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Experimental signature ambiguities in nuclear liquid-gas phase transitions

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Characteristics of intermediate mass fragment emission data advanced as experimental evidence of a hadronic gas to liquid phase transition are reviewed. Ambiguities in the interpretation of these characteristics are presented. The necessity of large Z fragments to tunnel through a large Coulomb barrier is shown to make extraction of a phase transition signature difficult.

I. INTRODUCTION

The internucleon separation dependence of the nucleon-nucleon interaction, attraction at long distances and repulsion at short distances, opens up the possibility that there may be a nuclear liquid-gas phase transition. Studies[1–5] of models for the nuclear equation of state show that the critical temperature of nuclear matter is probably in the 10—20 MeV range (with some estimates outside this range). This may allow such a nuclear gas to liquid phase transition to be observable experimentally, since thermal model analyses of intermediate energy nuclear reactions show that these temperatures can be achieved.

The search for experimental signals of the phase transition largely has concentrated on intermediate mass fragment emission in proton induced reactions, as researchers in this field were among the first to apply phase transition ideas to their data.[6–8] The yield of intermediate mass fragments $Y(A_F)$ is observed to decrease smoothly as a function of fragment mass A_F for $A_F \gtrsim 10$. Among other functional forms which approximate this decrease, the simple expression $Y(A_F) \propto A_F^{-\tau}$ has been used extensively. The exponent τ found in fitting the data was typically 2—3, in the range predicted by the thermal liquid drop model[9] for condensation around the critical point.

Of course, the whole idea of treating this kind of nuclear reaction as a phase transition has to be approached with caution. A sharp phase transition requires, firstly, a large number of particles, secondly, a long time scale relative to the time required for the phase transition to take place, and lastly, of course, an equilibrated system. Thermal model analysis of intermediate mass fragment data in proton induced reactions[10] typically shows that the number of nucleons in the system which produces the fragment is about double A_F. For the fragment mass range often examined, $10 \lesssim A_F \lesssim 20$, this implies a system of the order 40 nucleons. Density fluctuations for such a small number of particles are quite substantial, and will certainly tend to soften the sharpness of the transition. Consider, for example, a part of the Van der Waals-type equation of state shown in the inset of Fig. 1. In the Maxwell construction, states A are thermodynamically favored over states B and C. The probability for being in these other states can be calculated[5,11] and is shown in Fig. 1 for a 40 nucleon system and, for comparison, a 10 000 nucleon system. Clearly, one must go to temperatures well below the critical temperature before the transition becomes at all sharp for small systems.

Similarly, the time scale involved is fairly short. Hydrodynamical[12] and other calculations[13,14] give estimates for the lifetime of the hot region in the 10—20 MeV temperature range of $5-10\times10^{-23}$ sec. These results are supported by analysis[15] of nucleon emission in muon and proton induced reactions. On the other hand, the disassembly time in the liquid-gas transition is estimated[16] to be in the 10^{-22} sec regime. In other words, the time for the phase transition to occur is calculated to be comparable to the cooling time of the system.

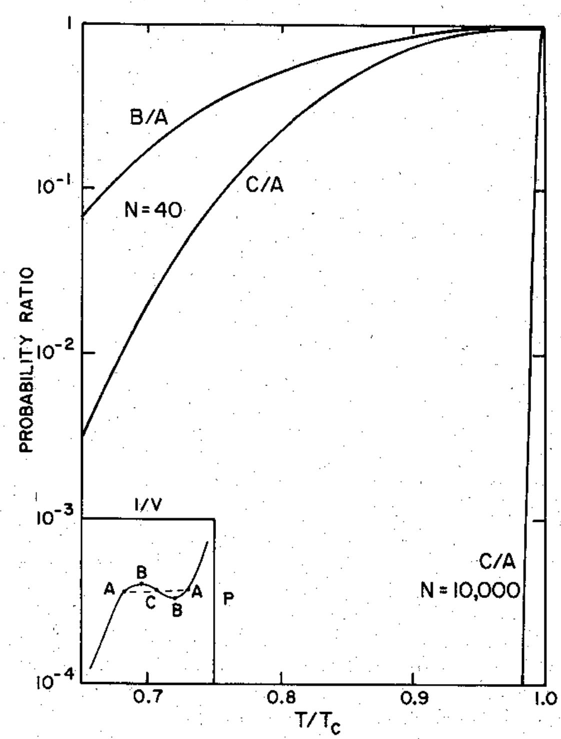

FIG. 1. Ratio of probabilities for a 40 nucleon system being in the states B and C compared to the thermodynamically favored states A. The inset shows the state labels in the Maxwell construction in a liquid-gas equation of state. See Ref. 5 for the calculational technique.

