Topic 7a: Biological Beams

Overview:

What gives a cell it’s internal structure?

There are biofilaments that have structural rigidity

It costs energy to bend a beam – define bending energy

Applications: Packing DNA into viruses, Nucleosome formation

Beams are everywhere:

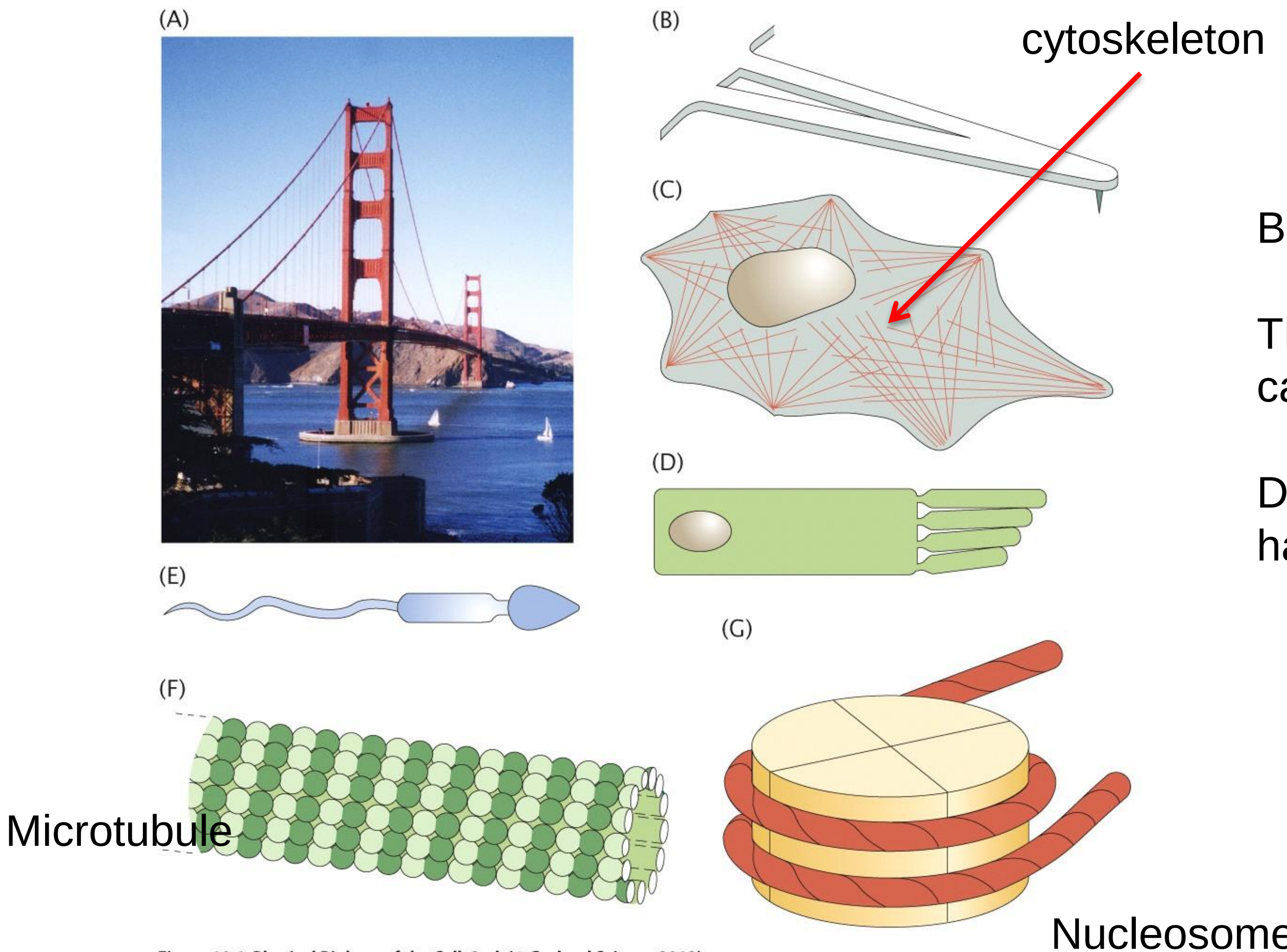


Figure 10.1 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

Beams provide structure

They resist bending and can support a load

DNA, actin, microtubules all have different rigidities

Deformations of a beam:

