Topic 10: Physics of charged solutions: Applications
(chapter 9 in book)

Outline:

- What’s the energetic cost of packing a protein that has charge?
- What’s the energetic cost of packing charged DNA into a viral capsid?

Electrostatic energy:

How much energy is there to bring two charges together?

Energy/work to bring Q2 in vicinity of Q1 is Q2 V2

The total energy to bring these two charges together is: ½(Q1 V1 + Q2 V2)

So in general:

$$U = \frac{1}{2}\sum Q_i V_i = \frac{1}{2}\int V(\boldsymbol{r})\rho(\boldsymbol{r})d^3\boldsymbol{r} = \int V(\boldsymbol{r})dQ$$

Example: assembling a charged sphere

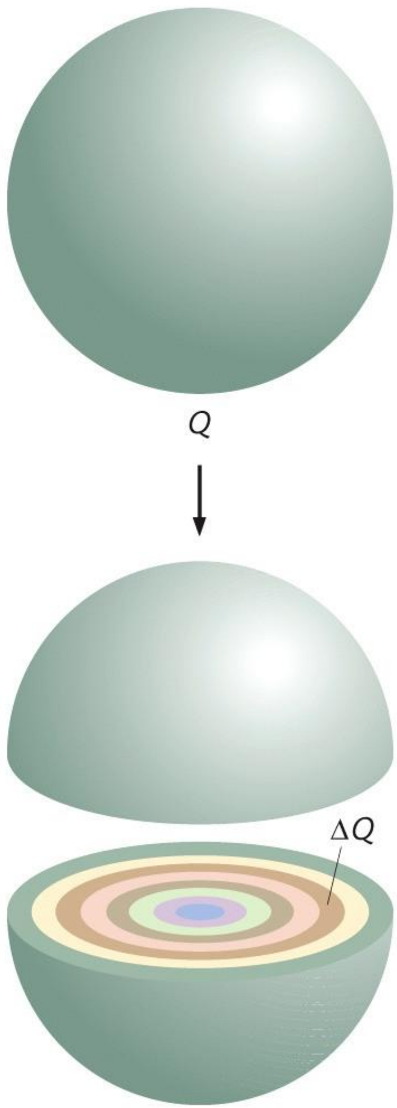


Figure 9.11 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

Let's use our formula to evaluate U for a sphere that has charge Q distributed uniformly over its volume.

The sphere gets assembled layer by layer. When we are about to add a layer at radius, r, with a charge, q enclosed inside the potential at this distance is:

$$V(r) = \frac{1}{4\pi D\epsilon_o}\,\frac{q}{r} = \frac{1}{4\pi D\epsilon_o}\,\frac{\rho 4/3\pi r^3}{r}$$

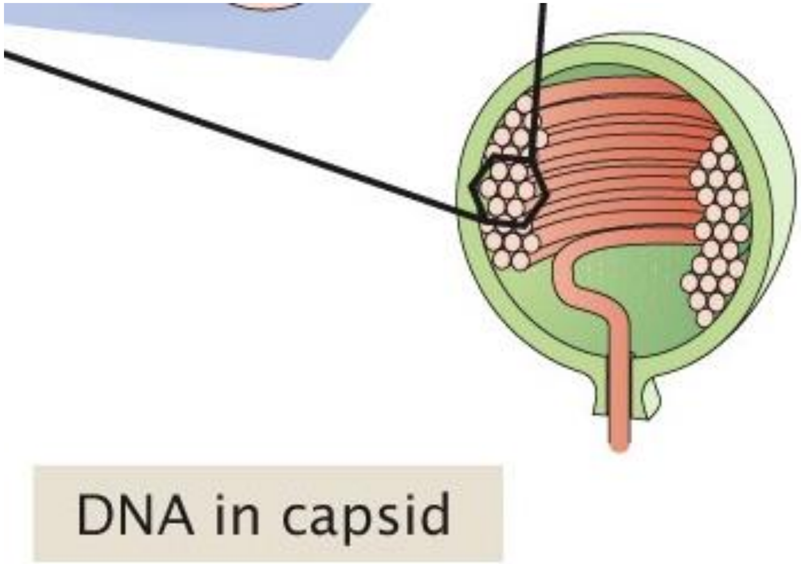
So the energy to assemble is:

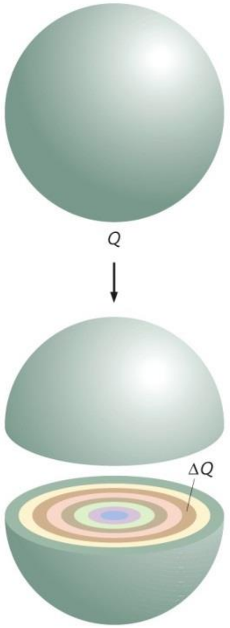
$$U = 4\pi \int \frac{1}{4\pi D\epsilon_o}\,\frac{\rho(\frac{4}{3})\pi r^3}{r}\,\rho\, r^2 dr$$

so

$$U = \frac{1}{4\pi D\epsilon_o}\left(\frac{4}{3}\right)\pi 4\pi\rho^2\,\frac{R^5}{5} = = \frac{1}{4\pi D\epsilon_o}\,\frac{3}{5}\,\frac{Q^2}{R}$$

First pass calculation: DNA in capsid





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If there is no screening, how much energy to put DNA into capsid? Assume charge Q of DNA is uniformly distributed in volume of capsid, so we can use our equation

For DNA in φ29 virus that has 20000 bp and there are 2 e / bp for DNA

Q = 20000 * 2e = 400000 e and R = 20 nm using D = 80 for H20 gives

so $U = \frac{1}{4\pi D\epsilon_o}\frac{3}{5}\frac{Q^2}{R}$ = 10^8 pN nm >> 200000 pN nm that was measured

Resolution: screening due to counterions greatly reduces the potential energy

Energy to assemble a protein:

uncharged hyrdophobic residues in core

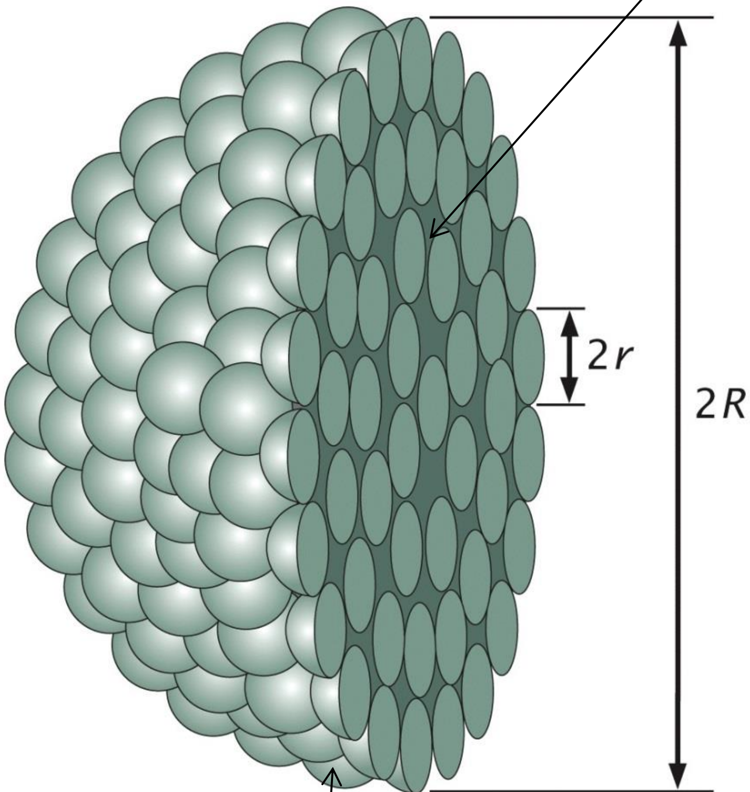

Figure 9.13 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

charge polar residues just on surface, Qtot

How does the energy to assemble the protein depend on it's size (i.e. the number of polar residues on it's surface)?

$$U = \frac{1}{2}\int V(\boldsymbol{r})\rho(\boldsymbol{r})d^3\boldsymbol{r} = \frac{1}{2}\int \frac{1}{4\pi D\epsilon_o}\frac{Q_{tot}}{R}\frac{Q_{tot}}{4\pi R^2}dA$$

so

$$U = \frac{1}{2}\frac{1}{4\pi D\epsilon_o}\frac{Q^2}{R}$$

Now Q = N e where $N = (4\pi R^2)/\pi r^2$

So the energy is

$$U = \frac{1}{2}\frac{e^2}{4\pi D\epsilon_o}\frac{R^3}{r^4}$$

Assembling proteins:

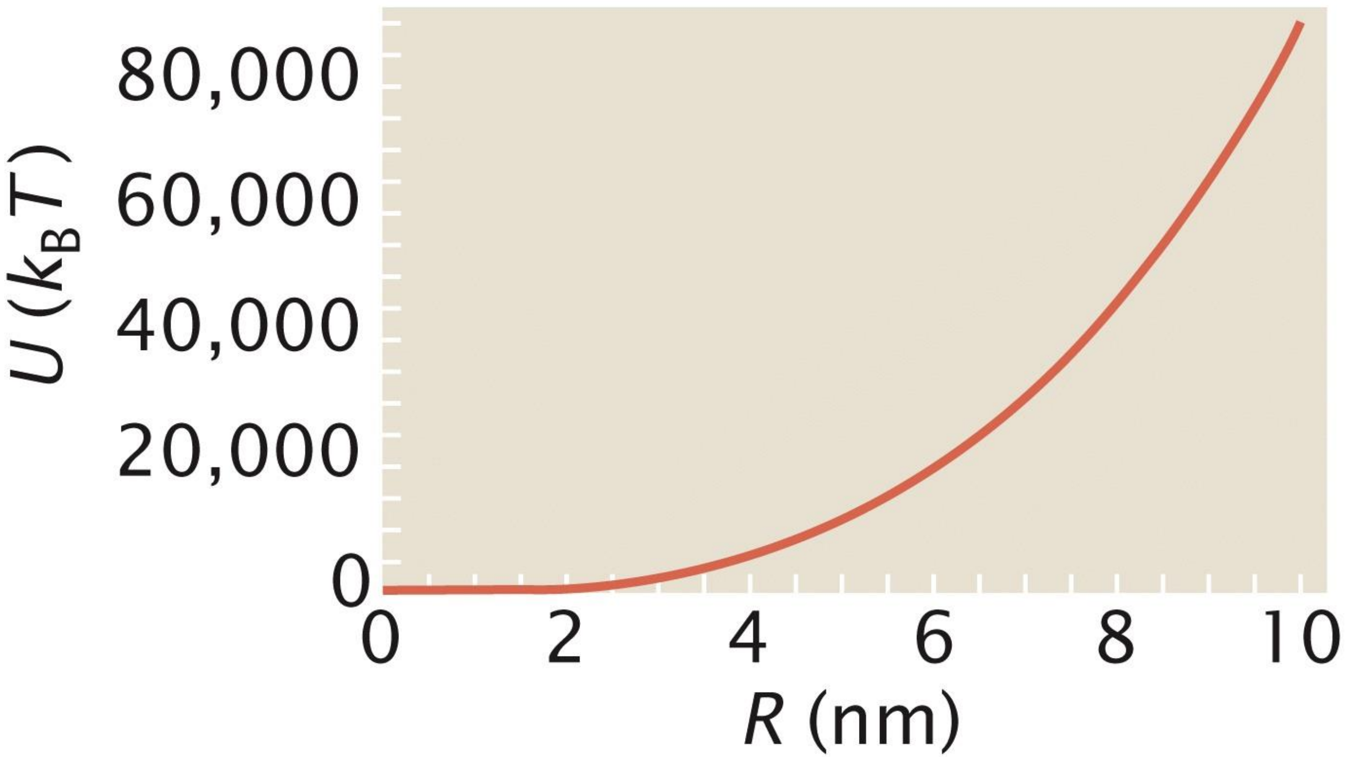

Figure 9.14 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

Key is that for proteins of typical size ~ 2 nm, it does not cost much charging energy to assemble the object. Whereas larger objects ~ 10 nm and bigger are more costly.