Of course, the fact that the mass yield curve can be described by a parametrized function is not confirmation of a phase transition. Several other[12,17–20] models which have nothing to do with phase transitions are equally successful in describing the data. The author has shown,[17] for example, that the mass distribution of a system of interacting nucleons which is allowed to form clusters will evolve to a form similar to that observed experimentally in about $5-10\times10^{-23}$ sec. In another approach,[12,19] the entropy per nucleon extracted from the mass yield curve is about what one would expect for the minimum entropy characteristic of the disassembly of a system in thermal and chemical equilibrium, without the extra entropy expected from a phase transition. Evaporative models[20] and ones involving the breakup of cold nuclear matter[18] can also reproduce the data.

In a recent paper,[21] the temperature dependence of the exponent τ was investigated in the analysis of several data sets.[6,7,10,22–26] As published, the temperature dependence appears to show discontinuous behavior, perhaps suggesting a phase transition. We wish to review here several of the model assumptions which go into the analysis of intermediate mass fragment data, and demonstrate the difficulties inherent in combining many different data sets together. The problem of extracting a reasonably unique temperature to be associated with a yield curve at a given bombarding energy is addressed in this paper, and an estimate of the large effect of Coulomb tunneling, omitted in most analyses, will be given. We will show that these effects cannot be ignored in the search for phase transition signatures. Lastly, we suggest where to search for signatures such that the effects are minimized.

II. SOURCE SIZE AMBIGUITIES

The first step in the thermal model analysis[27] of intermediate mass fragment data (for a general review of energetic fragment emission, see Ref. 28) is the determination of the velocity v_s and temperature T of the emitting region. The inclusive data are fitted by assuming that there is a single source for each fragment species, although the data show systematic deviations from the behavior expected in such a model. It will be assumed here (as elsewhere) that the deviations are not so large that the model is inapplicable.

We will concentrate on the results obtained in proton induced reactions, since they have been used most extensively in the search for phase transition signatures. The values for v_s and T found in these reactions vary strongly with ejectile mass, decreasing with increasing mass. The general trend can be interpreted to mean that heavier fragments are emitted from larger sources. Certainly, this behavior is required in the thermal model approach, since analysis[29] of the (p,p′) reaction indicates that the source size for nucleon emission is in the 5–10 nucleon range, certainly not what is needed for emission of a mass 20 fragment.

However, there may be fragment mass regions in which the source size is relatively constant. Consider the temperature dependence on fragment mass shown in Fig. 2 (data from Ref. 30). It has been pointed out[6–8] that fragment emission from a source of constant size A_s would lead to a linear relationship between T and A_F if recoil effects are included. That is, if one includes the recoil kinetic energy as well as the fragment kinetic energy in the $\exp(-E/T)$ term, then the apparent temperature of a given fragment will vary as

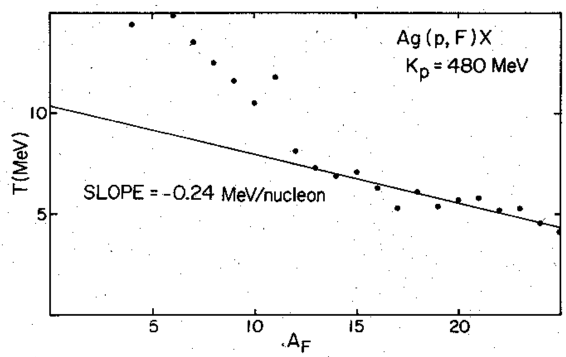

FIG. 2. Dependence of the fragment temperatures on mass. Data from proton induced reactions (Ref. 30).

