Nuclear Physics A447 (1985) 479c-488c
North-Holland, Amsterdam

THE NUCLEAR LIQUID-GAS PHASE TRANSITION

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The temperature and density regions where the nuclear liquid-gas phase transition is expected to occur are reviewed. By means of numerical simulations of the reaction trajectories of both proton and heavy ion induced reactions, it is shown that the mechanical instability region should be easily accessible. The change in entropy expected in crossing the isentropic spinodal at low initial entropy per nucleon is estimated and shown to be in agreement with that observed experimentally.

1. INTRODUCTION

The NN interaction exhibits a short distance repulsion and a long distance attraction, which suggests that nuclear matter has a liquid-gas phase transition similar to that of a Van der Waals fluid[1]). Finding an experimental signature for this phase transition in a nuclear reaction is difficult because many different aspects of a reaction mechanism may lead to similar values of an experimental observable as would be predicted by the existence of a phase transition[2]). One observable which may show large changes in the phase transition region is the entropy[3]). We give an example of where the entropy production should be significant, predict the entropy and, of course, compare with experiment. For the system chosen, the entropy change is shown to be several times the initial entropy, thus providing a good signature for a phase transition.

We begin with a review of the properties of the phase transition region. As a model calculation, we choose a zero-range Skyrme-type interaction[4]). The corresponding phase diagram is shown in fig. 1. The liquid-gas coexistence (LGC) line is determined by the Maxwell construction. The isothermal spinodal (ITS) curve is defined by $(\partial P/\partial \rho)_T = 0$ while the isentropic spinodal (IES) curve is defined by $(\partial P/\partial \rho)_S = 0$. The stable liquid and gas phases lie outside the LGC curve. The metastable phases are between the LGC and ITS curves while the mechanically unstable phase lies inside the ITS curve. The isentropes are indicated by dashed lines. The critical temperature *etc.* obtained using this equation of state are similar to those found in refs. [3]) and [5]). This calculation is for infinite matter and contains no coulomb term. The latter could lower the critical temperature significantly.

*Work supported in part by the Natural Sciences and Engineering Research Council of Canada.

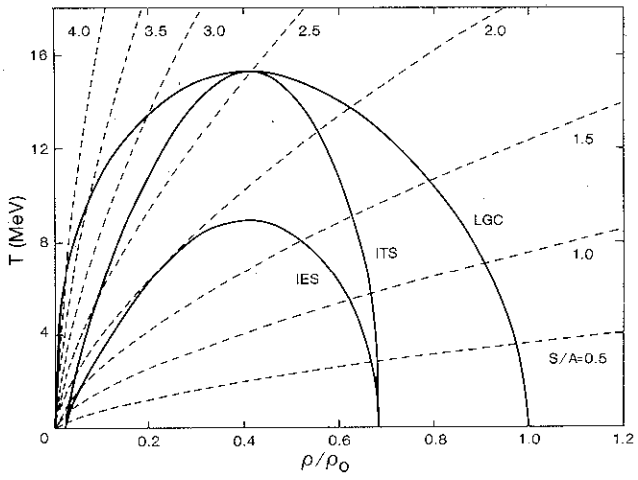

FIGURE 1
Phase diagram for uniform nuclear matter from ref. [4]). The dashed curves are isentropes with the value of S/A indicated for each. Shown as well are the liquid-gas coexistence boundary (LGC), isothermal spinodal (ITS) and isentropic spinodal (IES).

One can see that the temperatures and densities characteristic of the phase transition region are similar to those found in both proton and heavy ion induced reactions. However, these reactions do not provide us with an equilibrium system at a fixed T,ρ. Rather, the reactions evolve with time through the transition region complicating the extraction of an experimental signature. In order to make progress, we need to know the collision dynamics.

