LOW TEMPERATURE PHASE TRANSITIONS *

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ABSTRACT

Calculations of the equation of state of nuclear matter strongly suggest the existence of a liquid-gas phase transition. However, how sharp the transition will appear in finite nuclei, and what the experimental signatures will be are questions which evoke considerable debate. The current status of these issues, particularly the experimental signature ambiguities, is reviewed here.

1. INTRODUCTION

It has been found[1,2] that statistical ideas play an important role in heavy ion reactions and even proton induced reactions providing the target is sufficiently heavy. Indeed, it may prove possible to use these reactions to probe the nuclear equation of state. One of the more intriguing aspects of the equation of state[3-8] would be the existence of a nuclear liquid-gas phase transition. That such a phase transition should exist for infinite nuclear matter (finite systems will be considered later) has support from many detailed calculations. However, the essentials can be obtained from the following simple model.

The internucleon separation dependence of the nucleon-nucleon interaction, attraction at long distances and repulsion at short, is of the same general form as the intermolecular force, and hence may lead to a phase transition for nuclear matter similar to the liquid-gas phase transition of the molecular world. As is well known, an

* Invited talk

interparticle potential of the square well form shown in Fig. 1 generates a van der Waals-type equation of state (neglecting spin)

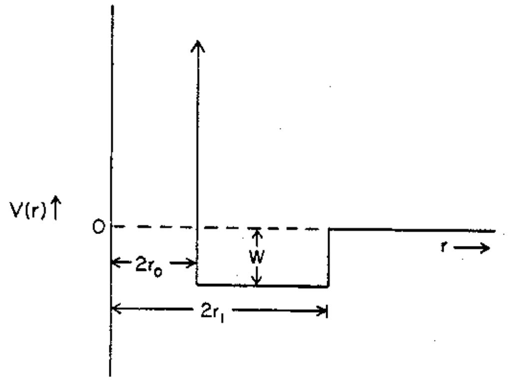


Fig. 1. Square well form for NN potential used to generate van der Waals-type equation of state.

$$\left(P + \frac{a}{\tilde{V}^2}\right)(\tilde{V}-b) = T \tag{1}$$

where

$$a = \frac{V_1 - V_0}{2} W \qquad b = \frac{V_0}{2} \qquad V_i = \frac{4\pi}{3}(2r_i)^3 \ . \tag{2}$$

Here, Boltzmann's constant has been absorbed into the temperature T, and $\tilde{V}$ is the volume per particle. This equation of state possesses a liquid-gas phase transition, with the critical point occurring at

$$T_c = \frac{8}{27}\frac{V_1 - V_0}{V_0} W \qquad \tilde{V}_c = \frac{3}{2} V_0 \ . \tag{3}$$

For the case at hand, choosing $2r_0$ = 1 fm, $2r_1$ = 1.64 fm and W = 10 MeV (these nucleons are bosons, so the potential well is not very deep), one finds that T_c = 10 MeV and $\rho_c = 0.16\ fm^{-3}$. This is admittedly very crude but shows the T,ρ range expected for the critical point. More sophisticated calculations[5,8] have also been performed. One of these[5] shows the critical temperature dropping from 22-28 MeV for infinite nuclear matter to 16-20 MeV for finite nuclei. Another approach[8] gives even lower temperatures. The critical density found is typically in the range 1/2 ρ_0. The remaining part of the phase diagram can also be calculated (for example, by using the Maxwell construction in the van der Waal's example) and has a form shown schematically in

Fig. 2. The regions marked superheated and supercooled are metastable regions which are not thermodynamically favored but nevertheless may be accessible, depending in part on the time scales involved.

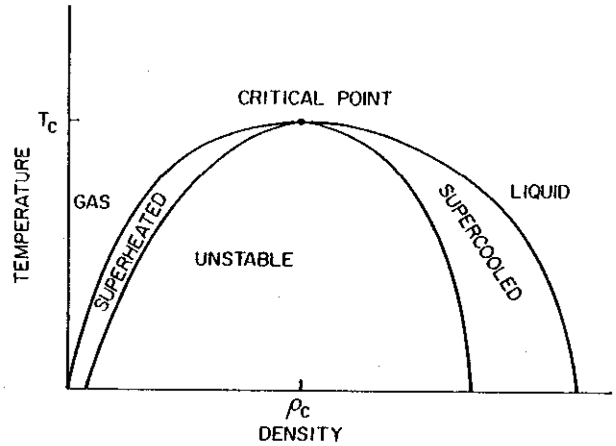


Fig. 2. Schematic representation of the liquid-gas phase transition region.