$$T_F=T_0\left[1-\frac{A_F}{A_s}\right], \tag{1}$$

where T_0 is the "true" temperature. Hence, a plot of T_F vs A_F should yield a straight line with intercept T_0 and slope $-T_0/A_s$, so that both A_s and T_0 can be determined. In the example shown in Fig. 2, a best fit to the data gives $T_0=10.4$ MeV and $A_s=43$. In other proton induced reactions at 80–350 GeV, A_s is found to be equal to 75 in p + Kr and 110 in p + Xe. The source size found by this method is several times the fragment size, as is also found by estimating the size by using the temperatures and source velocities themselves[29] (for bombarding energies less than 1 GeV). That is, if one assumes that all of the energy and momentum of the incident proton is dumped into the source, then conservation of energy and momentum can be used to specify the number of nucleons in the source if v_s and T are known.

Clearly, the temperatures associated with smaller fragment masses do not fit on the straight line used to fit the mass 12–25 region. One could fit a straight line to part of this lower mass region as well. The steeper slope would tell one that the source size had to be smaller. The fact that several different source sizes are needed to describe the temperatures raises the possibility that the source size is actually a continuous function of fragment size: Rather than there being three of four discrete source sizes in these reactions there is a continuum of sizes. Suppose the source size is parametrized by

$$A_s(F)=xA_F\ . \tag{2}$$

Then, if a constant amount of energy E^* (on the average) is put into the source, the temperature associated with each fragment should vary as

$$T_F=\frac{2}{3}\frac{E^*}{A_s(F)}=\frac{2E^*}{3x}\frac{1}{A_F}\ , \tag{3}$$

where Maxwell-Boltzmann statistics have been assumed

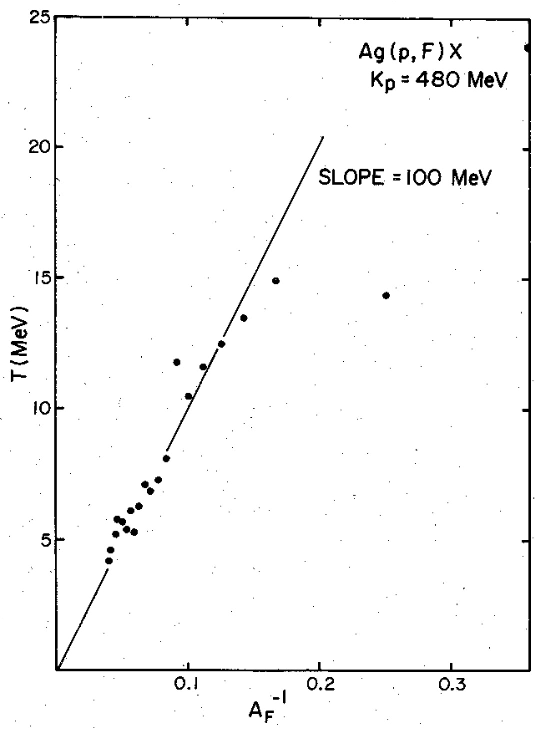

FIG. 3. Dependence of the fragment temperatures on the reciprocal of the mass. Data from proton induced reactions (Ref. 30).

and recoil effects have been dropped. Hence, a plot of T_F vs A_F^{-1} should yield a straight line. In fact, when the data shown in Fig. 2 are plotted in this way, straight line behavior is indeed observed, and it is observed for a much larger range of fragment masses than is fitted by the constant source approach of Eq. (1). To see if the parametrization is meaningful, it will be assumed that, for the data in Fig. 3, 400 MeV of the bombarding proton's kinetic energy goes into thermal energy, the rest goes into the momentum of the source. Then the experimentally determined value of the slope (100 MeV) implies that $x \simeq 2.7$: the source is double to triple the size of the fragment it emits. It should also be pointed out that this model is not the only one which can fit the temperature behavior over a wide range of fragment masses; the cold matter breakup calculations[18] are similarly successful.[31]

This picture of the source temperature decreasing by accretion of nucleons, as distinct from simply cooling by expansion, is intuitively what one would expect for proton induced reactions. The original source, which is involved in the fast emission of nucleons, is fairly small: about 6–10 nucleons. The source heats up the surrounding nuclear matter, itself slowing down and cooling in the process. One would expect a different scenario to apply for heavy ion reactions, where far more nucleons are involved in the first stages of source formation. There should be much less accretion than in proton induced reactions, and the temperatures should be far more constant as a function of fragment mass. Evidence that this is indeed the case can be found in Ref. 32, where the temperatures for the Ar + Au reaction at 42, 92, and 137*A* MeV were studied. The results of this analysis show an approximately constant temperature for $1 \leq A \leq 14$.