2. REACTION TRAJECTORIES

2.1. Experimental Information

The advent of the measurement of multiparticle correlations opens up the possibility that one will be able to map out segments of the reaction trajectory experimentally. By measuring two particle correlations, one can determine a length and possibly time scale associated with the interaction region. This approach was first applied to pion[6]) and proton[7]) emission, and has been extended to deuteron[8]) emission and, most recently, non-identical particle correlations[9]). An example of a two-proton correlation function[9,10]) is shown in fig. 2. The protons are assumed to be emitted from a source region which has a gaussian shape in space, with a length scale r_0. Predictions for the correlation function with various values of r_0 are shown for comparison. One should find that the apparent source size increases with increasing cross section of

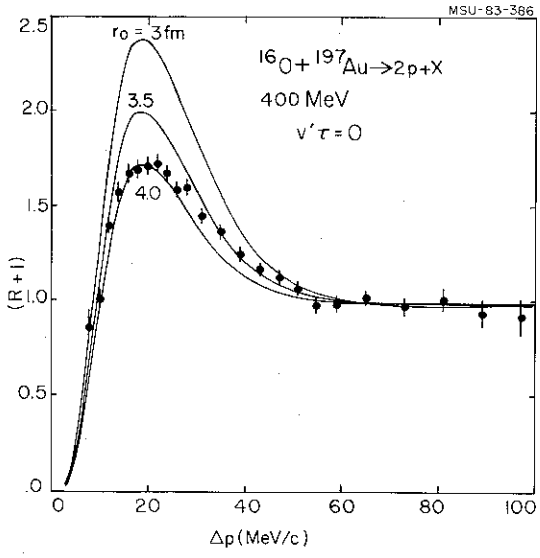

FIGURE 2
Two proton correlation function measured[10] in ^{16}O + ^{197}Au. Theoretical predictions for several values of r_0 are shown. From ref. [9].

the particles used in the correlation measurement; i.e. the larger the cross sections, the longer a species will remain in equilibrium and consequently both the lower the local temperature and the larger the volume. The volume trend is in fact observed in proton vs deuteron correlations[8,9]: the length scale associated with the source region measured by deuterons is of the order 50% larger than that observed by using protons.

Chemical temperature measurements are also consistent with these observations. As the system expands, the temperature measured in a local co-moving frame should decrease, as thermal motion is converted into radial motion. (The energy spectra, however, should not change significantly: the kinetic temperature is a measure of the energy in the system.) One can use isotopic ratios and abundances of excited states to measure the chemical temperature, and it is observed that for fragments in the mass range under consideration here, the chemical temperatures are consistently lower (usually by at least a factor of four) than the kinetic temperatures[11].

2.2. Heavy Ion Reactions

Until multiplicity selected correlation measurements become available in quantity, one will have to live with knowledge of only selected temperatures and

volumes. To determine the actual trajectory, one will have to use a model for the reaction region which is consistent with the data available so far. Our choice is to use a cascade model [by which we mean independent NN scatterings[12])]. Comparison with data can be accomplished by calculating the kinetic and chemical freeze-out points for various reactions. This involves numerically following the reaction time, τ_{rxn} and the expansion time, defined here as

$$\tau_{exp} = \langle r^2 \rangle^{1/2} / (d\langle r^2 \rangle^{1/2}/dt) \qquad (1)$$

in a Monte Carlo simulation of the reaction. Space does not permit a detailed description of the model[9]); however, the various source radii both as a function of species and multiplicity, are in satisfactory agreement with experiment. An example of the model's prediction is shown in fig. 3. There, the gaussian source parameter predicted by the model (assuming that the nucleon multiplicity is double the charged particle multiplicity) is compared with p-p correlation measurements[13] in 400 A•MeV Ca + Ca.