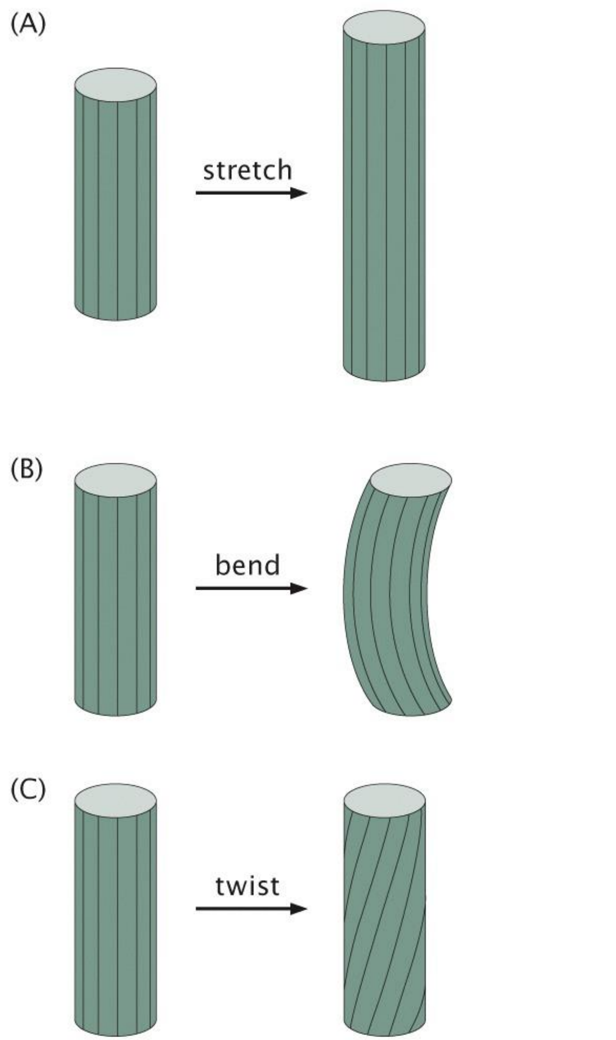


Figure 10.2 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

There are 3 ways to deform a beam

All involve some deformation of bonds in the material

Each has it’s own associated spring constant

Elastic deformations: F = k x

Stress Strain

$$\sigma = E\,\varepsilon$$

Deformation Energy:

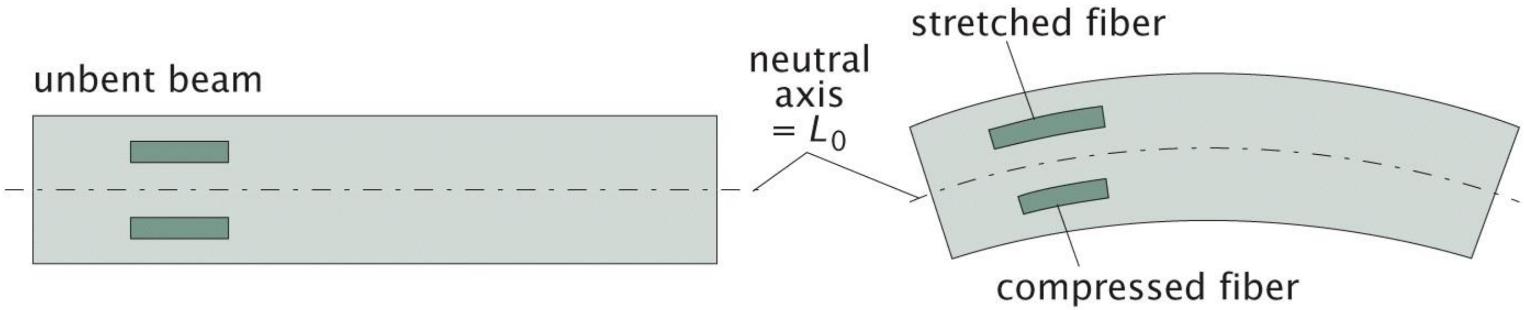


Figure 10.3 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

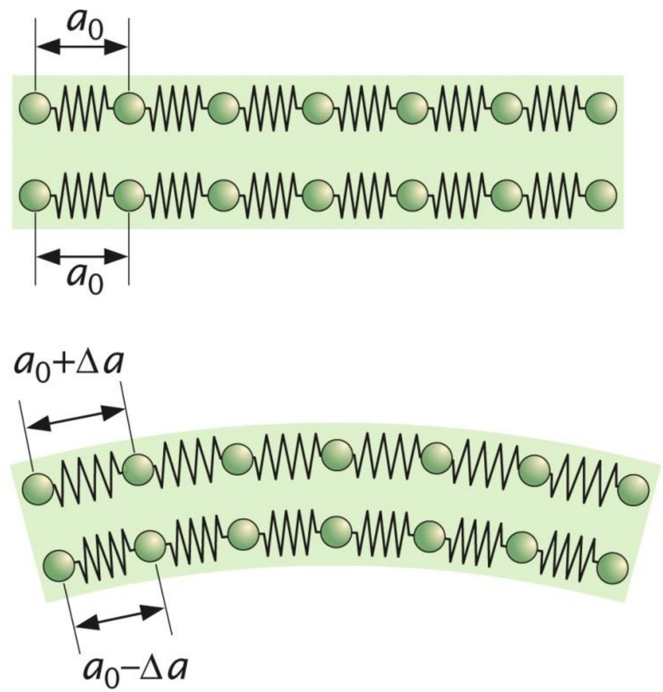
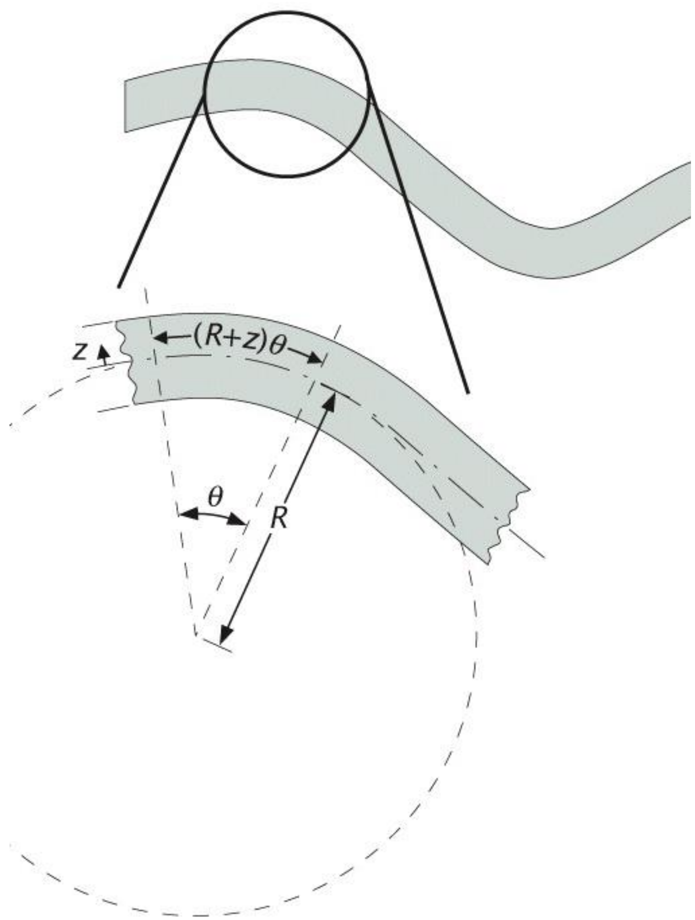
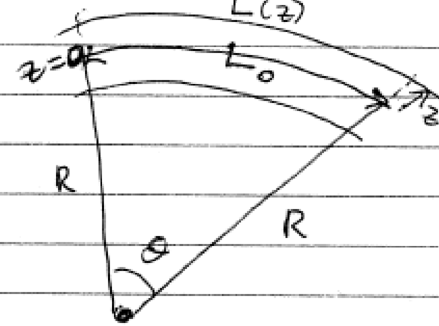


Figure 10.5 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

Deformation Energy



$\therefore$ Locally, a bent region will fall on a circular arc of radius R ~~for~~



• What is $\Delta L(z) = L(z) - L_0$?

$$L_0 = R\theta \Rightarrow \theta = \frac{L_0}{R}$$

and

$$L(z) = (R+z)\theta = (R+z)\frac{L_0}{R}$$

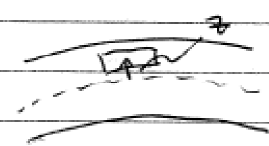
so

$$\Delta L(z) = (R+z)\frac{L_0}{R} - L_0 = \frac{z L_0}{R}$$

so

$$\text{strain}, \quad \varepsilon = \frac{\Delta L(z)}{L_0} = \frac{z}{R}$$

Energetic cost for small volume element



$$w(z) = \frac{1}{2} E\, \varepsilon(z)^2 = \frac{1}{2} E \left(\frac{\Delta L(z)}{L_0}\right)^2$$