2. TIME AND SIZE SCALES

The range of temperatures and densities found above for the phase transition region are typical of those obtained in the thermal model analysis of both proton and heavy ion induced reactions.[2] The path that the reaction region in the nucleus follows in the T,ρ plane might look something like that shown in Fig. 2, depending on the starting conditions: the interaction region is initially hot and dense and then cools as it expands. Although one can choose bombarding energies such that the reaction pathway passes through the phase transition region, whether there is a sharp phase transition depends on the size and lifetime of the interaction region. Further, if the interaction region remains in thermal and chemical equilibrium long after the phase transition has taken place, then experimental information about it may be lost.

The size problem will be dealt with first. In the thermal model analysis of energetic particle emission, the velocity of the source region is one of the parameters determined by the fitting procedure. Assuming that the projectile loses all of its momentum to the interaction region, one can then use conservation of momentum to determine the maximum number of nucleons in the source. As we will return to below, most of the data advanced as evidence for the phase transition involves proton induced fragmentation, and it is found[2] that the interaction region contains about ten nucleons for nucleon emission, perhaps forty for fragmentation.

Density fluctuations for such a small number of particles are quite substantial, and will certainly tend to soften the sharpness of the transition. Consider, for example, a part of the van der Waals-

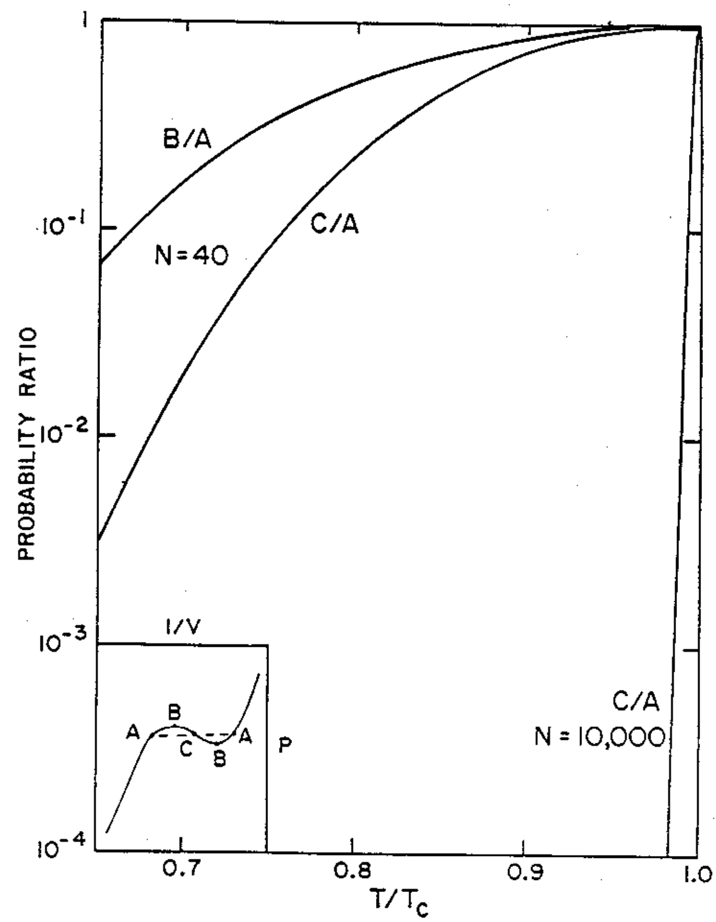


Fig. 3. Ratio of probabilities for a 40 nucleon system being in the states B and C compared to the thermodynamically favored states A. The inset shows the state labels in the Maxwell construction in a liquid-gas-equation of state. From Ref. 10.

type equation of state shown in the inset of Fig. 3. In the Maxwell contstruction, states A are thermodynamically favored over states B and C. One can calculate (see Ref. 7 and references therein) the probability of being in states B and C compared to A, and this is shown in Fig. 3 (from Ref. 10) for a 40 nucleon system and, for comparison, a 10,000 nucleon system. As is clear from this figure, one must go to temperatures well below the critical point before the transition becomes at all sharp for small systems.

