A CASCADE APPROACH TO THERMAL AND CHEMICAL EQUILIBRIUM*

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ABSTRACT

The temperature and density regions reached in intermediate and high energy nuclear reactions are investigated via the cascade approach. In one calculation, a nucleon-nucleon cascade code for proton induced reactions is used to find the reaction path near the liquid-gas phase transition region. It is shown that, for these reactions, fragmentation more resembles bubble growth than droplet formation, although the reverse may be true for heavy ion reactions. A quark-gluon cascade code based on QCD is applied to ultra-relativistic heavy ion collisions. It is shown that at least partial thermalization of the initial quarks and gluons is achieved. The energy density in the central region is found to be at least several GeV/fm^3.

I. INTRODUCTION

Computer simulations of nuclear reactions involving the production of energetic ejectiles[1] have been used for some time. The various simulations often involve differing assumptions about the mean-free path in their local environment of the particles being studied. Each approach has found domains of applicability, depending on the projectile, target, energy etc. under investigation.

The approach which we wish to use here is the cascade model, in which the particles are treated classically and their interactions are assumed to be describable as a sequence of independent interactions. There are two applications of the cascade approach which will be described here. The first is a traditional nucleon-nucleon cascade

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applied to intermediate energy proton induced reactions. There are two questions in proton induced reactions to which the simulations may provide answers:

i. Is there actually a dynamic equilibrium established in these reactions? In other words, is the mean time between NN collisions short compared to the lifetime of the system?

ii. What trajectories do the interaction regions take in approaching the liquid-gas phase transition region? Does fragmentation look like the condensation of vapor or the growth of bubbles?

The second application is the development of a cascade model of quark and gluon interactions. The physics of the simulation presented here is not as refined as it should be (and hopefully will be in the near future) but is nevertheless adequate to answer several important questions:

i. How rapidly is the bombarding momentum degraded?

ii. What is the energy and baryon number density in the central region?

Before moving on to discuss the results in more detail, a word should be made about computing time required. The calculations were performed on an IBM 3081-GX mainframe. For the nucleon-nucleon cascade, usually a few CPU hours were all that were required for the calculations presented here. For the quark-gluon cascade, things are dramatically more complicated: typical running times were one hour per event. Lastly, in this short note only the results will be presented; the details of the codes will be published elsewhere.

II. PROTON INDUCED REACTIONS

In simulating this reaction, it will be assumed that the target nucleons form an ideal Fermi gas filling to the top, a potential of depth 40 MeV as illustrated in Fig. 1. The boundary of the well is held fixed in time, since at the energies considered here, even if the projectile lost all of its momentum to the target, the resulting target velocity is so low that the well would not have moved appreciably in the time frame used. Those nucleons whose energy (energy defined here as kinetic plus potential) is less than zero collide elastically with the wall, while

those with energy greater than zero are free to leave. Parametrized p+p and p+n cross sections are used (assumed to be isotropic in the c.m. frame). Because of the relatively few number of collisions compared to a heavy ion reaction, Pauli blocking is handled in a simple way: if a collision between any two nucleons results in one of them having energy less than zero, the collision is considered blocked. Clearly, if one wishes to investigate detailed properties of the residual nucleus, or go to very low energies, a better job would have to be done.[2] For the observables considered here, the routine should be accurate to 8%.

The momentum space densities predicted for the collision of a 300 MeV proton with a mass 100 target (50 p's + 50 n's) is shown in Fig. 2. The collisions have been impact parameter averaged. The axes represent the momentum parallel and perpendicular to the beam direction, and the momentum space density is in units of particles per $(50\ \mathrm{MeV/c})^3$ [just for comparison, in the target nucleus there are 0.14 nucleons per $(50\ \mathrm{MeV})^3$]. Only those particles with $E > 0$ are shown, i.e., none of the nucleons in the residual nucleus are shown.

The calculation was stopped at a time of 8×10^{-23} sec after the projectile entered the target. On the r.h.s. of the figure one can still see an enhancement in the momentum space region about the projectile. The momentum distribution itself looks surprisingly thermal, showing a temperature and apparent source velocity similar to that found in thermal model analyses of proton inuduced reactions. For intimates of the thermal model, the calculation even shows the observed increase in apparent source velocity with increasing ejectile energy.

However, even if the energy spectrum has a thermal appearance, it does not necessarily hold that a dynamic equilibrium has been established. To answer that question, one must look at the nucleon-nucleon reaction rate. By looking at the spatial densities found in the code, it is easy to see that the reaction rate is going to be low. Shown in Fig. 3 are the coordinate space densities for a central 300 MeV p + (Z=N=50) system after 4 and 8×10^{-23} sec. Again, only those nucleons which have $E > 0$ are shown. One can see that, except for the region immediately around the projectile, the densities are low, resulting in a

low collision rate. The actual collision rate as a function of time is shown in Fig. 4. Although the collision rate goes as high as 4×10^{-23} sec in the figure, 3/4 of these collisions are blocked by the Pauli principle: the mean time between allowed collisions is long. Further, most of those collisions in Fig. 4 which are allowed involve the projectile: there is very little multiple scattering of the secondaries.

