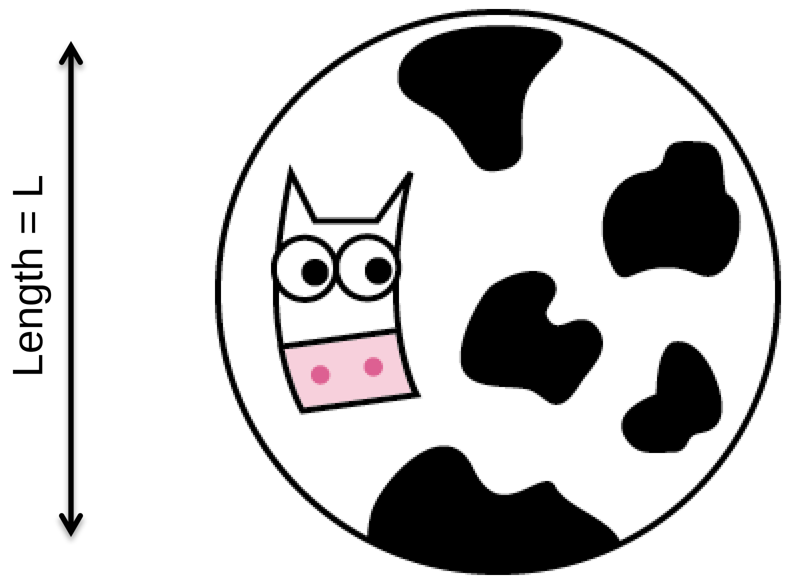
Topic I: Scaling laws in biology

Physics constrains Biology

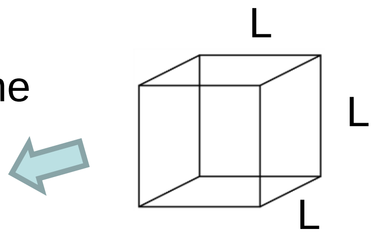
- Can big bird fly?
- Why are whales so big?
- Why are cheetah’s so fast?

How do things scale with size?

Consider a spherical cow … how does an organism's height depend on mass?



