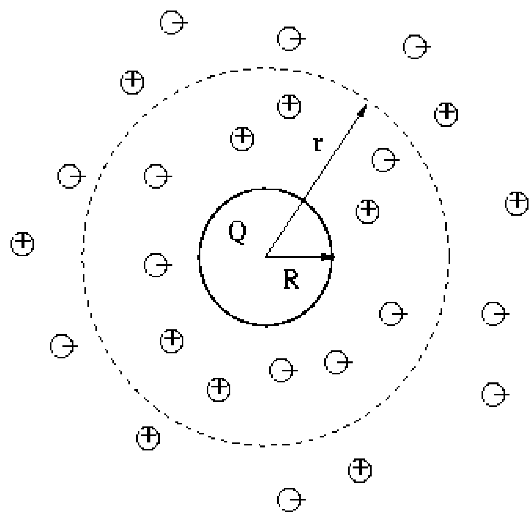
Physics 347 **Practice on diffusion, swimming, electrostatics**

Problem 1: Screening of a spherically charged object in salty solution:



In this problem you will calculate the screening of a spherical protein of radius, *R* that possesses a total surface charge, *Q* in the presence of positive and negative mobile ions.

a) Recall Gauss's law, $\int \vec{E}(r).\vec{dA}=q(r)/\epsilon$ where the left-hand side is the electric flux and the right hand side is the amount of charge enclosed out to a radius *r*. Use it to evaluate the magnitude of the electric field, *E(R)*, at the surface of the protein, where the radius is *R*. (Hint: your Gaussian surface is a sphere of radius, *R,* the electric field is uniform everywhere on this surface, and the charge enclosed is just *Q*).

b) In spherical coordinates, the linearized Poisson-Boltzmann (P-B) equation is: $\frac{d^2(rV(r))}{dr^2}=\frac{1}{\lambda_D^2}(rV(r))$, where the Debye length, $\lambda_D=\sqrt{\epsilon k_B T/(2e^2 c_\infty)}$ where c_∞ is the salt concentration far away from the protein. Show that $V(r)=(A/r)e^{(-r/\lambda_D)}$ is a solution to P-B equation.

c) Use the fact that $E(r)=-dV(r)/dr$ and your result for *E(R)* from (a) to find the constant, *A*.

d) For water, $\epsilon=80\epsilon_0$. Look up the value of the permitivity of free space, ϵ_0 to calculate the Debye screening length for a salt concentration, $c_\infty=100$ mM at room temperature, *T* = 293 K.

e) Linearizing the P-B equation is only valid if $eV(R)/(k_B T)\ll 1$. Using *Q* = *N e* and the above condition, derive a formula for the maximum number of excess charges *N* on the protein where the screening due to the salt solution derived above would be valid. Simplify your expression using the Bjerrum length,

$l_B = e^2/(4\pi\epsilon k_B T)$.

f) For a typical protein of radius R = 2 nm, use your result from (e) to calculate the maximum amount of charge N, using the Debye length from (d) and that the Bjerrum length in water at room temperature is l_B = 0.7 nm. Most proteins have less than this total amount of charge – Q is the total charge, i.e. the excess charge.

Problem 2: Stopping time of bacteria

Consider a bacterium, idealized to be a sphere with radius R=10^{-4} cm, with a density of of ρ = 1 g cm^{-3}, swimming in water which has a viscosity, η = 10^{-2} g/cm/s.

(a) Use Stoke's formula $c = 6\pi\eta R$ to calculate the drag coefficient, c. What are the units of c?

(b) There is a natural time scale given by $\tau = m/c$, where m is the mass of the bacteria. Calculate τ using your result from (a) and the mass of the bacteria, $m = \rho V$.

(c) Assume that the bacteria is initially swimming at a speed $v(0)$. It stops swimming and coasts to rest. Solve Newton's equation for the speed of the bacteria as a function of time, $v(t)$. Newton's equation, $m\,a = f$, for the bacteria can be written as $m\,(dv/dt) = -c\,v$. To solve this, bring all factors involving v to the LHS and everything involving t to the RHS. Integrate the LHS from $v(0)$ to $v(t)$ and the RHS from $t=0$ to t. You'll notice that the parameter τ appears in the solution, and can be thought of as a decay time.

(d) Use your result from (c) to calculate the distance that the bacteria travels in coming to rest if $v(0)$ = 2 x 10^{-3} cm/s. This requires integrating $v(t)$ from $t=0$ to $t=\infty$. Does the bacteria coast? Moving around is a tough job for a bacteria.

Problem 3: Finding Diffusing Food

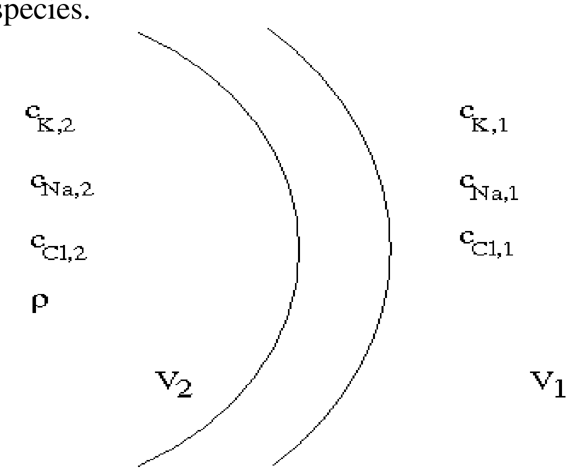
Bacteria swim to find food. Imagine that a bacteria is in a region of low food. For the bacteria to profit from swimming, it needs to get to the food source before diffusion spreads all the food into a uniformly lower concentration. You will calculate the minimum distance that a bacteria needs to swim so that it can outrun diffusion.

(a) On the same graph, plot the distance as a function of time, traveled by a bacteria if it is moving at a constant velocity of v = 30 μm/s, and the average distance traveled by food molecules diffusing in 3D with a diffusion coefficient of D=1000 $\mu m^2/s$.

(b) From the graph, what is the minimum distance from the source that the bacteria needs to be in order to outrun the diffusing food molecules? Express your answer in terms of the length of the bacteria whose length is ~ 2 μm long? What is the corresponding minimum time?

(c) The bacteria needs to generate a force $F = c\,v$, to overcome and balance the drag force that it experiences while swimming. Thus the power it generates (from 1st year physics) is $P = F\,v$. Use your result from Problem 3 for the drag coefficient, c. Calculate the power that the bacteria generates while swimming (express in Watts = J/s).

Problem 4: Membrane Rest Potential:
This is a redo of the calculation in the notes. Calculate the concentrations of Na+, K+ and Cl- inside an axon ($c_{Na+,2}$, $c_{K+,2}$, and $c_{Cl-,2}$) given that the outside concentrations are $c_{Na+,1}$ = 140 mM, $c_{K+,1}$=10 mM, $c_{Cl-,1}$ = 150 mM and that there is a excess negative charge inside that has a concentration of ρ = 125 mM. The net charge inside and the net charge outside must separately equal zero. The Nernst potential, $V_2 - V_1 = - k_BT/e \ln(c_2/c_1)$ must be the same for each species.



a) Using the equality of the Nernst potentials for each species, derive a condition relating the ratios of concentrations for each species.

b) Use the charge density equation inside the cell and your results from a) to find the concentrations inside. Hint: to arrive at an equation, let $c_{Na+,2} = x$ and arrive at an equation for x to solve.

c) Using the result in (b), evaluate the Nernst potential for each species showing that they are indeed equal.