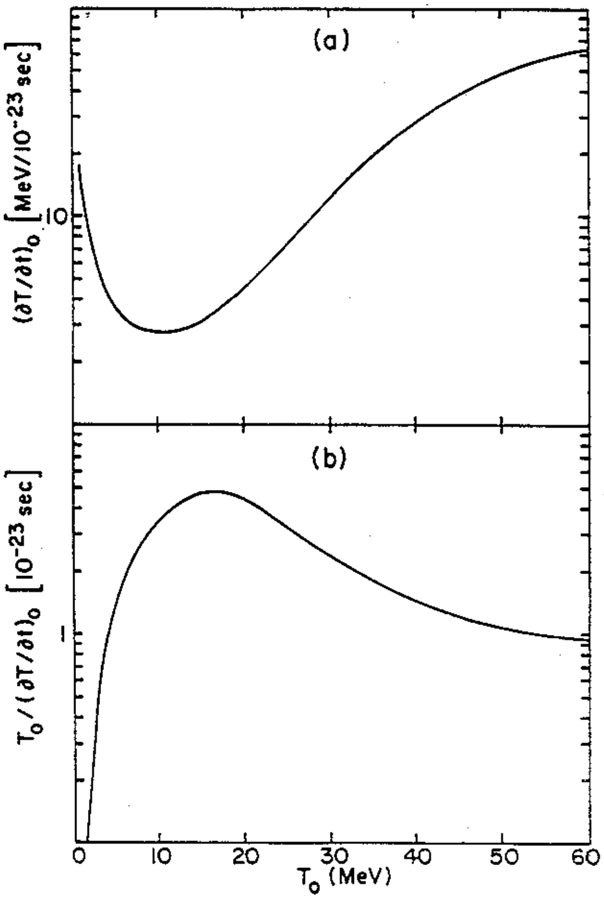
The time scale associated with these reactions may also be short compared to what one typically expects for a phase transition. Curtin, Toki and Scott[11] estimate that the phase transition has a time scale in the 10^{-22} sec regime. In contrast, the time for nucleon emission is probably considerably shorter. For example, in proton induced reactions, the ratio of the differential cross sections for the (p,p') and (p,n) reactions is far from its naive chemical equilibrium value,[12] and so can be used to estimate how far towards chemical equilibrium the nucleon emitting region has evolved. The calculations indicate[13] that the system has evolved for $\sim 10^{-23}$ sec.

For proton induced reactions, one can imagine that the projectile may deposit its energy in a small region of the target. One could then use the classical diffusion equation

$$\frac{\partial T(r,t)}{\partial t} = \frac{1}{C_p} \operatorname{div}\left(\frac{\kappa}{\rho} \operatorname{grad} T(r,t)\right) \tag{4}$$

to calculate the lifetime of the hot zone.[14] In Eq. (4), the temperature T is a function of r and t, while the quantities C_p, κ and ρ are the specific heat, thermal conductivity and density respectively. An approximate solution to this equation at r=0 for a gaussian source is shown in Fig. 4. Taking the central temperature divided by the cooling rate as a measure of the lifetime of the hot region, one can see that a lifetime in the few times 10^{-23} sec region is expected. This is similar to what is obtained from hydrodynamical[15] and other[16] estimates.

Although the nucleons may go out of equilibrium in the 10^{-23} sec range, heavy nuclear fragments may remain in equilibrium much longer because of their larger reaction cross sections. An analysis of two-



particle interferometry, which has conventionally been applied to two pion and two proton correlations, has been extended[17] to two deuteron and two triton correlations.

Fig. 4. (a) Estimated cooling rate of the r=0 region of a gaussian hot spot. (b) Central temperature divided by the cooling rate (from Ref. 2,14).