Hence, we see that thermal equilibrium is established, if at all, in only a restricted region of the target. The same is true of chemical equilibirum. The ratio (p,n)/(p,p') of the high energy part of the ejected nucleon spectrum is a measure of how close to chemical equilibrium the reaction has come. At chemical equilibrium, this ratio should be unity, whereas it is observed[4] to be ~1/2. The cascade simulation (Fig. 5) verifies that chemical equilibrium is not achieved (as was found previously in a rate equation analysis[5]). In summary, it may be that neither thermal nor chemical equilibrium is achieved among those nucleons knocked out of a proton induced reaction. This should not be taken to imply that the nucleons in the residual nucleus cannot re-equilibrate: that is a problem which is currently under investigation.

Let us now apply this simulation to the liquid-gas phase transition. One of the questions in the study of the transition is whether the approach to the mixed phase resembles droplet formation[6] or bubble growth.[7] Of course, which scenario is applicable depends on the reaction involved, and here we will concentrate on proton induced reactions, which historically were the first ones examined for evidence of the liquid-gas transition.[8] We have already seen that the densities of the nucleons struck from the target are low. In fact, on average in the reactions considered above, there are fewer than ten nucleons emitted from the target (as is observed experimentally). These are hardly enough to coalesce into a mass 15 fragment, typically the size used in looking at the droplet problem. Indeed, one can look at the multiplicity distribution found in the cascade. For more than 10 nucleons, the multiplicity drops very rapidly, like m^{-6} in Fig. 6. If the probability of finding a droplet of mass A increased with the number of nucleons available to form the droplet, then one would expect the droplet yield to drop at least as

fast as A^{-6}, in clear contradiction with experiment.[8] Hence, for a proton induced reaction the problem more resembles break-up of the residual system. Certainly, at least the temperatures seem to correspond. If one analyzes[9] the temperatures found from the isotopic abundance ratios of the medium mass fragments,[10] a temperature of 2-3 MeV is obtained.[11] Similarly, the residual excitation energy in the cascade approach also corresponds to a temperature of 2-3 MeV. Whether the mass distributions also correspond is currently under investigation.

III. QUARK-GLUON CASCADE

At much higher energies than what have been considered above, quark and gluon degrees of freedom will become much more important. To investigate what energy densities one should encounter in relativistic heavy ion collisions, we have constructed a quark-gluon cascade code. The physics of what goes into this code is still changing, so only an outline will be given:

i) The initial x-distributions of the quarks [q(x)] and gluons [g(x)] are taken from deep inelastic scattering. Similarly, the transverse momentum distribution of the partons is taken to be of the form $\exp(-p/p_0)$ with $p_0 = 0.35$ GeV/c.

ii) The quarks and gluons are randomly distributed in the nucleus, 5 gluons, 3 quarks and no antiquarks in each nucleon. The nuclei (equal mass) are then Lorentz contracted with respect to the c.m. frame. Lastly, the partons are spread out (in a Monte Carlo sense) about their (Lorentz contracted) coordinates by a Gaussian distribution with the width determined by the momentum.[12]

iii) First order QCD cross sections are used.

iv) The partons are allowed to scatter off-shell, then decay.

Obviously, the code must contain many low momentum cutoffs to avoid singularities in the non-perturbative regime.

It is found that, for (A=50) + (A=50) at $\sqrt{s_{NN}} = 50$ GeV and zero impact parameter there is significant thermalization. As a measure of this, the antiquark momentum distributions in the central region (defined

as a cylinder of length 1 fm and radius 3 fm centered on the center of mass coordinate of the two nuclei [necessarily stationary in the c.m. frame] were extracted. Antiquarks were chosen because they were initially absent in the colliding nuclei, and are therefore a relatively clean estimate of thermalization effects. Shown as a function of time in Fig. 7 is the antiquark central energy density for (A=20) + (A=20) and (A=50) + (A=50). One can see that it rises quite rapidly as the nuclei cross and reach a maximum of at least 1 GeV/fm^3. This corresponds to a temperature of at least 200 MeV (as has been found elsewhere[13]) demonstrating that these kinds of collisions should put one in the energy range required for the quark-gluon phase transition.

Shown in Fig. 8 are the relative q and $\overline{q}$ densities. In the initial stages, the region is mainly baryons so that the difference in quark ($\equiv Q$) and antiquark ($\equiv\overline{Q}$) densities is about equal to Q. With time, $Q - \overline{Q}$ drops off faster than $Q + \overline{Q}$, showing that one is left with a high density region with relatively low baryon number density.

The code is clearly going to be limited in its applicability to low momenta phenomena: it has difficulty handling with any accuracy the very large number of soft partons characteristic of the confinement region. Nevertheless, it should be useful for predicting large $p_\perp$ phenomena, baryon number rapidity distributions (to which it is now being applied) and other effects not dominated by small x partons.

IV. SUMMARY

Two applications of the cascade approach to nuclear reactions have been presented. In one, the reaction path for intermediate energy proton induced reactions is simulated, and it is shown that these reactions probably approach the liquid gas phase transition region via a low temperature break-up of the residual target nucleus. It is shown that complete equilibrium among the ejected nucleons is not achieved.

