fisher, Physics (N.Y.) 3, 255 (1967); see also D. Stauffer, Phys. Rep. 54, 1 (1979) for a percolation theory approach.

[7]For further work on phase transitions, see M. W. Curtin, H. Toki, and D. K. Scott, Phys. Lett. 123B, 289 (1983), and G. Bertsch and P. J. Siemans, *ibid.* 126B, 9 (1983).

[8]B. D. Anderson, A. R. Baldwin, A. M. Kalenda, R. Madey, J. W. Watson, C. C. Chang, H. D. Holmgren, R. W. Koontz, and J. R. Wu, Phys. Rev. Lett. 46, 226 (1981).

[9]For a discussion of proton induced reactions, see D. H. Boal, Lawrence Berkeley Laboratory Report No. LBL-16566; for heavy ion reactions, see A. Z. Mekjian, Nucl. Phys. A384, 492 (1982).

[10]For an application of the rate equation approach to light fragment formation, see A. Z. Mekjian, Nucl. Phys. A312, 491 (1978).

[11]D. H. Boal and J. H. Reid, TRIUMF Report No. TRI-PP-83-79.

[12]This decrease has been shown in an analysis of 200–500 MeV data by R. E. L. Green and R. G. Korteling (private communication). See also Ref. 4.

[13]Data on total reaction cross sections quoted in R. M. DeVries and J. C. Peng, Phys. Rev. Lett. 43, 1373 (1979) were used to obtain this formula. The formula underestimates the NN cross sections at low energies and overestimates them at high energies.