Data for the correlation function is shown in Fig. 5, along with calculations of the correlation function. For the d-d correlations, two sets of phase shifts were used for the nuclear part: a resonating group approach (RG) which generated attractive potentials, and an R-matrix approach (RM) which generated repulsive potentials. The data agree with the repulsive potential calculations. Parametrizing the particle emission region as a gaussian in space, the radius parameter r_0 is found to have the values 3-4, 6-8 and ~8 for p-p, d-d and t-t correlations under the same experimental conditions. This may imply that nuclear fragments, which have larger cross sections than protons, remain in equilibrium for a longer period of time.

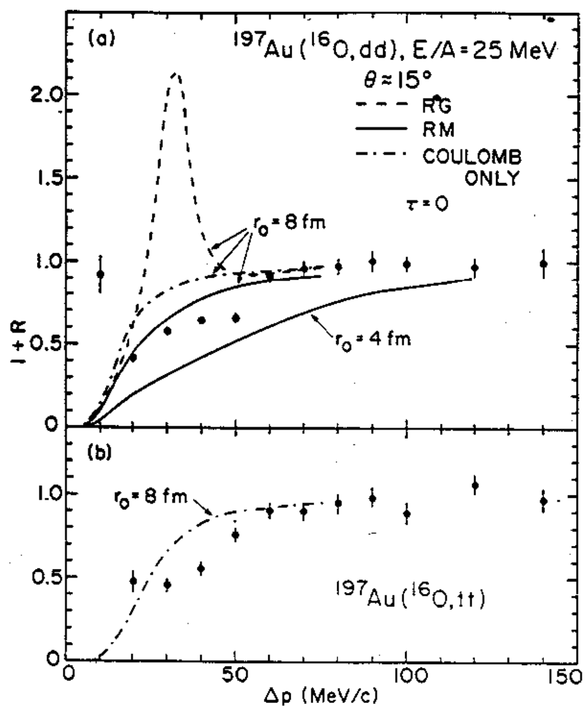


Fig. 5. Data for the two deuteron (a) and two triton (b) correlation functions shown in comparison with theory. Three sets of phase shifts were used in the theoretical analysis: coulomb only, resonating group calculation of nuclear part (RG) and R matrix calculation of nuclear part (RM). From Ref. 17.

Tentative support for this idea also comes from an interpretation of a novel measurement of fragment temperatures. The experiment[18] involves measuring the first excited state to ground state population ratio for ^{6}Li, ^{7}Li and ^{7}Be in a heavy ion reaction. The population ratios give temperatures in the 1/2 to 1 MeV range, whereas the single source thermal model fit to the differential cross section gives a temperature of 8–9 MeV. It has been argued[19] that the population ratio should reflect the local freeze-out temperature in a comoving frame with the expanding fireball: that is, the excited state population will be reduced by final state interactions among the hadrons as the fireball cools. The freeze-out temperature can be estimated by comparing the expansion time of the fireball with the reaction time for hadronic cooling calculated with experimentally measured N+Li (or Be, as required) cross sections. This is shown in Fig. 6. The freeze-out temperature so calculated is in the range indicated by the population ratio experiment. The time taken to freeze out (from the initial hot stage) is ~2×10^{-22} sec.

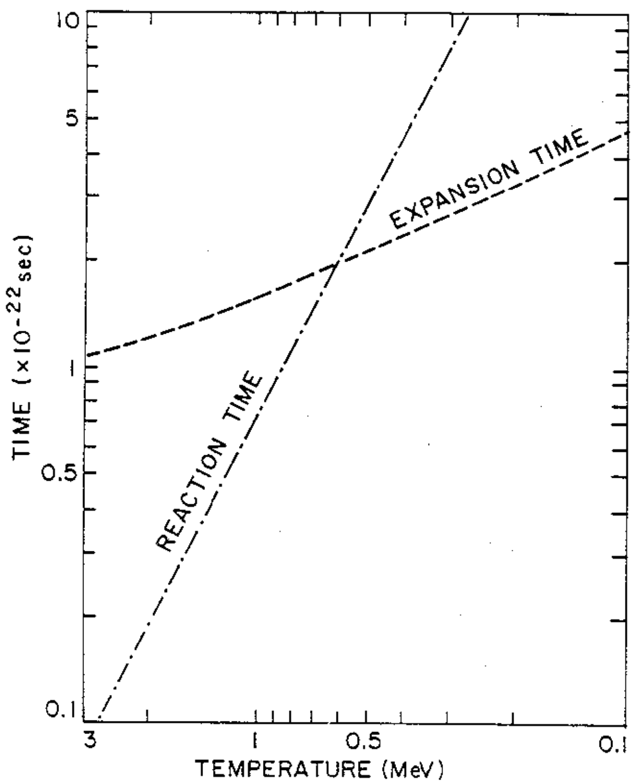


Fig. 6. Comparison of reaction time for lithium excited state to ground state transition (in a hadronic background) with expansion time for the fireball.