Mass is proportional to Volume



Mass ~ Volume ~ L^3

So

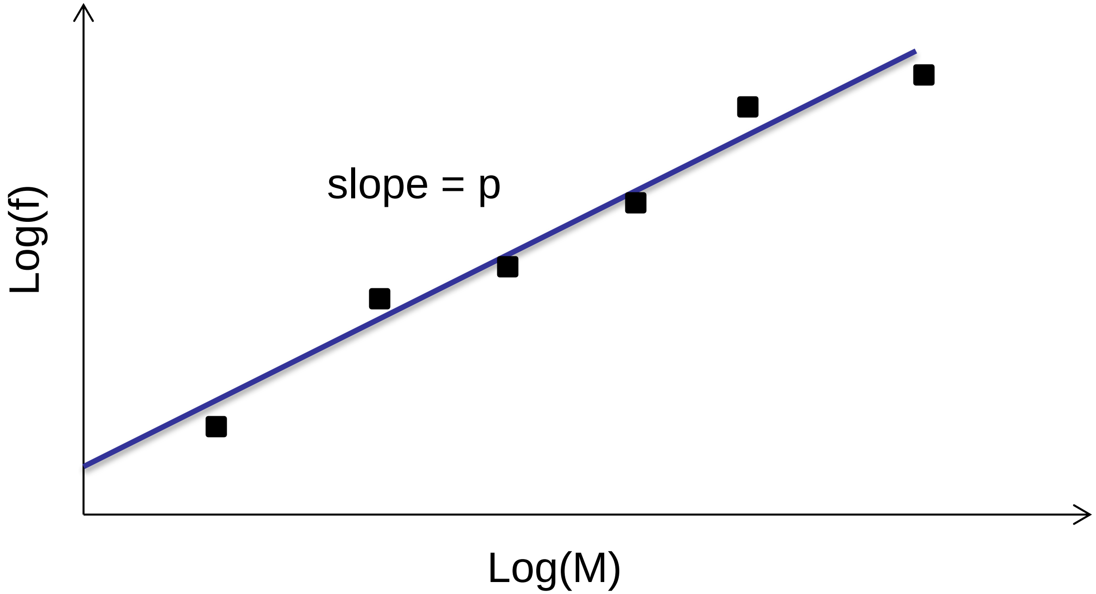
$$L \sim (\text{Mass})^{1/3}$$

Study of scaling behaviour = Allometry

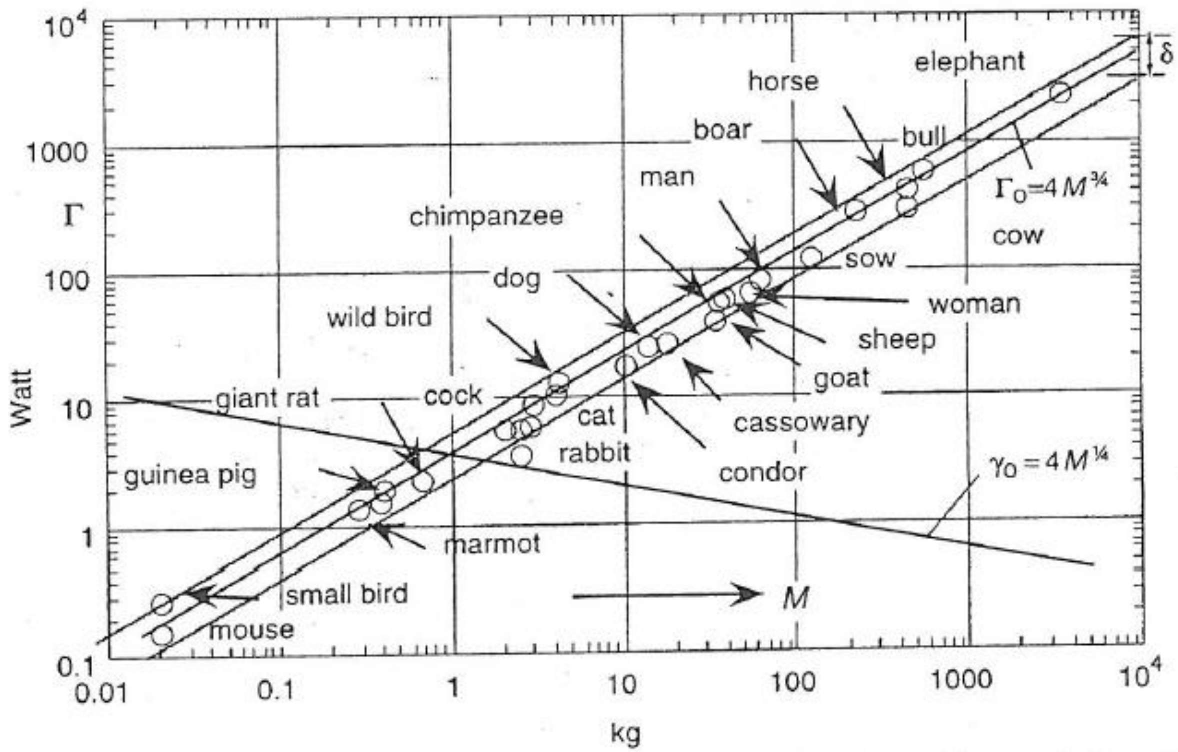
Physical properties of organisms that scale with mass = allometric scaling

$$f = a M^{p}$$

For f = Length, p = 1/3



How does energy consumption scale with mass?