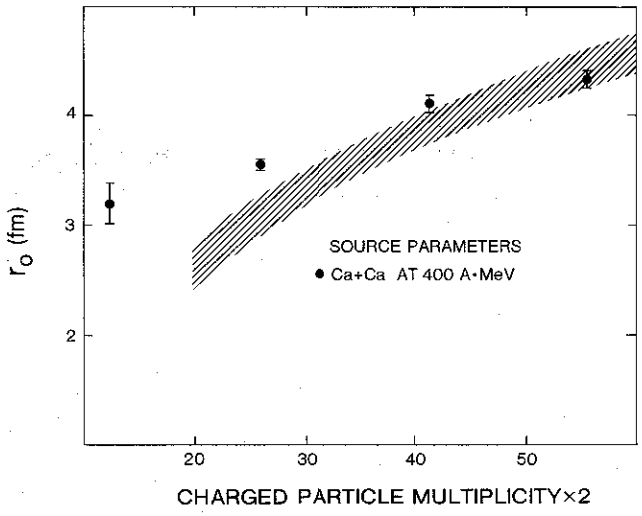

FIGURE 3
Model prediction for gaussian source radius compared with experiment. From ref. [9]).

Armed with this model, the trajectory of a heavy ion reaction through the transition region can be predicted. A typical result is shown in fig. 4, for a thermalized region having a temperature of 25 MeV at normal nuclear matter density. Typical freeze-out times are $0.5\text{-}1.0 \times 10^{-22}$ sec, showing that heavy ion

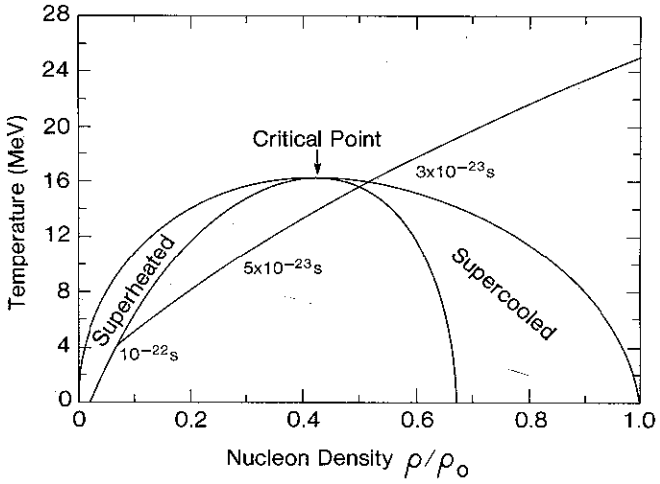

FIGURE 4
Model prediction for a heavy ion reaction trajectory in the phase transition region. From ref. [9]).

reactions go out of equilibrium in the phase transition region. The numerical solution is consistent with isentropic expansion.

2.3. Proton Induced Reactions

Intermediate and high energy heavy ion reactions generally show a trajectory which passes near the critical point. Of course, that is where density fluctuations, finite particle number effects *etc.* become troublesome and wash out many phase transition effects[1]). Dramatic changes caused by the phase transition are more likely to be found away from the critical region. Two options are available: entering the region with a large S/A resulting in droplet formation or entering at small S/A and reaching the mechanical instability region[14]). The former approach runs the danger that the system is out of equilibrium by the time the coexistence region is reached, and so we will concentrate on the latter.

Either low energy heavy ion reactions or intermediate energy proton induced reactions could be used to access this region. In order to minimize possible non-uniformities of the reaction region, we will concentrate on proton induced reactions.

As before, we need a model of the reaction trajectory. We choose a simplified Boltzmann-Uehling-Uhlenbeck approach[15]). In the model used here, target protons and neutrons form a system with single particle energy levels consistent with the equation of state. These nucleons collide elastically with the wall of

the spherical step function potential well in which they move. Because there are so few scatterings of the projectile as it traverses the target (producing a low nucleon multiplicity, as is observed experimentally) most of the nuclear levels remain Pauli blocked during the collision. Hence, we make the simplifying assumption that any scattering which results in a nucleon being scattered into a state below the Fermi surface is blocked. This model predicts particle energy spectra, multiplicities *etc.* which are in quantitative agreement with what is observed experimentally.