Hence it may be possible to measure later stages of the reaction by finding observables associated with heavy nuclear fragments. However, the sequential freeze-out aspect of heavy fragment emission may make fragment-to-fragment comparisons over large mass differences somewhat dangerous. Further, if the observable chosen for measurement is sensitive to a very late stage of the reaction, then information about the phase transition may be washed out. However, these calculations do indicate that fragments remain in equilibrium well into the phase transition region.

In summary, the facts that the system may be too small or short lived for there to be a sharp transition, or that the transition occurs at the same density range ($\rho_0/2$) as the reaction region goes out of thermal equilibrium, may mean that the phase transition idea may not be cleanly applied to nuclear reactions. Nevertheless, reaction conditions may be close enough to those required for a phase transition that there may be some experimental effect. The situation is similar to trying to discover the liquid-gas transition of water by watching 40 molecules for 10^{-12} sec.

3. EXPERIMENTAL SIGNATURES

Historically, the search for experimental signatures has concentrated on heavy fragment emission in proton induced reactions, (Refs. 20-22) and it is these reactions which will be dealt with here. One of the first observations put forward as evidence of a phase transition effect was the form of the heavy fragment mass yield curve. In the thermal liquid drop model of droplet formation,[23] it is found that at the critical point, the mass distribution of droplets has the simple form

$$Y(A_F) \propto A_F^{-\tau} \tag{5}$$

where τ has a value between 2 and 2.5.

As shown in Fig. 7, the yield does fall with increasing fragment mass, A_F, and can be approximately fitted with the parametrization of Eq. (5). In fact, at high energies, the phenomenologically determined value of τ is typically in the 2-3 range. The general form of $Y(A_F)$ away from the critical point has additional parameters and has been used to fit yield data for a wide range of bombarding energies.

Of course, the droplet model approach is only one of several[24-27] which have been used successfully to describe the same data, and so the agreement with Eq. (5) should not be taken as proof of a phase transition. For example, one alternate approach[24] adopts the same condensation picture (i.e., one starts with a system of nucleons which are allowed to collide and form fragments) but follows the explicit time development of the system by solving a simplified set of coupled rate equations. The temperature and density are assumed to be constant over the fragment formation epoch, the time required for fragment formation