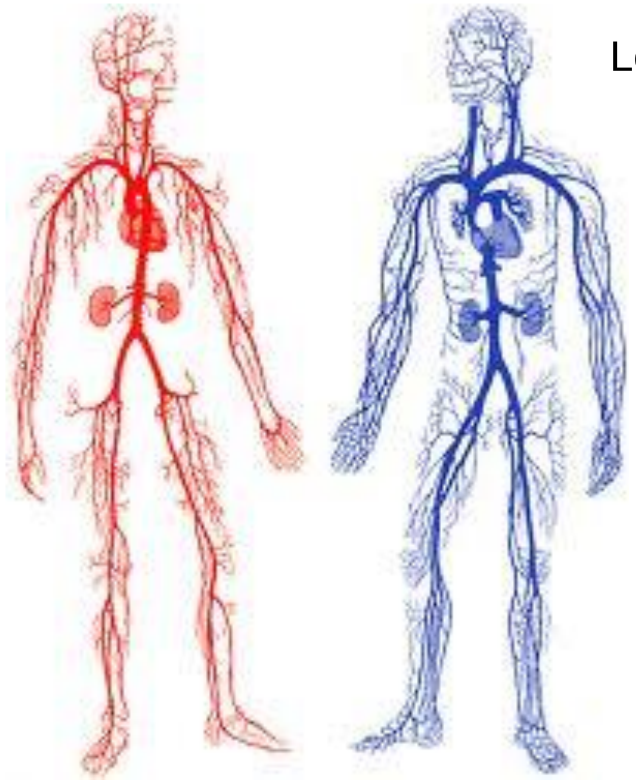


From *Zoological Physics*, Boye Ahlborn

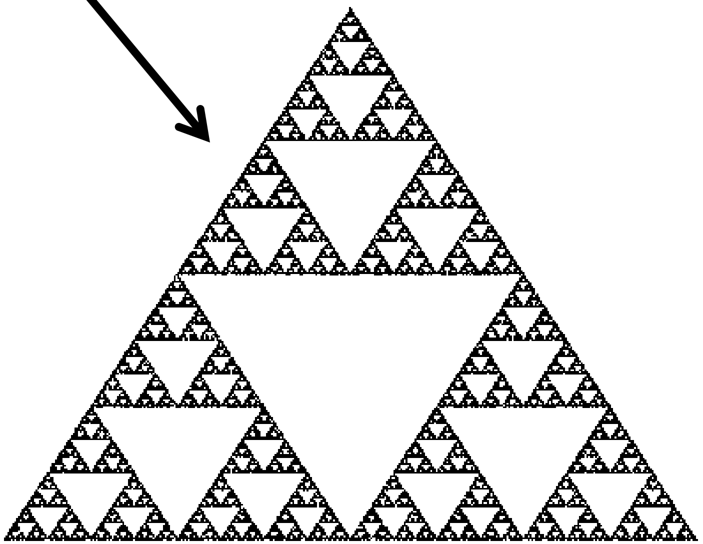
Metabolism = $\Gamma = a\, M^{3/4}$

BUT why is p = ¾ and not p = 1 that might be expected if Power ~ Mass ?

Circulatory system and fractals



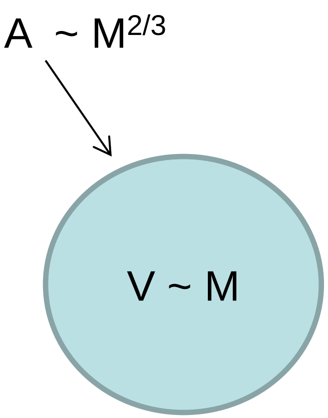
Looks the same on all scales = fractal



Circulatory system has a fractal character – not quite 3D, not quite 2D

Physicists showed that this leads to p = ¾ for energy consumption.

Advantages of different body sizes



Q: how long will it take an animal to burn through it's fat reserves?

$\Gamma = \frac{\Delta E}{\Delta t}$ and assume ΔE come from burning fat. so $\Delta E \propto M$

time taken: $\Delta t = \frac{\Delta E}{\Gamma} \propto \frac{M}{M^{3/4}} = M^{1/4}$

so larger animals can go longer without eating.

Q: how much heat does an animal lose based on its size?

- Heat loss: heat loss $\propto A \propto M^{2/3}$

  but surface area/unit mass $= \frac{A}{M} = M^{-1/3}$

- So large animals will lose less heat when it's cold. e.g. Deep diving whales.

Curious scalings I:

parameter, f	factor a	exponent α
body surface in m²	0.11	0.65
brain mass (man) in kg	0.085	0.66
brain mass (non primates) in kg	0.01	0.7
breathing frequency in Hz	0.892	−0.26
cost of transport (running) in J/m · k	7	−0.33
cost of transport(swimming) in J/m · kg	0.6	−0.33
effective lung volume in m³	$5.67 \cdot 10^{-5}$	1.03
frequence of heartbeat in Hz	4.02	−0.25
heart mass in kg	$5.8 \cdot 10^{-3}$	0.97
life time in years	11.89	0.20
metabolic rate in W	4.1	0.75
muscle mass in kg	0.45	1.0
skeletal mass (cetaceans) in kg	0.137	1.02
skeletal mass (terrestrial) in kg	0.068	1.08
speed of flying in m/s	15	1/6
speed of walking in m/s	0.5	1/6

$f \sim 1/M^{1/4}$

Aside: note that $f_H \propto M^{-1/4}$

if the heart were behaving as a simple harmonic oscillator, $f_H \propto \sqrt{M}$