If the "accreting source" picture of proton induced reactions is correct, then one cannot treat fragment emission as occurring simultaneously independent of fragment mass, and use the yield curve to clearly extract information about phase transition parameters. Heavy ion reactions are more appropriate for this kind of study because the larger number of nucleons in the source help sharpen the transition both in the sense of fluctuations as shown in Fig. 1 and in the sense of providing a more uniform source for fragment emission.

III. COULOMB TUNNELING

Panagiotou *et al.*[21] have pointed out that the exponent τ associated with the yield curves increases with decreasing temperature, perhaps in a rather dramatic fashion. Their plot of τ vs T is reproduced in Fig. 4(a), to which a few extra points have been added. A rather dramatic point at $\tau = 1.7$, originally included in Ref. 21, is not shown in Fig. 4(a) because this point was generated from data[26] which was later included in a more complete[25] data set, used to generate point *U*. We omit the earlier data to

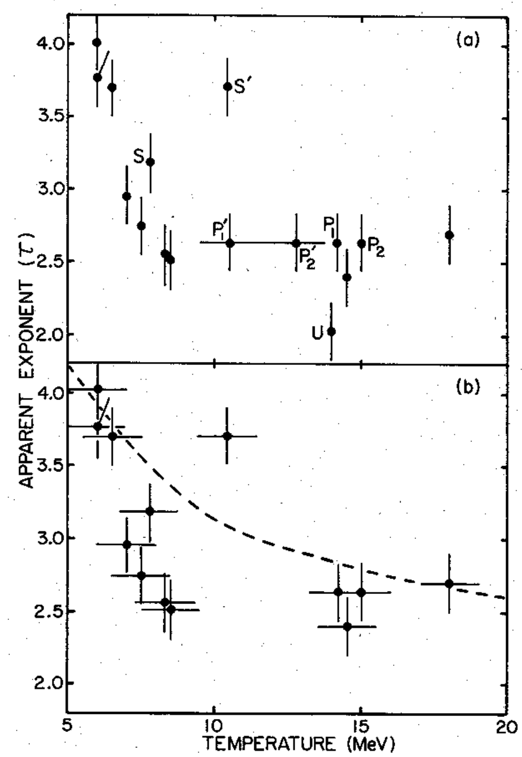

FIG. 4. (a) Temperature dependence of the exponent τ as given in Ref. 21. See the text for an explanation of labels. (b) Estimated dependence of τ on tunneling through the Coulomb barrier is given by the curve through the data.

avoid "double counting." Similarly, a data set[33] from a silver target at 5.5 GeV incident proton energy is omitted here (as it was in Ref. 21) because the data are included in the more complete experiment of Ref. 25.

The authors of Ref. 21 have tried to use as many data sets as possible in their analysis, to avoid the problem of fluctuations in a small number of data sets generating spurious behavior. Naturally, there are some drawbacks in doing this, and we will briefly comment on three. The first drawback involves the assignment of a temperature to a given yield curve (see also, Sec. IV). As was indicated previously, the actual fragment temperatures (as determined by thermal model analysis of the high energy fragment differential cross sections) are lower than the intercept temperature, as illustrated in Fig. 2. In Fig. 4(a) an average of the actual fragment temperatures associated with the intercept temperatures P_1 and P_2 are shown as points P_1' and P_2'. Clearly, some care must be taken to ensure that the quoted temperatures are extracted uniformly. Similarly, some of the yield data are available as $Y(A_F)$ while others are available only as $Y(Z_F)$. Using the charge instead of the mass in Eq. (1) leads to about a 10% shift in the intercept temperature. Again, caution must be exercised. Lastly, binding energy considerations will begin to play a large role in fragment emission for heavy targets. Hence, the point labeled U, which has a uranium target, has much more energy released than the other points, which are predominantly mass 100 targets.