Packing DNA into a virus: Redux

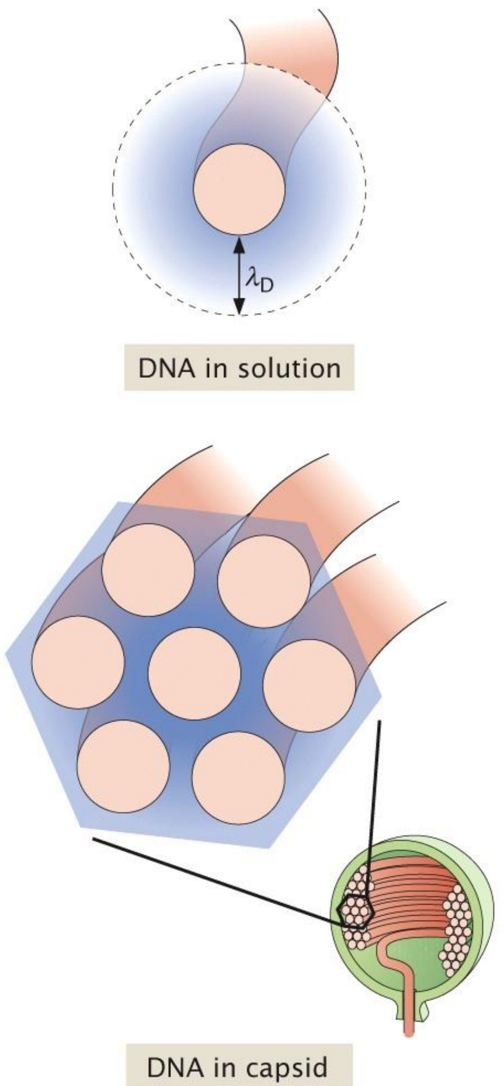

Figure 9.17 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

We know that counterions screen the DNA

The Debye screening length is given by λ_D

So given this, what is the energy required to assemble into the capsid?

Now the system is effectively screened, and so the object is neutral.

The energy cost is the loss of entropy of the cations in going from the outside to the inside

Recall our entropy for gas, S ~ N log(V)

So the work done is $dG = (-kT\ N \log V_{cap}) - (-kT\ N \log V_{cloud})$

or

$$\Delta G = N\ k\ T \log \frac{V_{cloud}}{V_{cap}}$$

Packing DNA into a virus: Redux

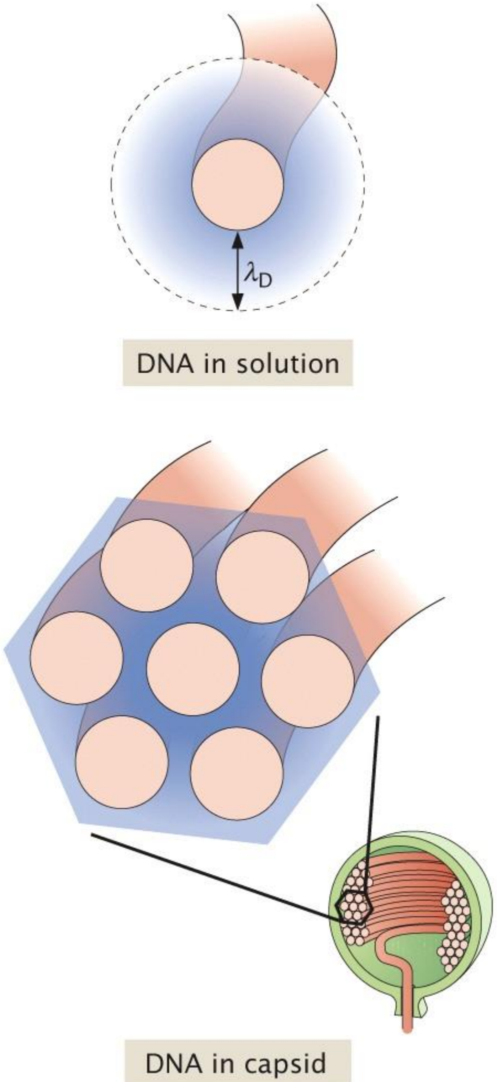

Figure 9.17 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

Since there were -40000 e for the DNA, there are N = +40000 e counterions

The volume of the cloud outside capsid is:

$$V_{cloud} = L\,\pi\,(\,(r + \lambda_D)^2 - r^2)$$

where r = 2 nm for DNA, and λ ~ 1 nm

The free volume for the counterions in the capsid is

$$V_{capsid} = \frac{4}{3}\pi R^3 - L\pi r^2$$

where R = 20 nm for the phi29 virus.

Using the previous equation and these numbers

ΔG = 265000 pN nm ~ 200000 pN nm that was

measured experimentally.

Summary:

- Looked at the energy required to assemble charge
- For DNA packed into capsid, without counterions a significant energy would be required to overcome electrostatic energy
- Assembling small charged objects like proteins is not that costly
- Counterions make DNA effectively neutral and the energy cost is mostly the entropic cost of moving the counterions into the capsid
- Compare that to the almost negligable entropic cost for the DNA polymer itself (you did that on A 5)