What is important here is the density of particles knocked out of the nucleus. Shown in fig. 5 are the nucleon densities predicted in this cascade model for a p + (Z = N = 50) central collision at 300 MeV bombarding energy. The densities are shown only for those particles which have enough energy to escape; bound nucleons are not included. In the top part of the figure, the time is 4×10^{-23} sec after the projectile has entered (from the left) and the largest density is in the center of the nucleus. At 8×10^{-23} sec, as shown in the bottom part of the figure, the density of nucleons which can escape is

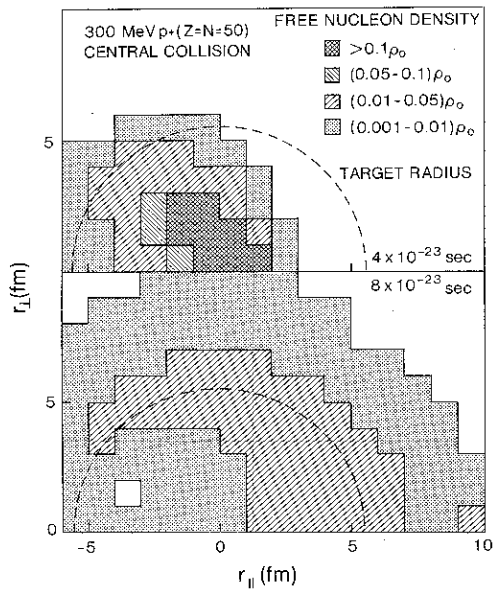

FIGURE 5
Model prediction for nucleon densities in a central 300 MeV p + (Z = N = 50) collision. The densities are shown 4×10^{-23} and 8×10^{-23} sec after the projectile has entered from the the left. Only those nucleons with enough energy to escape the target are included. From ref. [4]).

clearly low, typically 1/20 of normal nuclear matter density. The total number of nucleons knocked out of the sea is only 4.43 even in this central collision, hardly enough to form a mass 10-15 fragment. Further, the multiplicity drops off very rapidly above 10, showing that systems from which fragments could condense like droplets are not produced in any abundance in these reactions: fragmentation should involve breakup of the residual system. We assume that the system only breaks up if it reaches the isentropic spinodal. Otherwise the residual system will de-excite via evaporation.

It is straightforward to calculate the entropy achieved in these reactions via a numerical evaluation of

$$S = -\int d\varepsilon \, \mathcal{D}(\varepsilon)[f \ln f + (1-f) \ln(1-f)] \qquad (2)$$

where $\mathcal{D}(\varepsilon)$ is the density of states. One also needs to know whether the residual system possesses enough excitation energy E^* to enter the mechanical instability region[16], since the isentropic spinodal line does not go to $T = 0$, $\rho = \rho_0$. Again, this can be found from the numerical simulation. The results for the locus of E^*/A vs S/A found in the simulations is shown in fig. 6. The system moves into the metastable region easily and just reaches the unstable region delimited by the isentropic spinodal curve. Most, but not all, impact parameters allow the trajectory to pass reasonably close to the unstable region. For proton bombarding energies of less than 50 MeV, even central collisions will not reach the IES curve, so that evaporation should be much more important at lower energy.

The actual test of passage through the phase transition region is the calculation of the increase in entropy. First, from fig. 6 we will take the point at which the system enters the mechanical instability region to have $S/A = 0.4$. Applying the procedure of ref. [3], the increase in S/A from the equation of state is 0.3 and the new temperature is 4.8 MeV. (The dependence of this increase on the expansion trajectory chosen in a heavy ion reaction has been investigated in ref. [17]. The entropy analysis used therein is different from the approach to proton induced reactions presented here, since nucleon emission in these reactions may well be dominated by knockout and evaporative components rather than the "gas" liberated in the phase transition.) To calculate the total entropy associated with the surfaces and motion of the drops is more difficult since we do not have an expression at our disposal for the droplet mass distribution far from the critical point. [Numerical solutions should resolve this difficulty. See refs. [18,19])]. Estimates of the sum of these quantities are given in ref. [4]) and tend to be in the range $\Delta S/A \sim 0.7$-0.9 for intermediate mass fragments.