In the other application, preliminary results from a quark-gluon cascade simulation are given. As examples of what this code can generate, the energy and baryon densities of the central region in equal mass relativistic heavy ion collisions are shown. The antiquark energy

densities can reach 1 GeV/fm^3 even for mass 20 on 20 at 25 A•GeV (in the c.m. frame) corresponding to a temperature of more than 200 MeV.

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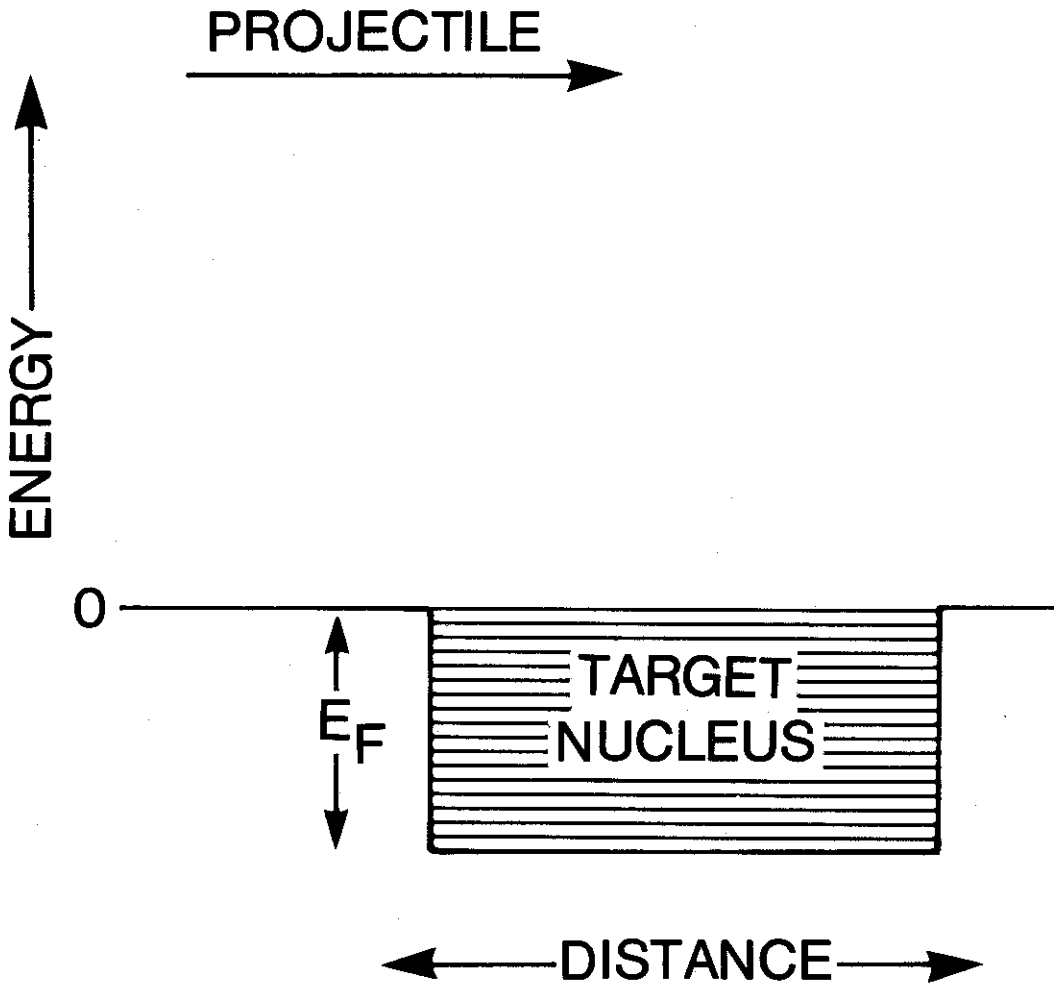
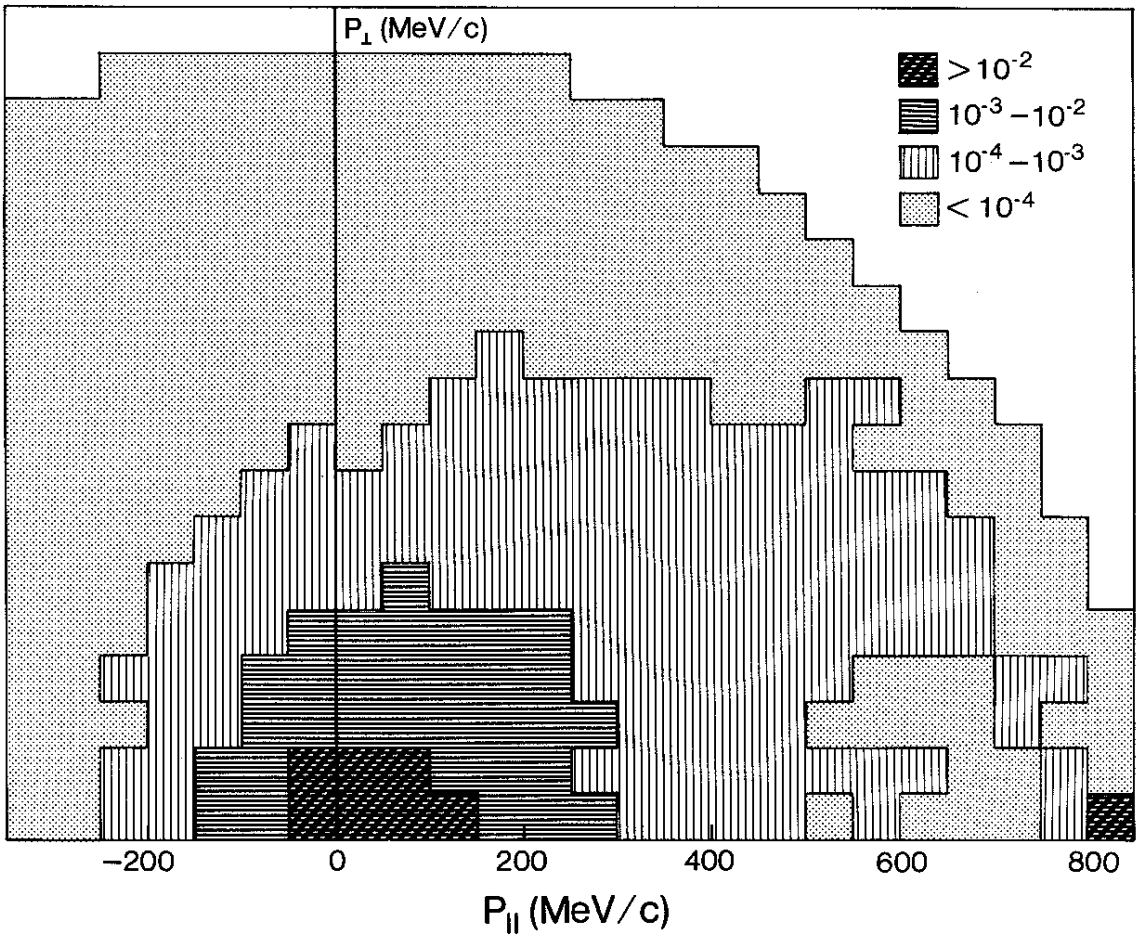
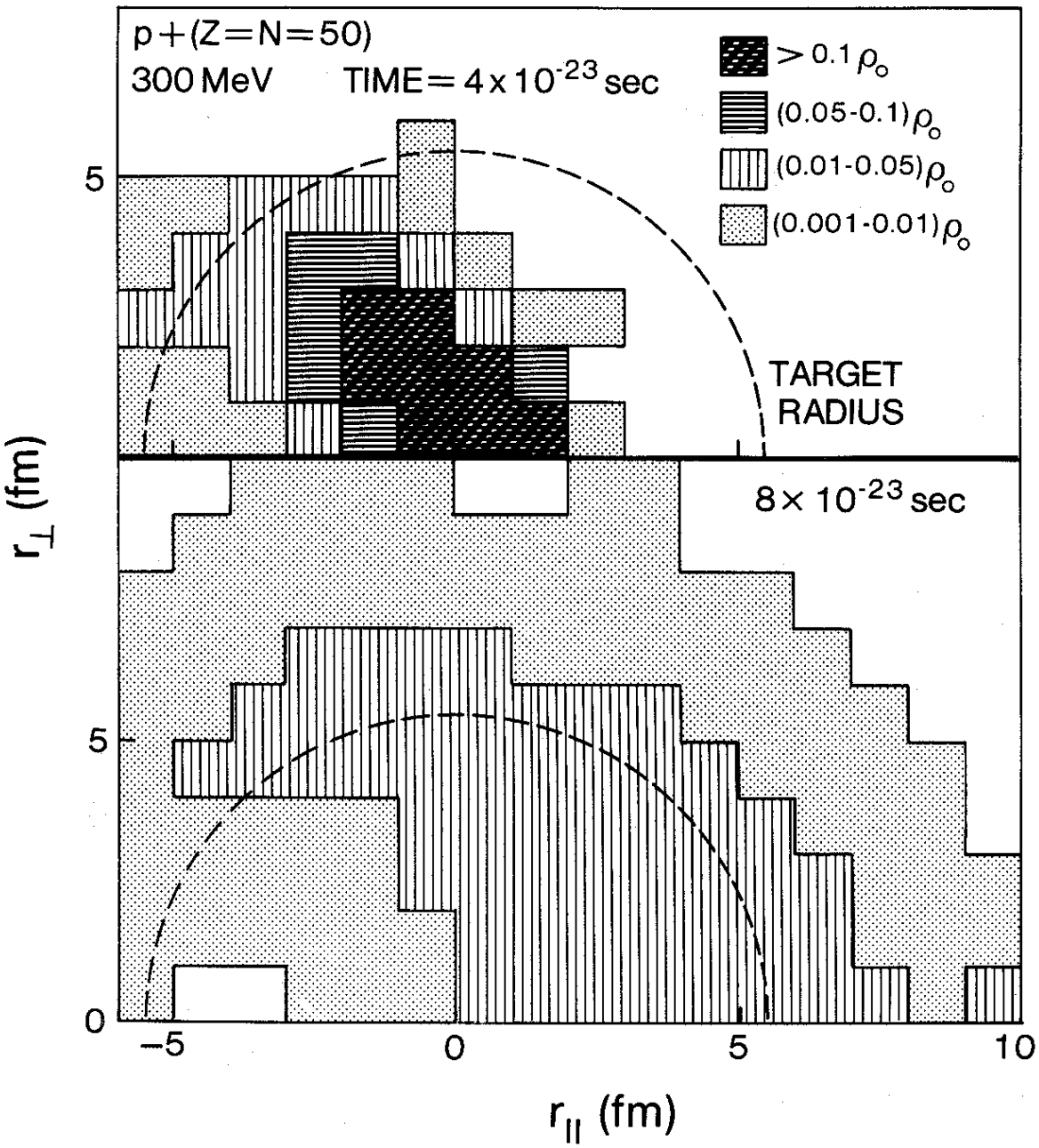


