Physics 347 **Assignment #5** **Due Nov 20 (in class)**

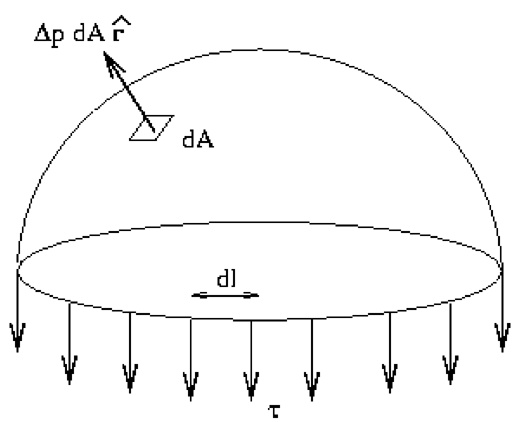
Reading: We covered up to 10.2.2 and included 10.4.1 and 10.4.2. For the topic of membranes we covered in Chapter 11: 11.1-11.3 inclusive. For the topic of diffusion we covered in Chapter 13: 13.1-13.2.0 (we did not cover 13.2.1 -), all of 13.3.1. Sections not mentioned above were not covered (though are interesting to read).

Problem 1: Persistence length of spaghetti

Consider a piece of uncooked spaghetti that is 2mm in diameter. If the Young's modulus of the dry pasta is $E = 1 \times 10^8$ J/m^3, what is the persistence length at T = 300 K? In class we worked out the relationship between the Young's modulus and persistence length as, $k_b T \xi_P = E I$. For a solid cylinder the cross-sectional moment of inertia is $I = \pi R^4/4$ where R is the radius of the cylinder. Will the spaghetti bend much just due to thermal fluctuations?