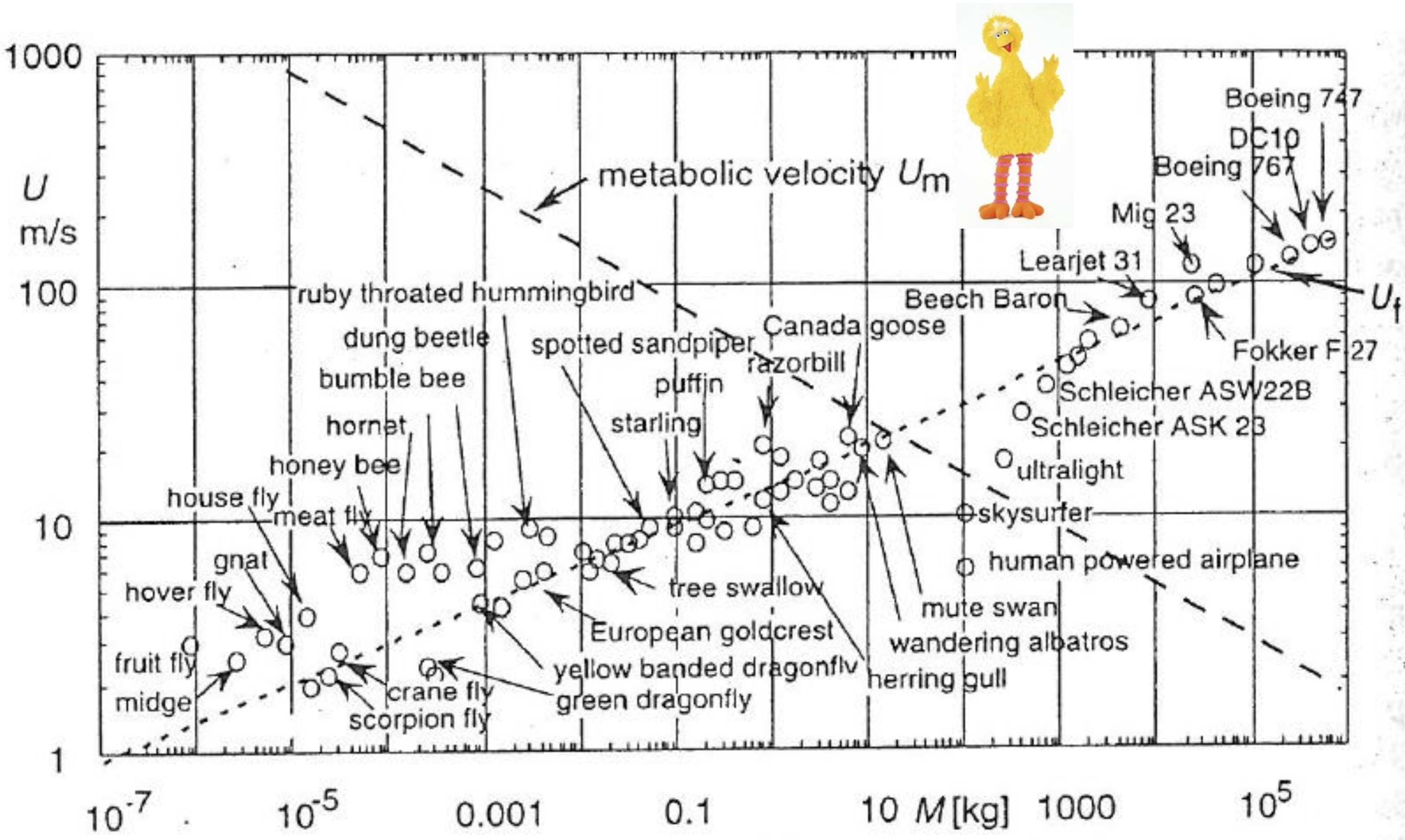
Curious scalings II:

parameter , f	factor a	exponent α
body surface in m^2	0.11	0.65
brain mass (man) in kg	0.085	0.66
brain mass (non primates) in kg	0.01	0.7
breathing frequency in Hz	0.892	−0.26
cost of transport (running) in J/m · k	7	−0.33
cost of transport(swimming) in J/m · kg	0.6	−0.33
effective lung volume in m^3	$5.67 \cdot 10^{-5}$	1.03
frequence of heartbeat in Hz	4.02	−0.25
heart mass in kg	$5.8 \cdot 10^{-3}$	0.97
life time in years	11.89	0.20
metabolic rate in W	4.1	0.75
muscle mass in kg	0.45	1.0
skeletal mass (cetaceans) in kg	0.137	1.02
skeletal mass (terrestrial) in kg	0.068	1.08
speed of flying in m/s	15	1/6
speed of walking in m/s	0.5	1/6

Number of heart beats in a lifetime = (frequency) x (life time) = 1×10^9 beats

a constant independent of size!!!

