Computer Simulation of Temperature Measurements in Heavy-Ion Collisions

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The observed differences in kinetic and chemical temperatures in heavy-ion reactions is addressed by means of a computer simulation. Focusing on ^{4}He nuclei emitted at wide angles, we find that the chemical temperatures show no strong variation with laboratory bombarding energy between 35 and 100 MeV/nucleon. Further, the chemical temperature is found to be in the 3–4-MeV range, much lower than the kinetic temperature and consistent with experiment. The difference in the two temperatures is established after a fairly short time scale roughly corresponding to the breakup time of the reaction region.

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The thermal model has been used with success for some years in describing the differential cross sections of particles emitted in heavy-ion reactions.[1] In the thermal model, the single-particle energy spectra are fitted with the functional form associated with a Maxwell-Boltzmann distribution of particles as observed from a moving frame. The fit then yields the velocity of the frame, as well as the temperature of the particles.

An alternative approach to this determination of the temperature was proposed several years ago: the measurement of excited-state populations. The first measurement by this means,[2] which we will refer to as the chemical temperature to distinguish it from the kinetic temperature associated with the energy-spectrum analysis, yielded values which were dramatically lower than the kinetic results, in some cases up to a factor of 10 lower. There have been a number of experiments[3] since the original measurement which have both verified it and provided other systems whose ratio of chemical to kinetic temperatures are closer to unity, typically about 1:4.

There are a number of factors which could affect the excited-state population ratios which, in turn, would alter the apparent temperature. One difficulty is that excited-state decays on a time scale much longer than the typical time frame of a few times 10^{-22} sec associated with the reaction will also change the populations.[4] However, even when these decays are taken into account (admittedly in a model-dependent way), the difference between the two temperatures remains.

On general grounds, we do not expect these two methods of temperature measurement to yield the same results. The distribution of kinetic energies is determined relatively early in the reaction and is mainly a function of the bombarding energy and geometry, the latter through the multiplicity of noncompound reaction products and the equipartition theorem. However, as the thermalized spatial region in a collision expands, the temperature observed in a frame comoving with a local region of coordinate space will decrease.[5] As long as a given species remains in chemical equilibrium, the population ratios will follow this locally decreasing temperature.[6]

To investigate this problem on a more quantitative basis than was possible previously (e.g., Ref. 6), we will use a computer simulation which possesses stable computational ground states and thus allows the calculation of excitation energy distributions. The details of the simulation, which we refer to as quasiparticle dynamics (QPD), can be found in Ref. 7. The essence of the model is that it uses a momentum-dependent potential —which we refer to as the Pauli potential—to represent the antisymmetrization effects inherent to fermions. Thus, one works completely within a consistent Hamiltonian formalism both for determining nuclear ground states and for propagating those states during a collision.

In order to associate a temperature with the excitation energy distribution of a particular reaction product, we must calculate the distributions associated with an equilibrated nucleus for a variety of temperatures. This is difficult to perform analytically because of the presence of the complex nuclear and Pauli potentials. Hence, we will use a Monte Carlo procedure to evaluate the energy spectrum. Our studies will focus on the ^{4}He nucleus in which all spins are paired because of its abundance in the reaction products and because its small size makes the Monte Carlo procedure computationally fast.

The method we choose for the Monte Carlo procedure is the following: An initial configuration is chosen for the four quasiparticles representing the nucleons, and then the phase-space coordinate of each quasiparticle is moved successively. The position is moved randomly within a box 0.6 fm to a side centered on the old position, and the momentum is moved in a similar box of dimension 60 MeV/c. After each change, the centers of mass and momentum are repositioned to zero and the energy of the new configuration is compared with the old. The new configuration is either kept or rejected accord-

ing to the value of a random number compared with the weight $\exp(-\Delta E/T)$. The configuration is stored after each 50 sweeps through the four quasiparticles, providing a largely uncorrelated sample. A total of 20000 configurations is kept at each temperature.

There is one technical point worth mentioning which makes a significant difference to the predicted density of states at high temperatures. In the QPD simulation of the reaction, a fragment is defined by searching through the final quasiparticle positions (after an elapsed time of hundreds of fm/c) and linking together those quasiparticles separated in space by less than 3.5 fm. It is important to place the same restrictions on the coordinate positions allowed in the Monte Carlo procedure; otherwise, one will be sampling a very different phase space than what is used in the reaction simulation.

From the Monte Carlo samples we calculate the distribution of excitation energies. Since the computational ^{4}He nuclei have a binding energy per nucleon of 6.24 MeV, the part of the excitation energy distribution which will be of most use in reaction studies is the region below 6 MeV/nucleon. The fractional distributions are shown in Fig. 1 for temperatures of 2, 3, and 4 MeV, the distributions being normalized to unity over the 0–6.0-MeV/nucleon range. As can be seen from the figure, the 2-MeV distribution has a peak below 6 MeV/nucleon and this peak shifts to higher excitation energies as the temperature increases. Although space does not permit us to show the full distributions, they do fall off exponentially at higher excitation energies as is expected.