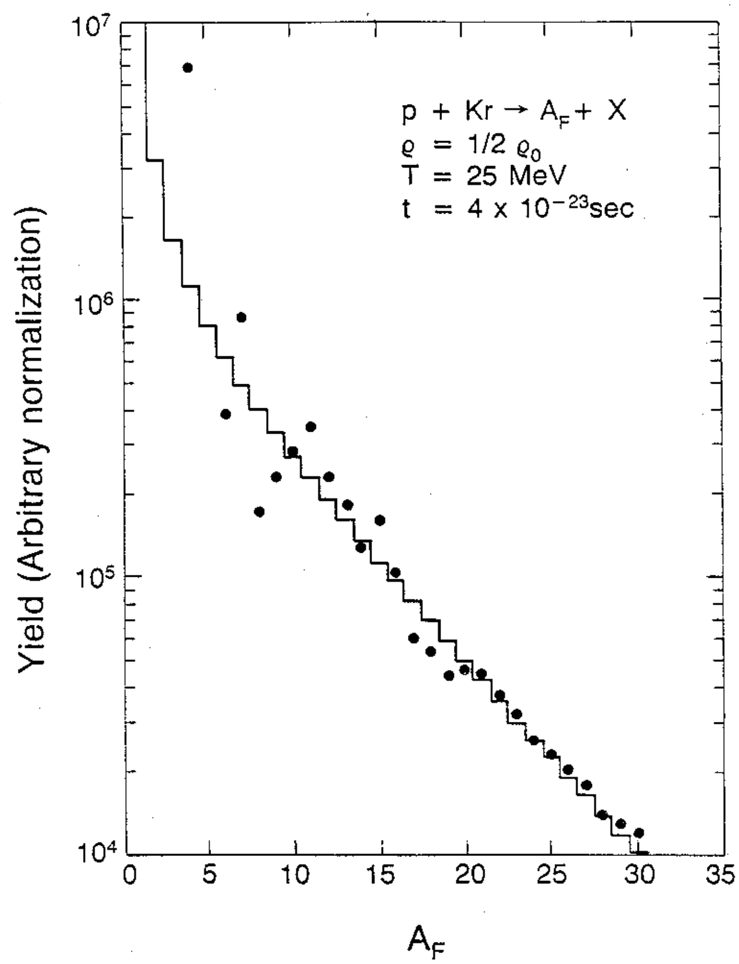


Fig. 7. Yield curve (from Ref. 22) for the p+Kr reaction at 80-350 GeV bombarding energy. The histogram shows the results of a rate equation approach to fragment formation (from Ref. 24) for the parameters shown.

then being determined phenomenologically. The time found by fitting the data in Fig. 7 is 4×10^{-23} sec for the parameters shown. Such a time is consistent with the other time estimates which have been made for the expansion times; however, the exponent τ does not show discontinuous behavior as a function of time in this approach. Other approaches will be discussed below.

Panagiotou et al.[28,29] have proposed that the temperature dependence of the apparent exponent τ in Eq. (5) might signal a phase transition. They observe a dip in the value of τ as a function of temperature (where τ has been determined by fitting the yield curves with $A_F^{-\tau}$ even away from the critical point and the temperature has been determined by fitting the differential cross sections). An example of the temperature dependence of τ (taken from Ref. 29, since the author feels Ref. 28 had certain consistency problems[10]) is shown in Fig. 8. Certainly one observes a definite rise in τ at low temperatures. Is this rise evidence for the phase transition?

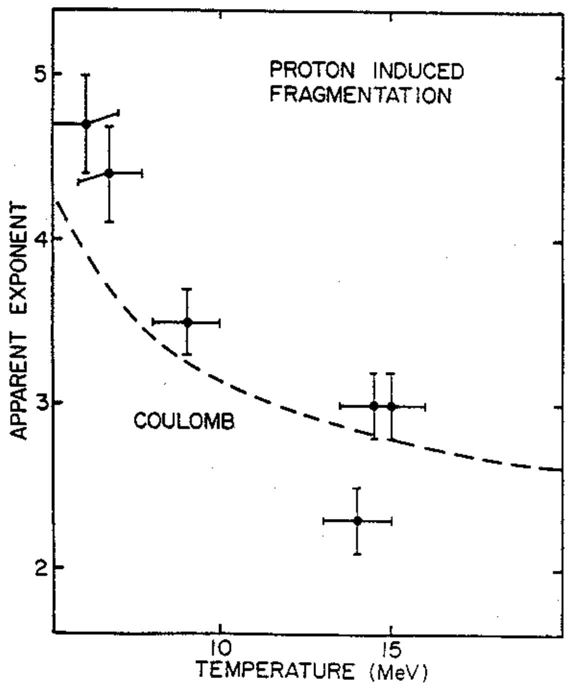


Fig. 8. Apparent behavior of power law exponent τ as a function of temperature ("Data" from Ref. 29). Shown for comparison is an estimate of the Coulomb effect on τ (Ref. 10).

The yields themselves are obtained by numerically integrating the inclusive differential cross section, which has its largest values at low energy in the region of the Coulomb barrier. An example of the differential cross section for two mass 7 isobars in the 480 MeV p+Ag reaction[30] is shown in Fig. 9. Combinatorial and binding energy effects may account for a factor of two difference in the high energy tails.