energy density

$$= \frac{1}{2} E \frac{z^2}{R}$$

Deformation Energy: Integrating over beam cross-section

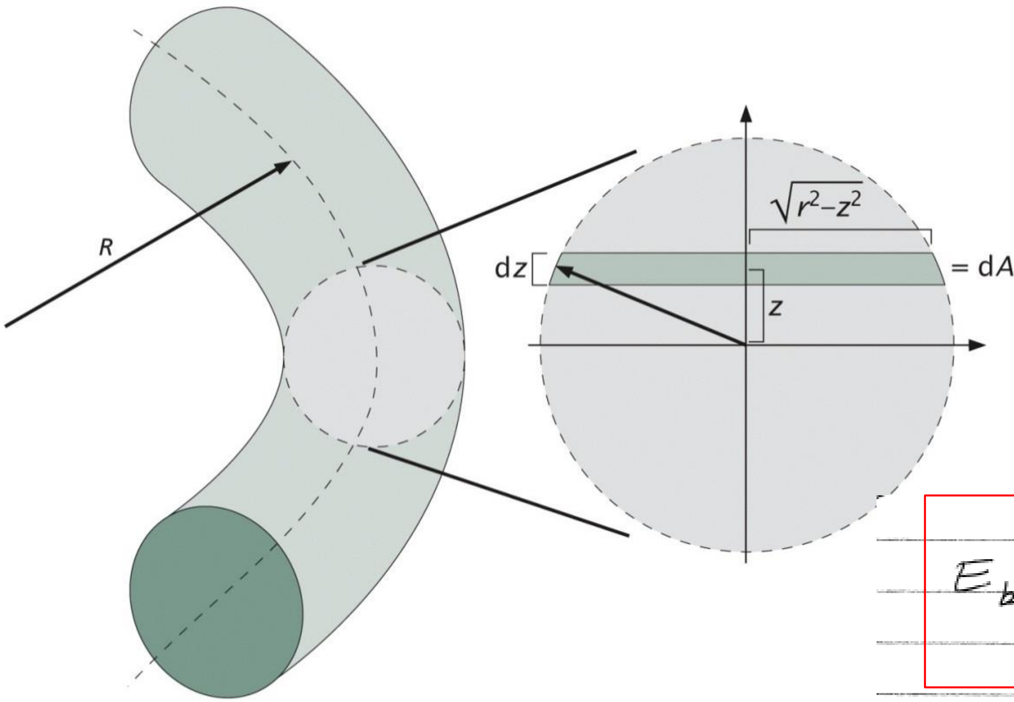


Figure 10.6 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

Integrate over cross-sectional area

$$E_{bend} = L_0 \int dA \, \frac{E}{2R^2} z^2$$

$$\boxed{E_{bend} = L_0 \frac{EI}{2R^2}} \; ; \quad I = \underbrace{\int dA \, z^2}_{\text{moment of material}}$$

Bending Energy: General Result

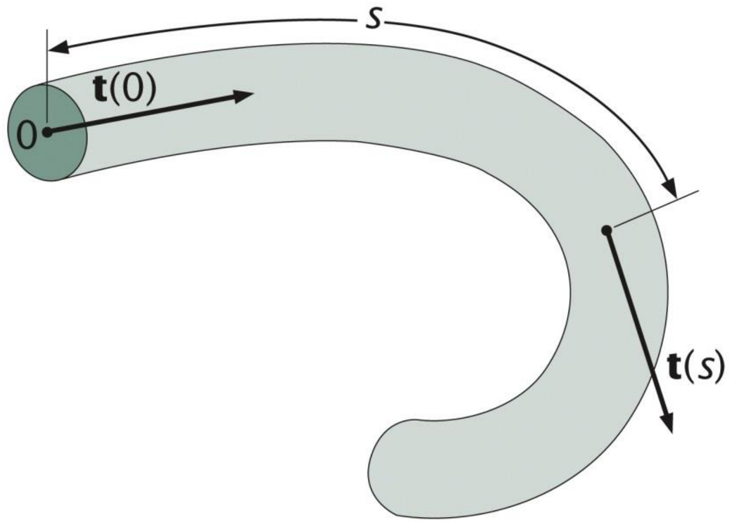
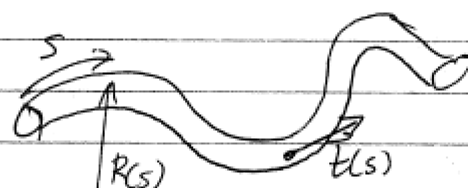


Figure 10.7 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

Now we consider that at each point on the beam it can be curved in it's own way

Each point s, has it's own curvature R(s) (i.e. a different radius on which the beam is bent)



local curvature

$$\Rightarrow \boxed{E_{bend} = \frac{K_{eff}}{2}\int_0^L ds \, \frac{1}{R(s)^2}}$$

where $K_{eff} = E \cdot I \equiv$ material constant

- If we have $\vec{t}(s) \equiv$ tangent vector to the curve then

$$K(s) = \frac{1}{R(s)} = \left|\frac{d\vec{t}}{ds}\right| \equiv \text{curvature}$$

Connection to persistence length:

Recall that the persistence length is the distance over which the polymer/beam begins to become uncorrelated (i.e. form loops)

In other words, the beam is curving over distances of it's length, so L ~ R ~ ξ_P

How can we connect this to the bending energy?