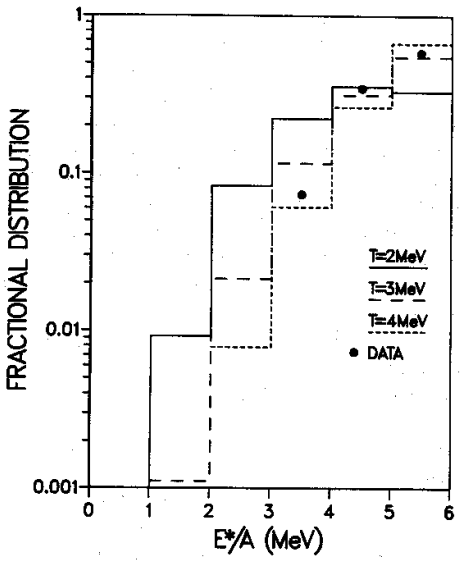

FIG. 1. Fractional distribution of excitation energies per nucleon predicted for the computational zero-spin ^{4}He nuclei (subject to cluster constraint) for temperatures of 2, 3, and 4 MeV (histograms). The points are from the simulated Ca+Ca reaction at 100 MeV/nucleon and $b=0$ fm. All distributions have been normalized to unity over the 0–6.0-MeV/nucleon range in excitation energy.

Having determined the form of the excitation energy distribution as a function of temperature, we now evaluate these quantities in a simulated collision. The reaction which we choose to investigate is Ca+Ca at bombarding energies of 35 and 100 MeV/nucleon and an impact parameter b of 0 fm. These reactions are assumed to be typical of those for which the temperature measurements have been made. For each reaction, a sample of 4500 events was generated. The total event sample including analysis took more than 200 CPU hours to generate on an IBM 3081 mainframe.

The differential cross section which the simulation predicts for the reactions is strongly peaked in the forward and backward directions in the c.m. frame. The experimental measurements are made away from these regions, so the approach taken here will be to average the fragment angular distributions over the range of 60°–120° in c.m. angle. If a wider angular range were chosen, then the average kinetic energy is found to rise, although the average excitation energy remains roughly the same. The predicted ^{4}He excitation distribution for the 100-MeV/nucleon reaction (evaluated at about 100 fm/c after the time of maximum overlap) is shown as the data points in Fig. 1.

In the figure, the ^{4}He distribution in the reaction is most closely matched to the calculated distribution between a temperature of 3 and 4 MeV. The method which we use for quantitatively determining the temperature is the following: first we use the average excitation energy (subject to the same 6-MeV/nucleon bound described above) as a maximum-likelihood estimator (see Ref. 8) of the chemical temperature. This involves comparison of the expectations of the excitation energy from the QPD simulation with that obtained in the Monte Carlo method as a function of temperature. The maximum-likelihood-estimator method was also used to determine the variance of the temperature. Second, a χ^2 test was applied to the distributions to ensure that the temperature was, in fact, a meaningful parameter. The results of the second test will not be included in this paper. The value obtained for the chemical temperature under the same conditions as for Fig. 1 is 3.3 ± 0.3 MeV.

This value is much lower than the temperature extracted from the ^{4}He cluster kinetic-energy spectra in the same simulation and is in the range observed experimentally[9] in hadronic decays of ^{6}Li in similar reactions. Further, in simulations of the Ca+Ca reaction at 35-MeV/nucleon laboratory bombarding energy, we find that the chemical temperature decreases by perhaps 20% even though the kinetic temperature drops by a factor of 2.

From the reaction-mechanism point of view, the question of interest is at what time did the chemical and ki-

netic temperatures begin to differ? To investigate this question, we stopped the simulation every 10 fm/c and performed the same evaluations of the excitation energy distributions as above on whatever clusters are present. Obviously, this approach will yield no information until distinct ^{4}He nuclei have begun to emerge from the reaction region and this time will be close to the breakup time at about 50 fm/c in the simulation. Further, the statistics will not be particularly good for early times in the collision when few ^{4}He fragments are present.

The time dependence of the temperatures is shown in Fig. 2 for the Ca+Ca reaction at 100-MeV/nucleon laboratory bombarding energy. The same angular averages have been performed as before. One can see that the temperatures show little variation after 100 fm/c elapsed reaction time, meaning that their values are fixed near the breakup time. This does not preclude there being a further change in the temperatures arising from long-time-frame evaporative emission or other decay processes, but it does indicate that the low chemical temperatures are set early in the reaction.

Also shown in Fig. 2 is the kinetic temperature of the ^{4}He clusters determined here as $\frac{2}{3}$ of the average cluster kinetic energy (i.e., the kinetic energy associated with the cluster's c.m. motion) as observed in the c.m. frame of the colliding nuclei. One can see that the kinetic temperature drops rapidly with time, then becomes constant. The initial clusters observed at around 50–100 fm/c with high kinetic energy also have a high excitation energy. As measured by their average excitation energy, highly excited nuclei can completely break apart, or may lose energy through collisions. At present, our statistics prevent us from saying whether one of these processes is dominant.

