

## The Arithmetic of Monetary Growth and Inflation

We know that a money demand function can be described as:

$$(M/P) = y \cdot e^{-\alpha R}$$

where  $M$  is the nominal quantity of money;  $P$  is the price level,  $y$  is the level of real income, and “ $R$ ” is the nominal interest rate.

For the moment we will ignore the interest rate so that the money demand equation can be written as:

$$2. (M/P) = y$$

This is a most simple form of money demand that allows us to see what happens when the rate of the money supply growth is increased.

Rewrite 2 so that the initial relationship is:

$$3. M_0 = P_0 y_0$$

Now assume that  $M$ ,  $P$  and  $y$  are each displaced by a small amount,  $\Delta M$ ,  $\Delta P$ , and  $\Delta y$ . This leads to a new version of 3:

$$4. (M_0 + \Delta M) = (P_0 + \Delta P) \cdot (y_0 + \Delta y) \text{ which can be multiplied out to leave:}$$

$$5. M_0 + \Delta M = P_0 y_0 + \Delta P \cdot y_0 + P_0 \cdot \Delta y + \Delta P \cdot \Delta y$$

Now, first notice that from 3,  $M_0 = P_0 y_0$ , so that *the first term on both sides of 5 cancel*. Second, if  $\Delta P$  and  $\Delta y$  are both small, then multiplying two small things together leads to a product that is *very* small – so small in fact that we shall assume that it disappears! (That is, assume that  $\Delta P \cdot \Delta y = 0$ .)

This leaves us with 6:

$$6. \Delta M = \Delta P \cdot y_0 + P_0 \cdot \Delta y$$

Now recall that  $M_0 = P_0 y_0$ . Divide the term  $\Delta M$  by  $M_0$  on the left hand side (LHS), and both terms on the right hand side (RHS) by  $P_0 y_0$ . (This is OK because they are equal.) Now what we have is 7:

$$7. \left( \frac{\Delta M}{M_0} \right) = \left( \frac{\Delta P \cdot y_0}{P_0 y_0} \right) + \left( \frac{P_0 \cdot \Delta y}{P_0 y_0} \right).$$

But since the  $y_0$  cancels in the first term on the RHS, and the  $P_0$  cancels in the second term on the RHS, we are left with the expression:

$$8. \left( \frac{\Delta M}{M_0} \right) = \left( \frac{\Delta P}{P_0} \right) + \left( \frac{\Delta y}{y_0} \right).$$

Since each of these terms can be read as a percentage change, equation 8 reads as: the percentage change in the money supply is equal to the percentage change in prices plus the percentage change in real income.

If we now divide by a *change in time*,  $\Delta t$ , so that the change in M,  $\Delta M$  is per period,  $\Delta t$ , then we have an equation like 9 where everything is expressed in *rates of growth* per year (or per month or whatever time period you like):

$$9. \left( \frac{1}{M_0} \frac{\Delta M}{\Delta t} \right) = \left( \frac{1}{P_0} \frac{\Delta P}{\Delta t} \right) + \left( \frac{1}{y_0} \frac{\Delta y}{\Delta t} \right).$$

Equation 9 reads as: the percentage *rate of growth* of the money supply is equal to the percentage *rate of growth* of prices plus the percentage *rate of growth* of real income.

If the rate of growth of the money supply is called,  $\rho$  (rho), and the rate of growth of the price level (otherwise known as the rate of inflation) is called,  $\pi$  (pi), and the rate of growth of real income is called,  $\lambda$  (lambda), then we can write the relationship binding the long-run growth rates of money, income and prices as:

$$10. \pi = \rho - \lambda.$$

This is a very important relationship. We have a simple expression tying the inflation rate to variables about which we know something.

We assume that the government can control the quantity of money and therefore the growth rate of the money supply,  $\rho$ . We know that the growth rate of real income changes by at most (in normal time) between +4% and -4% per year.

The rate of inflation, however, as we saw in Boliva, can be many thousands of percent per year. The culprit? Changes in the rate of growth of the money supply. It is in this sense that inflation is everywhere and always a monetary phenomenon since it reflects the difference between the rate of growth of the money supply and the rate of real income growth. As a matter of historical interest, the money supply growth rate is far more variable than the growth of real output (income) for most countries over long periods of time.