Recall, that bending is an internal degree of freedom and that each degree of freedom has ½ k T of energy – and there are 2 bending degrees of freedom

So

$$k\,T = \frac{E\,I\,L}{2\,R^2} = \frac{E\,I}{2\,\xi_P}$$

gets smaller as T goes up

Or

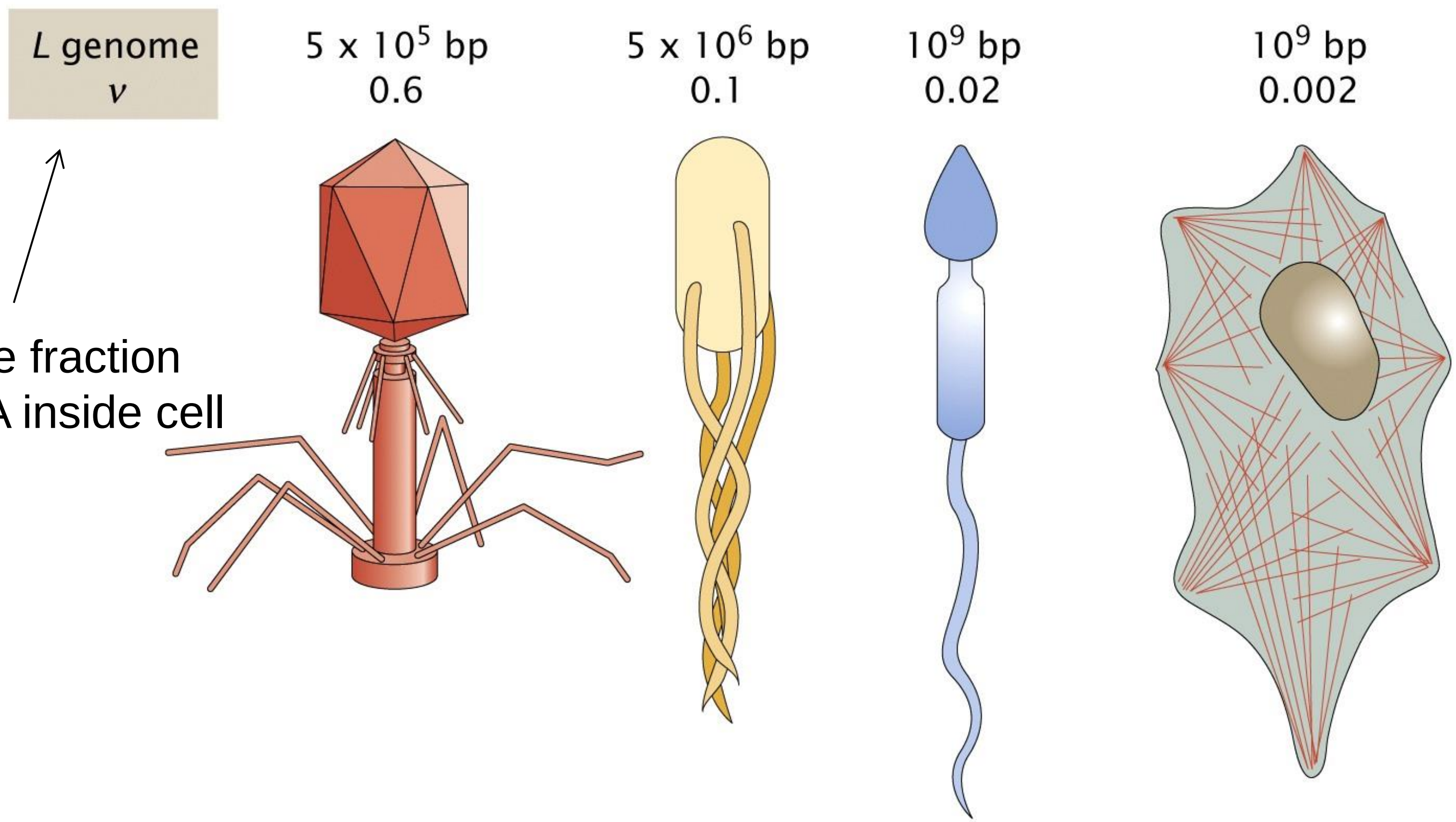
$$\boxed{\xi_P = \frac{E\,I}{k\,T} = \frac{K_{eff}}{k\,T}}$$