However, a clue can be obtained from the predicted lifetime of the states. One would expect that ^{4}He nuclei below the first decay threshold would be stable on the hadronic time scale, while those nuclei with excitation energies between 3 and 6 MeV/nucleon would decay on a time scale of hundreds of fm/c. Above 6 MeV/nucleon, the lifetimes should be very short. The breakpoint in the behavior at 3 MeV/nucleon comes from the threshold for decay: The threshold in excitation energy per nucleon is 2.97 MeV for $p+{}^3$H, 3.11 MeV for $n+{}^3$He, and 3.53 for ^{2}H+^{2}H. To obtain a quantitative value for the lifetimes, the configurations generated in the Monte Carlo simulation of the density of states were propagated for 250 fm/c and run through the cluster search code at 10-fm/c intervals. The initializations are grouped according to excitation energy into bins of 1 MeV/nucleon. The resulting populations are shown in Fig. 3 as functions of time.

From the figure, one can see that configurations with excitation energy below the decay threshold (3 MeV/nucleon) are stable, as required. At higher excitations, there is a component which decays in an intermediate time frame (220-fm/c lifetime for the 4–5-MeV/nucleon example) and one component which decays on a very

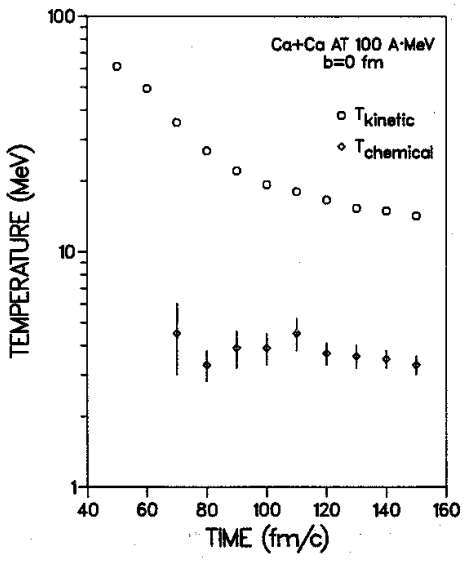

FIG. 2. Time dependence of the ^{4}He kinetic and chemical temperatures calculated for the Ca+Ca reactions at 100 MeV/nucleon and $b=0$ fm. See text for method of determining the temperatures.

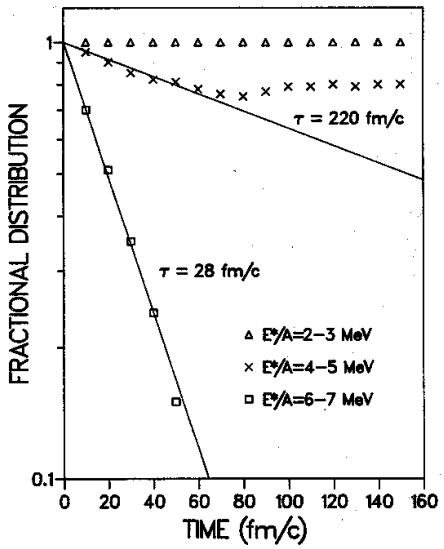

FIG. 3. Time dependence of the population of ^{4}He nuclei with various initial excitation energies 2–3 (triangles), 4–5 (crosses), and 6–7 MeV/nucleon (squares). The initializations were chosen randomly from the phase space associated with a temperature of 2 MeV subject to the cluster connection constraint.

long time scale. Finally, for energies above the vaporization threshold, the configurations are short lived—28 fm/c in the example shown. These values for the lifetimes indicate that the initially highly excited states probably break apart rapidly into nucleons. Although we have not shown it, the abundance of ^{4}He at this time is fairly low. Most of the ^{4}He nuclei which subsequently break up on the time scale of hundreds of fm/c are probably emitted somewhat, but not much, later.

In summary, we have used a computational model which possesses well defined nuclear ground states to investigate the problem of temperature measurement in heavy-ion collisions. The model was used to evaluate the fractional distribution of excitation energies as a function of temperature for the computational ^{4}He nucleus. Next, a simulation was performed of the Ca+Ca reaction at two bombarding energies and fixed impact parameter. The temperatures extracted from this simulation showed the same characteristics as are observed experimentally: The chemical temperature had a value of 3–4 MeV (much lower than the kinetic temperature) and it varied only slowly with bombarding energy. Lastly, it was established that the low value of the chemical temperature is established around the breakup time of the reaction. Decays on the time scale of 10^{-21} sec are not necessary to produce this effect, although they may contribute to it.

However, our limited statistics did not allow us to follow the rates for the formation and decay of the excited states in any detail. In particular, the changes in these rates in $(1-2)\times10^{-22}$ sec after the period of maximum overlap deserve more study. To make further progress on this problem from the computational point of view, a much larger event sample will have to be generated. In addition, the analysis will have to be extended to heavier fragments than ^{4}He, both because most of the experimental measurements have been made with mass 6 and 7 fragments, and because such nuclei have a richer excited-state structure (even in the computational model) than ^{4}He. Such calculations are in progress and will be reported elsewhere.

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