Some of these effects have been taken into account in plotting Fig. 4(b). The U point has been dropped because of the grossly different Q value. Some nominal error bars have been restored. A new point, labeled S', has been included. This point is based on a more complete experiment[30] than point S, which allows use of Eq. (1) with mass rather than charge. Green, Korteling, and Jackson fit[30] the same range of masses as in Ref. 21, namely $A \gtrsim 6$.

One sees from Fig. 4(b) that the dip structure hinted at in Fig. 4(a) is less convincing. Nevertheless, the interesting rise in the exponent at low temperatures remains. Is this evidence for a phase transition, or is it explicable in terms of more conventional ideas? The yield curves used in determining the exponents in Fig. 4 are found by numerically integrating the inclusive differential cross section $d^2\sigma/dE\,d\Omega$. The differential cross section has its largest value at relatively low energies, in the region of the Coulomb barrier. For example, the authors of Ref. 30 show that Coulomb effects strongly cut off their heavy fragment yields in this energy region.

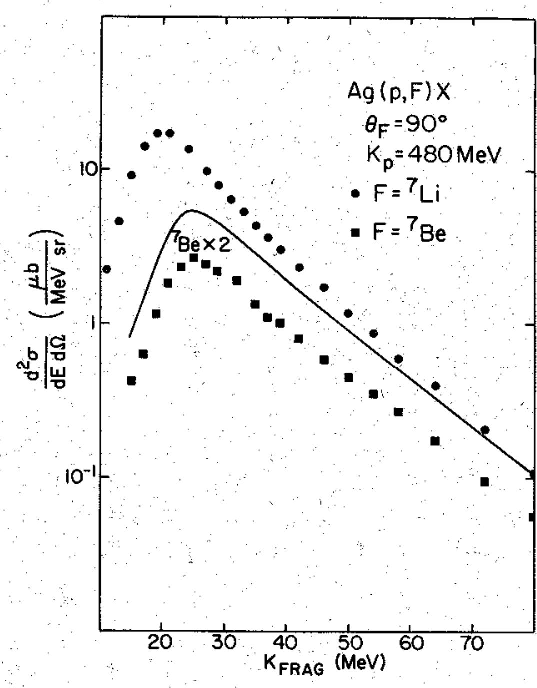

FIG. 5. Comparison of ^{7}Li and ^{7}Be energy spectra in the p + Ag reaction at 480 MeV and 90°. Data are from Ref. 30.

To illustrate the effect, shown in Fig. 5 are the energy spectra of ^{7}Li and ^{7}Be at 90° in the p + Ag reaction at 480 MeV. Combinatorial (the relative probability of selecting the protons and neutrons of the fragment from the silver target) and binding energy effects (the two nuclei have similar binding energies) could account for perhaps a factor of 2 in the relative yields. However, at low energies it is clear that ^{7}Be is substantially suppressed compared to ^{7}Li. One possibility, which we will return to in Sec. IV, is that the temperature associated with the low energy part of the spectrum is much lower than that of the tails. Another is that the necessity of the ^{7}Be to tunnel through a larger Coulomb barrier than the ^{7}Li suppresses the low energy part of the spectrum. If this effect is present for mass 7, then it should be even stronger for heavier masses.

To see if Coulomb effects can quantitatively explain the behavior shown in Fig. 4, we perform the following simple calculation. Suppose that there is a mass distribution given by $Y(A_F)=CA_F^{-\delta}$ (where C is a normalization constant) which describes the yield independent of temperature before Coulomb effects are included. The individual fragment spectra we will characterize by a Maxwell-Boltzmann distribution in energy. Now, at high temperatures, the existence of the Coulomb barrier will only have a major effect on the lowest energy part of the fragment spectrum. As the temperature is lowered, an increasing fraction of the large Z fragments will be subject to a Coulomb barrier in leaving the nucleus. The calculation consists of taking the spectrum inside the Coulomb barrier and modifying it by multiplying by a barrier penetration factor $P_A(q)$, where q is the fragment momentum. In other words,