1. Energy level diagram for a proton induced reaction.

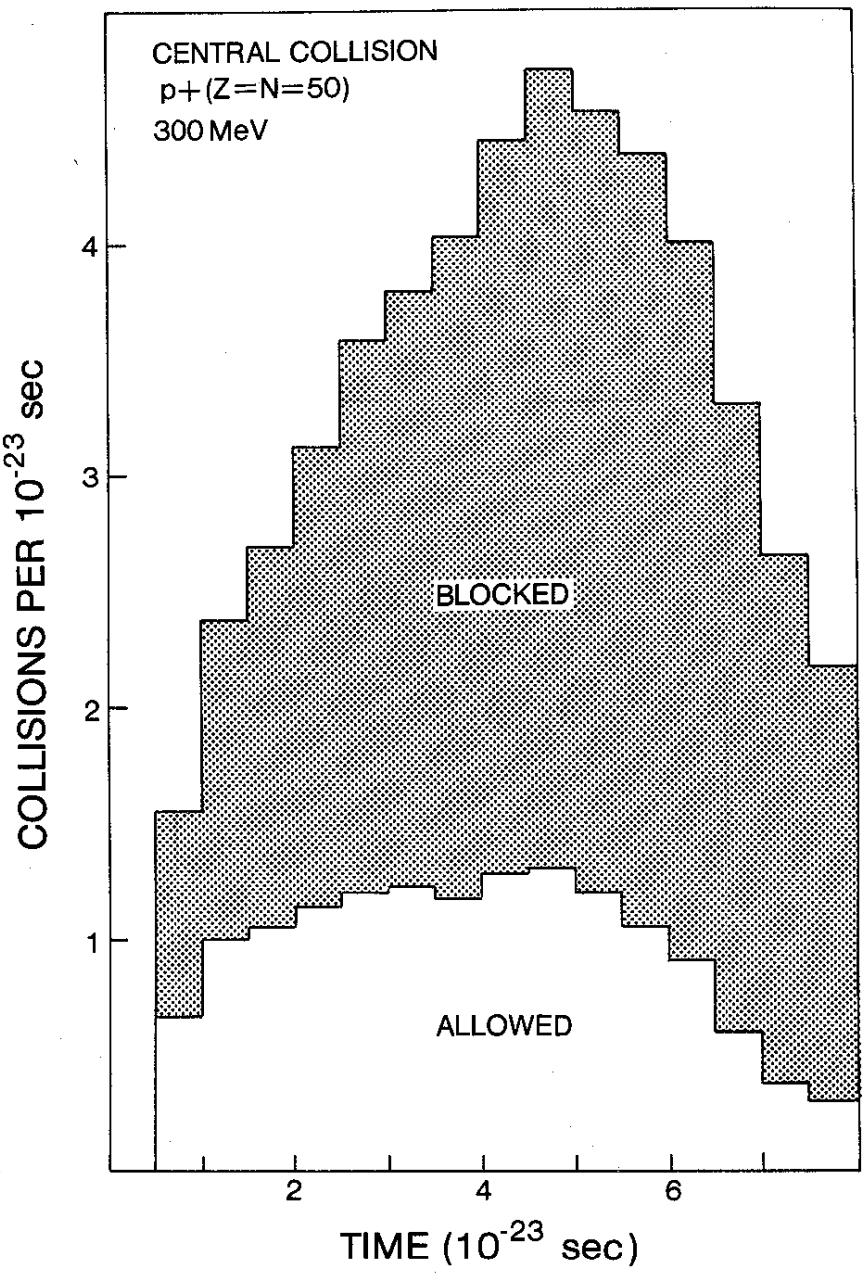
Figure 2



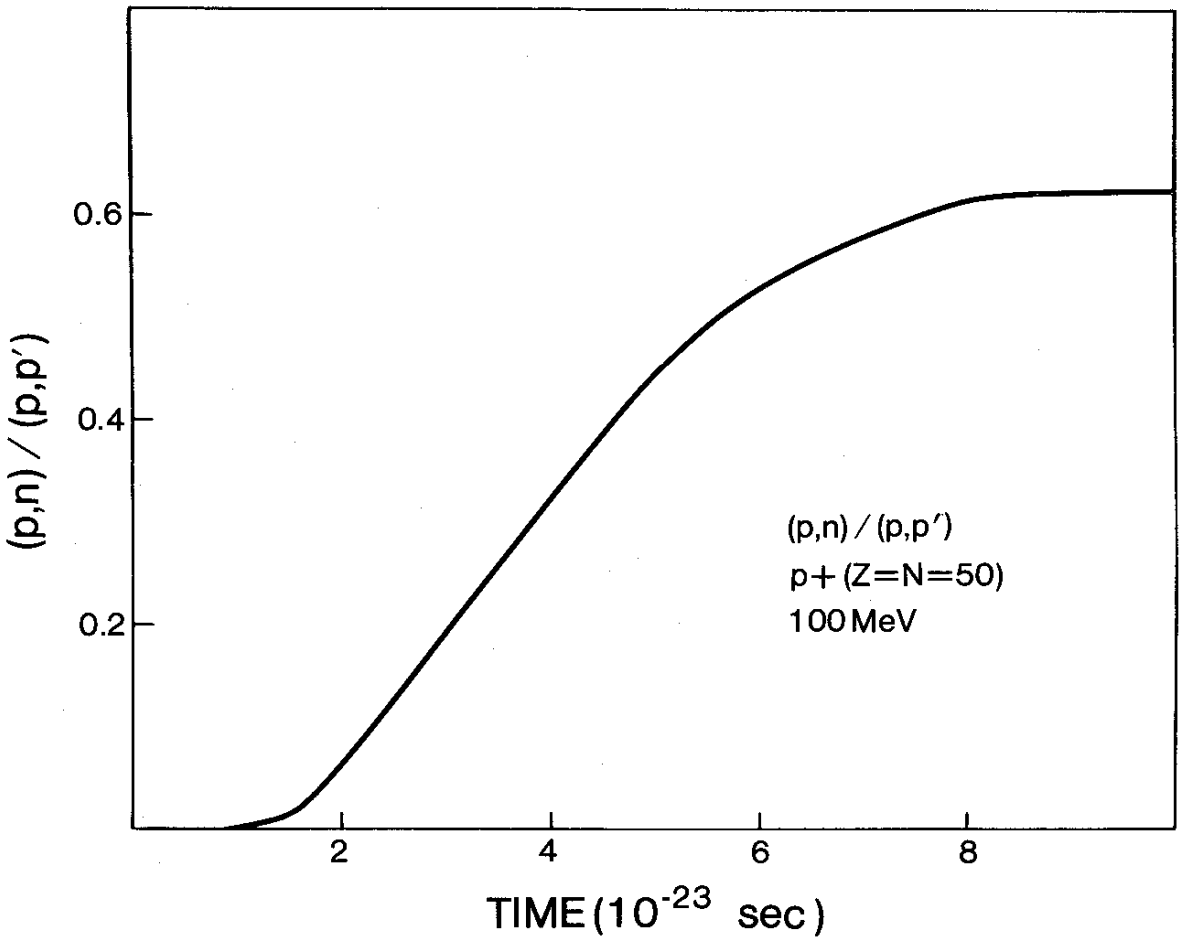
2. Momentum densities found in cascade simulation of a proton-nucleus reaction. The densities are quoted in particles per $(50\ \mathrm{MeV/c})^3$. Shown is an impact parameter averaged collision of a 300 MeV proton on an N=Z=50 target. The time is taken