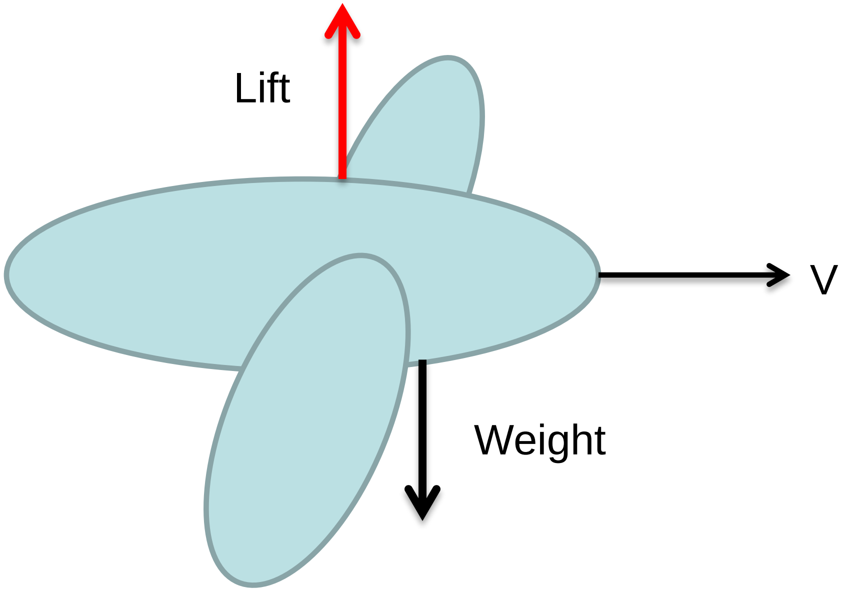
Can Big Bird fly???



Flight velocity = speed needed to fly ~ M $^{1/6}$ WHY???

Metabolic velocity = speed that body can supply ~ M $^{-1/4}$ WHY???

Speed to fly



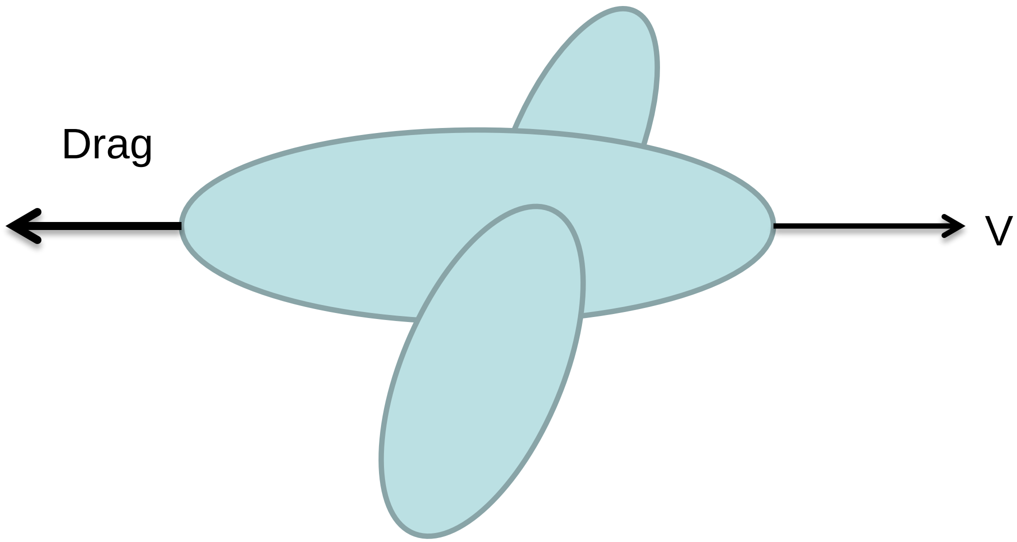
To just fly, the weight needs to be balanced by lift.

Lift ~ (Area ~ $M^{2/3}$) x (V^2) = (Weight ~ M)

So

$V_{fly} \sim M^{1/6}$

Speed body can generate



Body must supply power to overcome the drag force

$$\text{Power} = (\text{Drag}) \times V = (\text{Metabolic Power}) \sim M^{3/4}$$

Now when flying

$$\text{Drag} \sim (\text{Lift}) \sim (M)$$

So

$$V \sim M^{-1/4}$$

Summary:

Many of an organism’s characteristics can be related to its mass

Body plans must operate under physical constraints

We have seen how physics constrains the size of flying animals

So, Big Bird can not fly, but the Kory Bustard can