gets larger as K goes up

Or the spring constant

$$K_{eff} = k\,T\,\xi_P$$

Applications I: Packing DNA into a small space



Volume fraction
of DNA inside cell

Figure 10.13 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

Recall that to perfectly pack spheres into a space, the packing fraction ~ 0.75

DNA packing into viruses

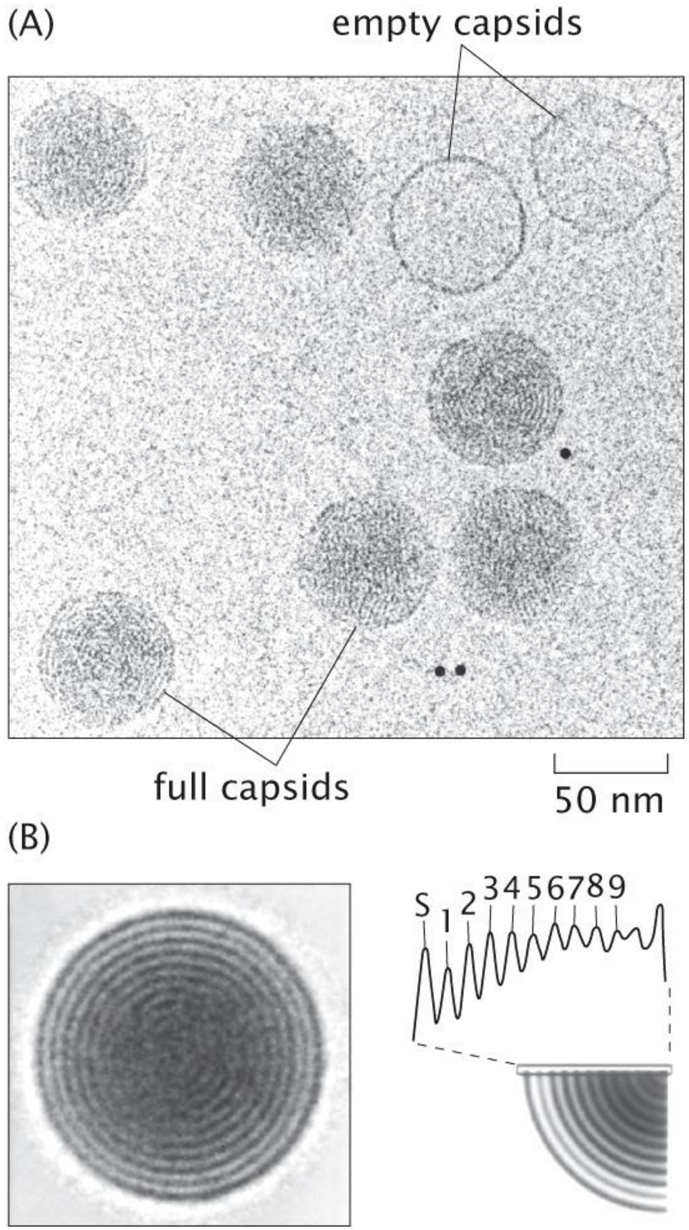


Figure 10.14 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

Viruses are made of a capsid which is a container that stores its DNA

Capsids are on the order of 50 – 150 nm in size

The images on the left show the DNA packed into the virus

It looks like a coiled up string, forming loop after loop

Q: How much energy does it cost to bend DNA into all these loops?

This will be the amount of work that the virus needs to do to pull the DNA into it's capsid

It has a protein motor to do this

Viruses come in many shapes:

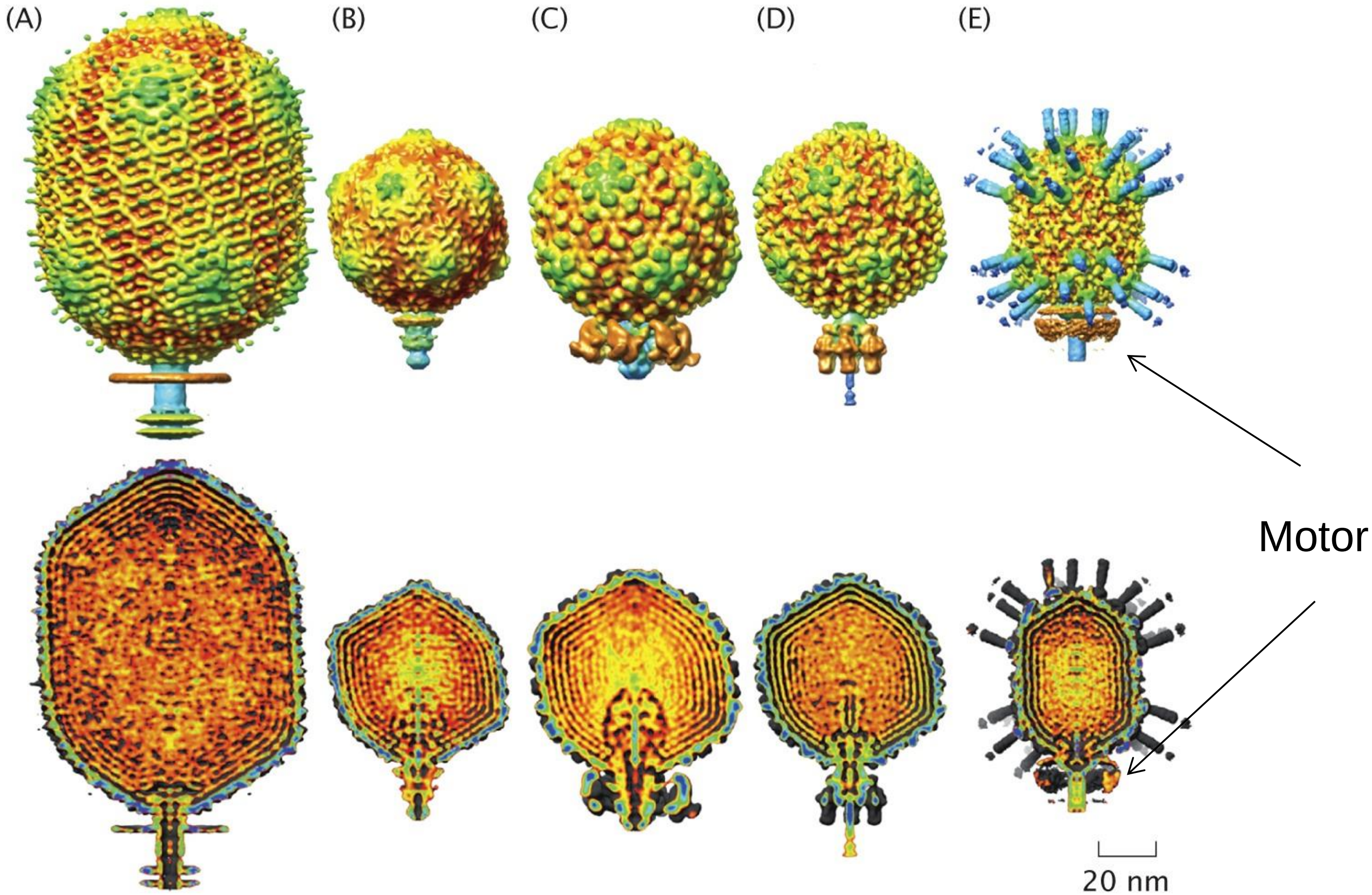


Figure 10.15 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

Different shapes cause the DNA to pack differently

Is one geometrical shape better than another for packing? i.e. sphere vs cylinder?