to be 8×10^{-23} sec after the proton has entered the target. Only those nucleons with $E > 0$ are shown.



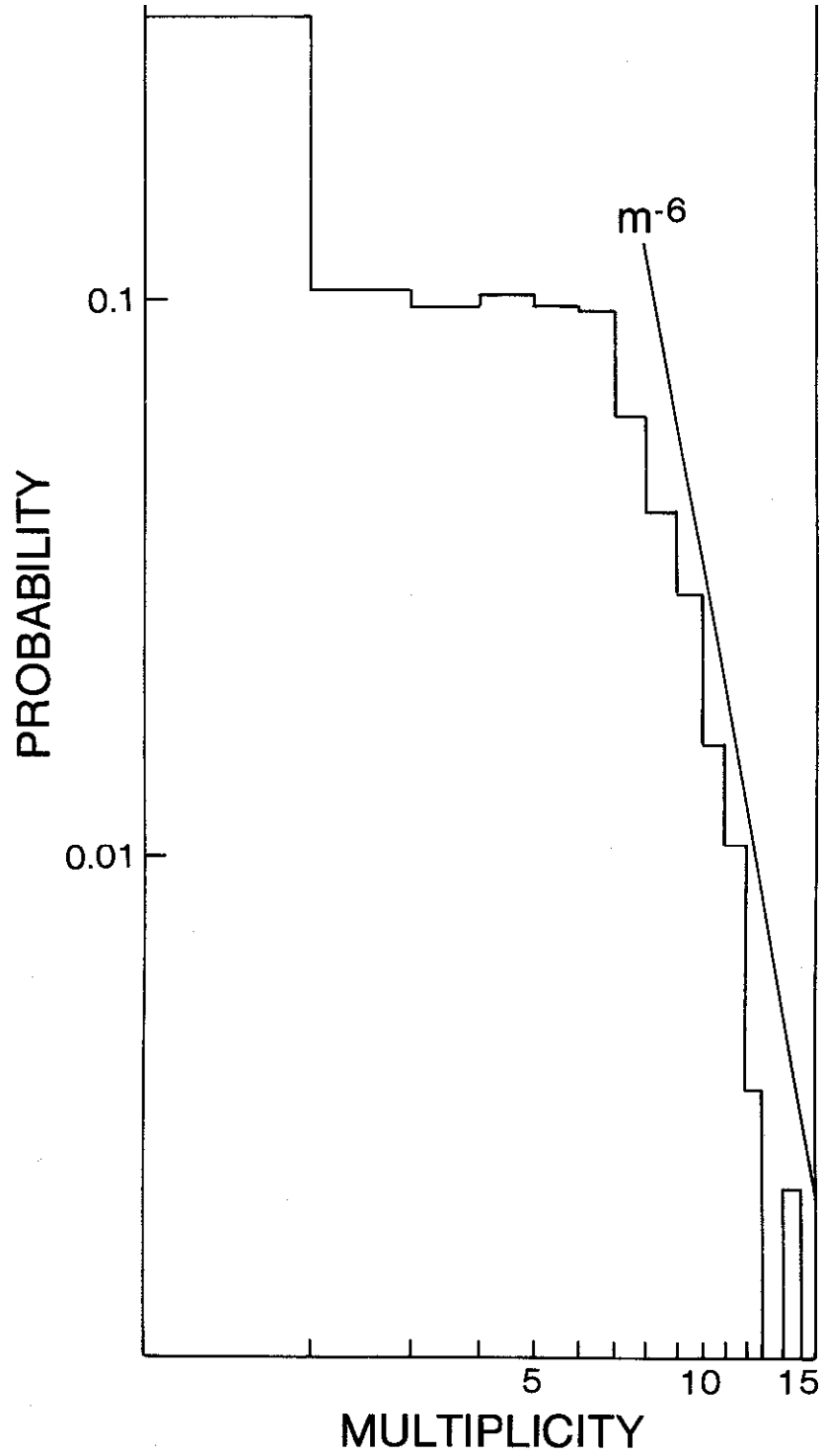
3. Coordinate space densities for the same reaction as Fig. 2, except that the impact parameter has been set equal to zero. The proton enters from the left, and the densities are shown after 4 and 8 × 10^{-23} sec. Only nucleons with E > 0 are shown.