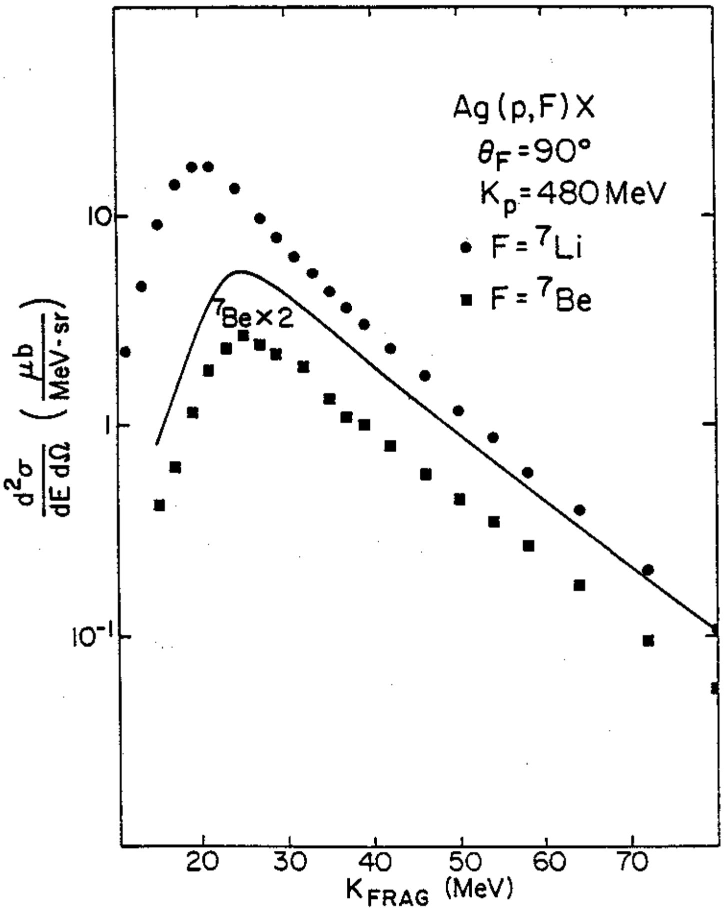


Fig. 9. Comparison of ^{7}Li and ^{7}Be energy spectra in the p+Ag reaction at 480 MeV and 90°. Data are from Ref. 30.

At low ejectile energies, ^{7}Be production is suppressed with respect to ^{7}Li, perhaps because of Coulomb effects and/or the presence of a low temperature component to the cross section which would favor ^{7}Li over ^{7}Be on binding energy considerations. The yields, which are strongly affected by this region, are observed to differ by about a factor of five, not the factor of two expected from the tails.

A simple calculation to see if Coulomb effects could account for the rise at low temperature has recently been performed.[10] The energy distribution of a particular fragment was assumed to be Maxwell-Boltzmann inside the Coulomb barrier, but reduced by a barrier penetration factor outside. The results of the calculation are shown in Fig. 8. One can see that the predicted variation in τ is similar to what one finds experimentally.

However, the temperature which should be associated with the yield curve may not be the temperature associated with the high energy tails as has been done in Fig. 8. For example, if one attempts to fit the yields of a group of isobars with an expression like $\exp(-\Delta BE/T)$ where ΔBE is the difference in binding energies, then much lower temperatures are obtained than those formed by analyzing the energy spectra tails. Hence, what dominates the yields may be a low temperature component of the energy spectra. Perhaps this low temperature component arises from hadronic cooling as was suggested for the excited state population ratios. Alternatively, it may mean that the yield curves largely reflect the breakup of cool nuclear matter. Models involving[25-27] the statistical breakup of cool matter in fact have been successfully applied to fragmentation. Consequently the critical temperature may be significantly lower than 10-15 MeV.

4. SUMMARY

There is impressive calculational evidence that nuclear gas to liquid phase transitions should be expected in large, long lived assemblies of nucleons. Further, there is also tentative evidence that final state interactions may keep nuclear fragments in equilibrium well into the phase transition region in the p,T plane during disassembly. There are indications that the nuclear interaction region involved in intermediate mass fragment emission may be both too small and too short lived to support a sharp transition. The mass yield curves themselves can be explained by several models which do not invoke a phase transition. The change of the yield curves with temperature is approximately what one expects from the necessity of the higher Z fragments to tunnel through a substantial Coulomb barrier at low temperatures.

Better experimental signatures (or disproof of the alternate models proposed for the current signatures) are required before phase transitions can be said to be established.

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