The produced fragments have been treated so far here on a purely classical

level. The occupation of excited states in the quantum picture has become

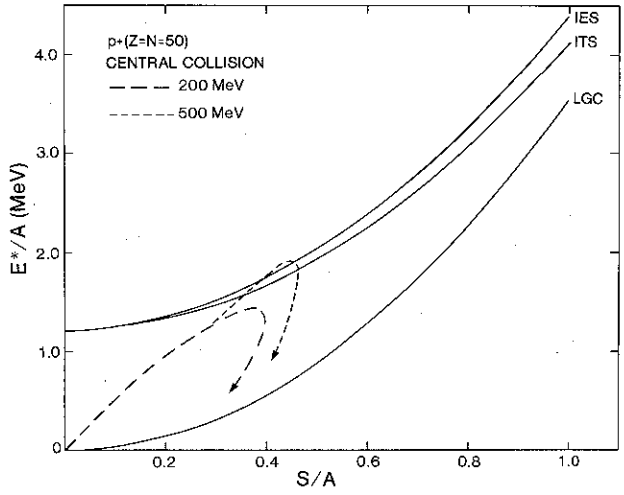

FIGURE 6
Isothermal spinodal (ITS), isentropic spinodal (IES) and liquid-gas coexistence curve (LGC) shown as a function of excitation energy and entropy per nucleon. The reaction trajectories for the excited nuclear system predicted by the collision model for 200 and 500 MeV p + (Z=N=50) central collisions are shown by the dashed lines. From ref. [4].

surface and bulk entropy in the continuum picture. An alternative approach, then, which not only includes the excited states explicitly but also solves the problem of not knowing the droplet distribution would be the following: assume that the system breaks up at the isentropic spinodal as before [ref. [3]], but now compare the number and energy densities with those of an ideal gas of nuclei (including the excited states) in chemical equilibrium. With equal proton and neutron chemical potentials then the fragment distribution is characterized by μ, V and T. These parameters are fixed by the energy and particle number constraints, and the condition that the entropy must be a maximum. For the case under consideration here, this procedure yields T ~ 3-4 MeV and S/A = 1.6-1.7. Hence, we find that S/A has increased from 0.4 to 1.7 in passing through the instability region.

Use is also made of the chemical equilibrium model in obtaining S/A from the observed fragment distributions. The technique is similar to that of ref. [20]; the main difference is that the assumptions made in ref. [20]) regarding volume *etc.* are removed, thus allowing each of T,V,μ_p and μ_n to be fit. This accomplished by incorporating not only the low-lying excited states of light nuclei

(about 500 were included) but also estimating the important contribution of the heavy residual systems (not much of the total baryon number is contained in the light nuclei) by using a phenomenologically determined density of states. In other words, all nuclei up to the target are included so a more accurate estimate of S/A can be made. Fitting 500 MeV p + Ag data[21]) (motivated by fig. 6) we find a chemical temperature of slightly under 3 MeV, in agreement with the prediction. The entropy per nucleon is 1.6 also in good agreement with what is expected from the phase transition.

3. SUMMARY

It has been shown by means of numerical simulations that both heavy ion reactions and proton induced reactions have reaction trajectories which enter the nuclear liquid-gas phase transition region. Further, the systems appear to go out of equilibrium around the phase transition boundaries, which implies that an experimental signature for the transition will not necessarily be masked by re-equilibration.

Although previous searches[22,23]) for a signature of the transition have concentrated on the region around the critical point, the approach explored here was to look away from the critical point for discontinuous behavior of the entropy. The entropy was shown to increase several-fold in moving through the mechanical instability region. Whether this signature demonstrates the existence of the phase transition depends in part on whether the data possess an alternate explanation, as has been put forward for the mass yield signature[24,25]).

ACKNOWLEDGMENT

The author wishes to thank J.C. Shillcock and A.L. Goodman, who collaborated with him on the work presented here, as well as the many colleagues with whom he has enjoyed often heated discussions of the phase transition question.

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