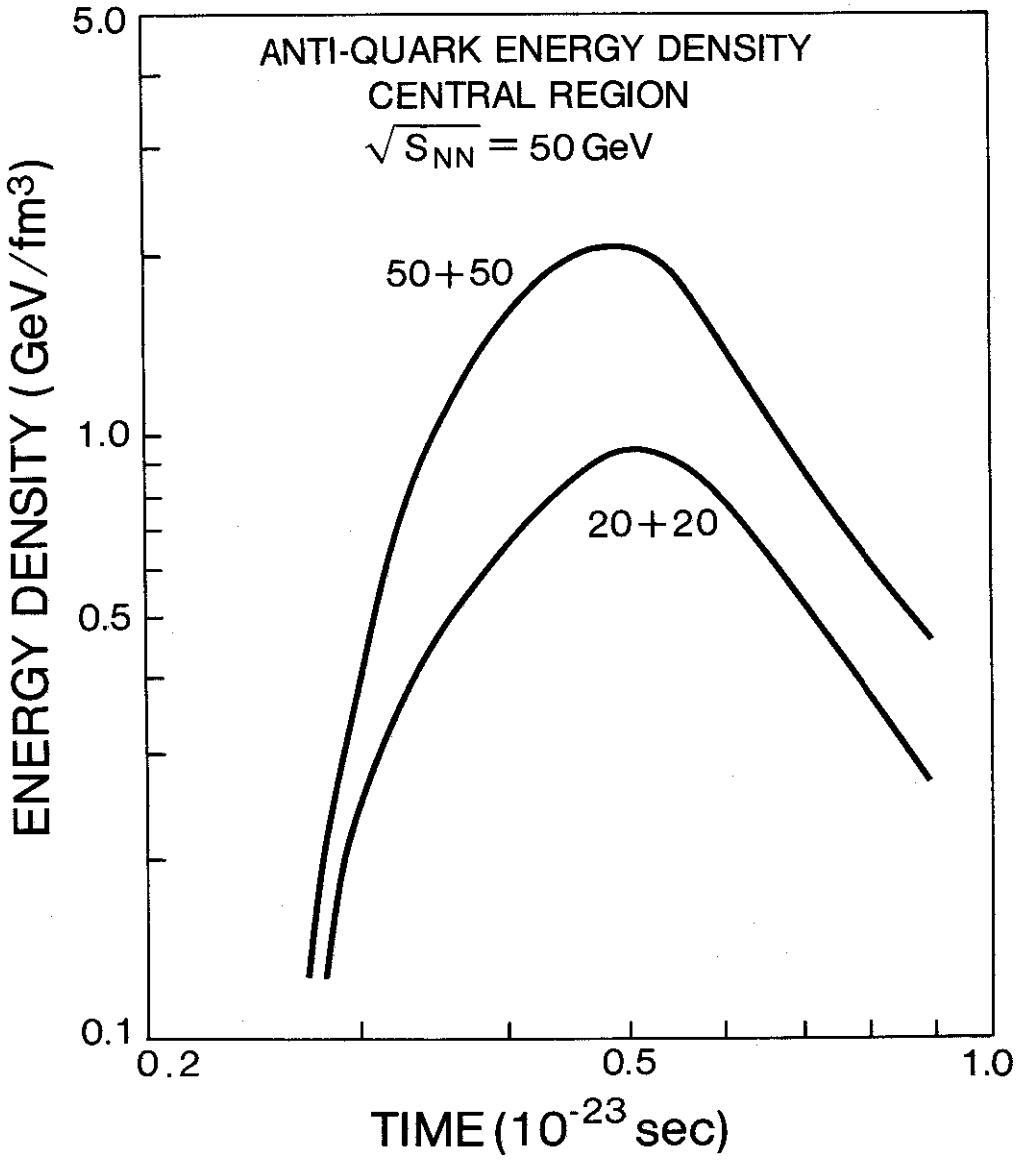
4. Collision rate for the central collision of Fig. 3 showing the relative percentage allowed or blocked by the Pauli principle.