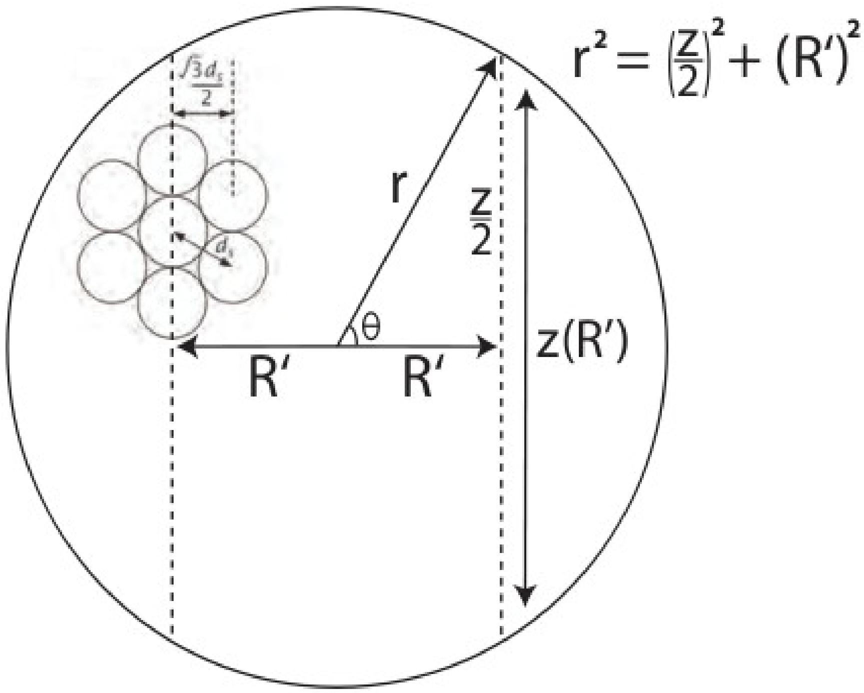
Problem 2: Laplace-Young Relation

Here you will derive the Laplace-Young relation using balance of force arguments. Consider a spherical membrane with an inside pressure p_{in} and an outside pressure p_{out} and a radius R. Because of the pressure difference, Δp, there will be an outward normal force at all points on the membrane. This outward force will be balanced by the tension, τ, in the membrane. To balance the forces, imagine that the sphere is cut in half – there will be a net force up due to the pressure difference which is balanced by the surface tension (force/length) acting down at the equator. Show that balancing these forces leads to the Laplace-Young relation $\Delta p = (2\tau)/R$. Hint you need to do the surface integral over the sphere for the pressure contribution. Recall that the unit vector in the r direction is given in cartesian coordinates as $\hat{r} = (R \sin\theta \cos\phi, R \sin\theta \sin\phi, R \cos\theta)$ and the infinitesimal area element is $dA = R^2 \sin\theta \, d\theta \, d\phi$.



Problem 3: Packing DNA into a Spherical Capsid (Textbook P10.5):

Repeat the problem done in class of packing DNA into a virus. In this case the virus is a sphere of radius, r. For a given length, L, of DNA packed into the DNA, it forms closed loops inside the capsid to an inner radius $R'(L)$.



a) Calculate the length L of DNA inside the capsid given that it packs to a final radius R'.

b) Calculate the bending energy of the DNA in the capsid as a function of the innermost radius, R'.

Now for some numbers. Use a plotting program like matlab, maple, mathematica or excel to generate the following graphs. In class we derived the bending energy for packing DNA into a cylindrical capsid (look up in notes) – there's a formula for $E(R)$ and $L(R)$.

c. For the cylindrical capsid evaluate $L(R)$ and $E(R)$ over a range of R from R = 0 to $R = R_o$. Use the values $R_o = 27$ nm, $z = 36$ nm and $d_s = 2$ nm. Using these results, now make a plot of E vs L for the cylindrical capsid.
d. Now use your result from above for a spherical virus that has the same volume as the cylindrical virus. Calculate what the radius, r, of the sphere needs to be to

have same volume. Using this value for the spherical radius, the same length d_s, plot E vs L for DNA packed into the spherical virus.

e. Assuming that both viruses have DNA with a length $L = 50000*0.34$ nm packed into both viruses, based on your plots of E vs L, which virus has DNA exerting a greater force due to the bending energy?

Problem 4: Entropy of packing DNA into ϕ29 (Textbook P10.6)

a) Estimate the work that needs to be done by the motor in ϕ29 to overcome the free energy due to the entropy loss of packing the DNA into the virus. Consider that once inside the virus the DNA only exists in one well defined packed configuration. Outside the virus, the DNA is a freely jointed chain (random polymer) and assume each link in the DNA chain can take on six possible orientations corresponding to the six directions in 3D. Using this information, calculate the number of possible configurations the DNA can have from which you can evaluate the entropy and then the corresponding free energy outside the capsid. The DNA has a length L = 6800 nm, and recall it has a persistence length of 50 nm.

b) What is the total work done by the motor when packing the DNA into ϕ29 as measured by Smith et al. (2001). Use Figure 10.18 to estimate the work. Given this amount of work, what percentage is needed to overcome the entropy loss? Is this a big or small percentage?

Problem 5: Einstein relation & drag force

In our discussion of the random walk we defined the diffusion constant to be $D = \delta^2/(2\Delta t)$. This depends on the step size, δ, and the time between collisions, Δt. Clearly these depend on the type of substance that the object is diffusing through, the object's size and on temperature. Einstein was able to come up with a connection between the diffusion coefficient, D, the drag coefficient of the object, c, and temperature, T, namely $Dc = k_B T$.

(a) For a spherical object with a radius R, the drag coefficient is $c = 6\pi \eta R$. Use the Einstein relation to calculate the diffusion coefficient in water at room temperature of a lipid vesicle that has a radius of 50 nm where the viscosity of water is $\eta = 10^{-3}$ kg/m/s at STP.

(b) If the vesicle is being dragged by a motor protein with a speed of v = 10 μm/s, what is the drag force if the flow is laminar?

(c) The motor protein must exert this amount of force to balance the drag of the vesicle. How much power is it generating to move at the above speed?