$$\left.\frac{d^3\sigma_A}{d^3q}\right|_{\text{outside}} = P_A(q)\left.\frac{d^3\sigma_A}{d^3q}\right|_{\text{inside}}, \tag{4}$$

where the integral of the "inside" distribution obeys $Y=CA_F^{-\delta}$. The integral of the left-hand side then gives the yield. The penetration factor will be approximated by $\exp(-2\int \kappa\, dx)$, where $\kappa=\sqrt{2m[V(x)-K]}$, $V(x)$ being the Coulomb barrier and K the kinetic energy.

The change in the apparent exponent for the $10 \le A_F \le 20$ region is shown as the curve on Fig. 4(b). The high temperature exponent is chosen as $\delta=2.2$ here, although it can be obtained from the calculations of Refs.

17–20. The results depend upon the charge and mass of the emitting system, chosen to be 25 and 50, respectively, here, similar to what is found in analysis of the fragment differential cross sections.[8,10,30] No attempt was made to tailor the Coulomb barrier to get a "best fit" to the data. Similarly, no temperature dependence is given to the power law exponent before the Coulomb correction is applied. It should be clear from the curve that if nothing else, the magnitude of the Coulomb effects is in the same range as the variation in τ observed at low temperatures.

IV. DISCUSSION AND CONCLUSIONS

In summary, in spite of the growing calculational evidence that nuclear gas to liquid phase transitions should be expected in large, long-lived assemblies of nucleons, there are indications that the nuclear interaction region involved in proton induced intermediate mass fragment emission is both too small and too short lived to support a sharp transition. The mass yield curves themselves can be explained by several models which do not invoke a phase transition (including condensation without a phase transition). The change of the yield curves with temperature is consistent with what one expects from the necessity of the higher Z fragments to tunnel through a substantial Coulomb barrier at low temperature.

It should also be pointed out that the most appropriate temperature to be associated with a given yield curve may not be the temperature found in the thermal model analysis of the high energy tails. The yield has a very large contribution from the low energy part of the spectrum, which may have a substantially lower temperature than the tails. This was pointed out in Ref. 8, wherein the yields of several isobars were fitted and found to show a low temperature. For example, in Fig. 5, combinatorial effects alone indicate that the $^7Li/^7Be$ ratio should be about 1.3 assuming that the fragments originate from a 40 nucleon parent with the same p/n ratio as silver. If the remaining difference between the differential cross sections in the tails is attributed to $\exp(-\Delta BE/T)$, where ΔBE is the difference in binding energies of the fragment plus residual nucleus for each fragment (averaged over ^{107}Ag and ^{109}Ag parents), then $T\sim9$ MeV. However, the ratio of the yields is about 5, much higher than the ratio of the differential cross section tails. If the same argument is used, then the temperature associated with the yields is 3 MeV compared to $T\approx14$ MeV from Fig. 2. Presumably this low value is an indication that the low energy part of the spectrum, which contributes a sizable fraction of the yield, has a large evaporative component. Whatever the origin of the effect, it demonstrates that the temperature scale in Fig. 4 might be much lower.

Clearly, for experimental evidence of the liquid-vapor transition to be convincing, the ambiguities outlined above should be avoided. Heavy ion reactions would be preferable to proton induced reactions as the former both would provide a larger multiplicity which, in turn, should give a sharper transition, and would give a more uniform freeze out condition. In proton induced reactions it remains troublesome that the source appears to accrete nucleons with time, and is only about double or triple the fragment mass: fragmentation in this picture looks more like fission. Because of the large change in Q value as the target mass enters the mass 200 range, a mix of targets which includes this mass range should obviously be avoided.

It would also be useful to limit the effects of Coulomb tunneling as much as possible. One procedure would be to fit the high energy tails of the differential cross section with a single source model and then determine the yield from the normalization, as was done for (p,p′) in Ref. 29. This would also allow a cleaner association of a temperature with the yield curve, since the temperature ambiguity associated with the low energy part of the spectrum could be avoided.

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