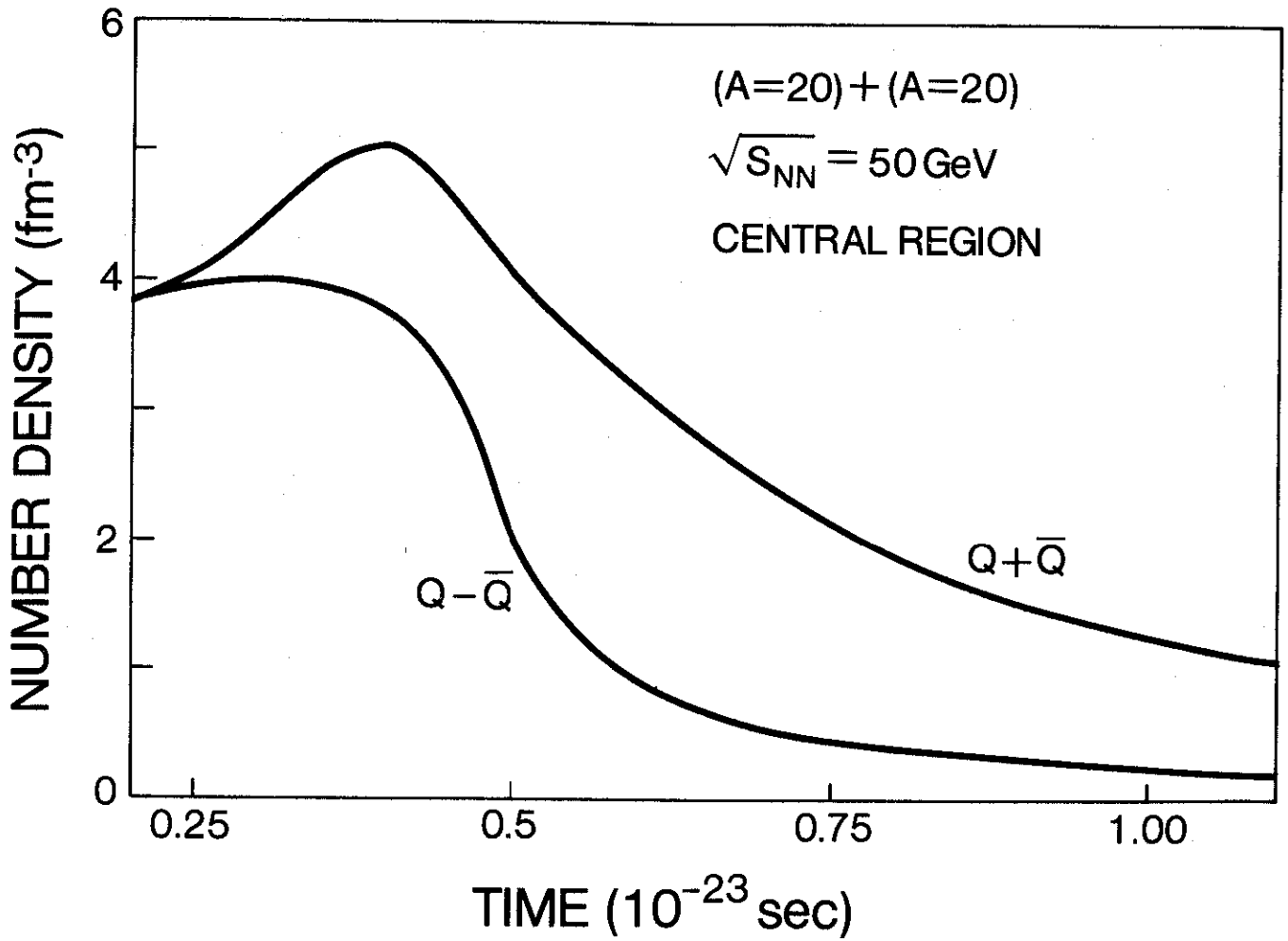
5. n/p ratio for p + (Z=N=50) at 100 MeV bombarding energy.



6. Multiplicity distribution for free nucleons found for the simulation described in Fig. 2.



7. Antiquark central energy density achieved in (A=20) + (A=20) and (A=50) + (A=50) collisions at $\sqrt{s_{NN}}$ = 50 GeV.



8. Sum and difference of the quark (Q) and antiquark ($\overline{Q}$) number densities predicted for the central region by the cascade calculation with (A=20) + (A=20) at $\sqrt{s_{NN}}$ = 50 GeV.