DNA Packing in viruses: our model

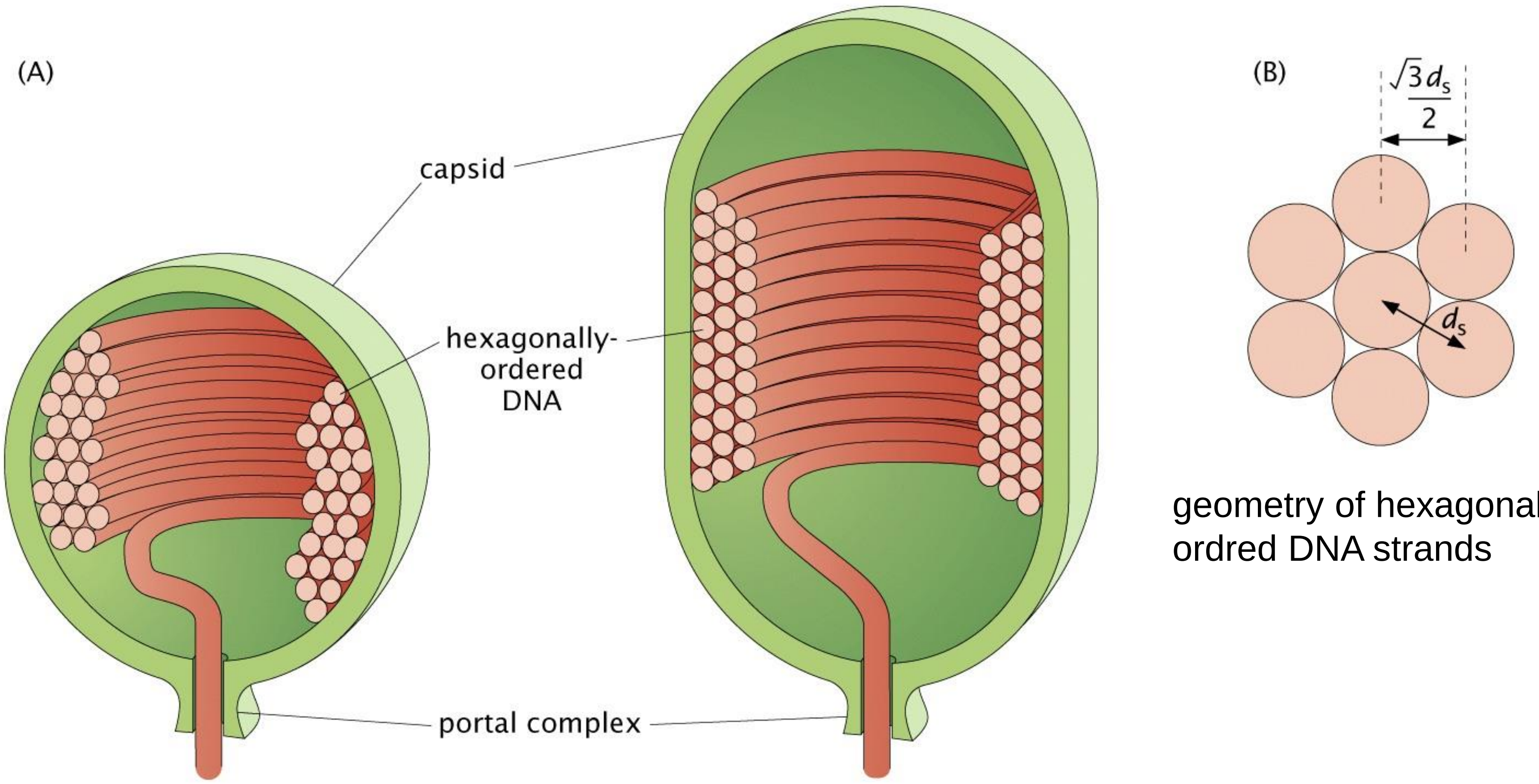


Figure 10.16 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

geometry of hexagonally ordred DNA strands

Based on observations, DNA packs from the outside in, gradually filling in the capsid volume

DNA strands pack in a hexagonally closed pack configuration

Separation of strands scaling:

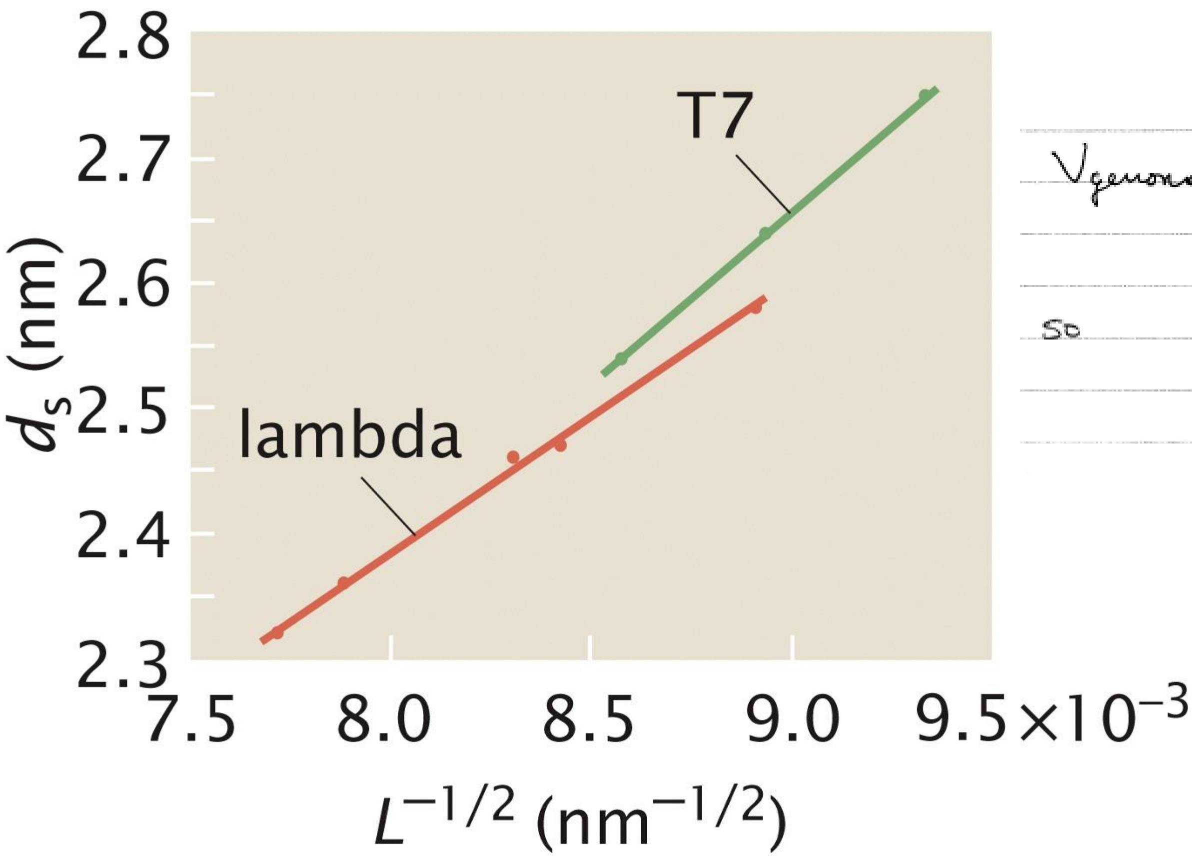


Figure 10.17 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

$V_{genome} \sim L\, d_s^2$ (volume of cylinder)

$\approx V_{capsid}$

so

$d_s \approx \sqrt{\frac{V_{capsid}}{L}} \sim L^{-1/2}$

Force needed to pack DNA

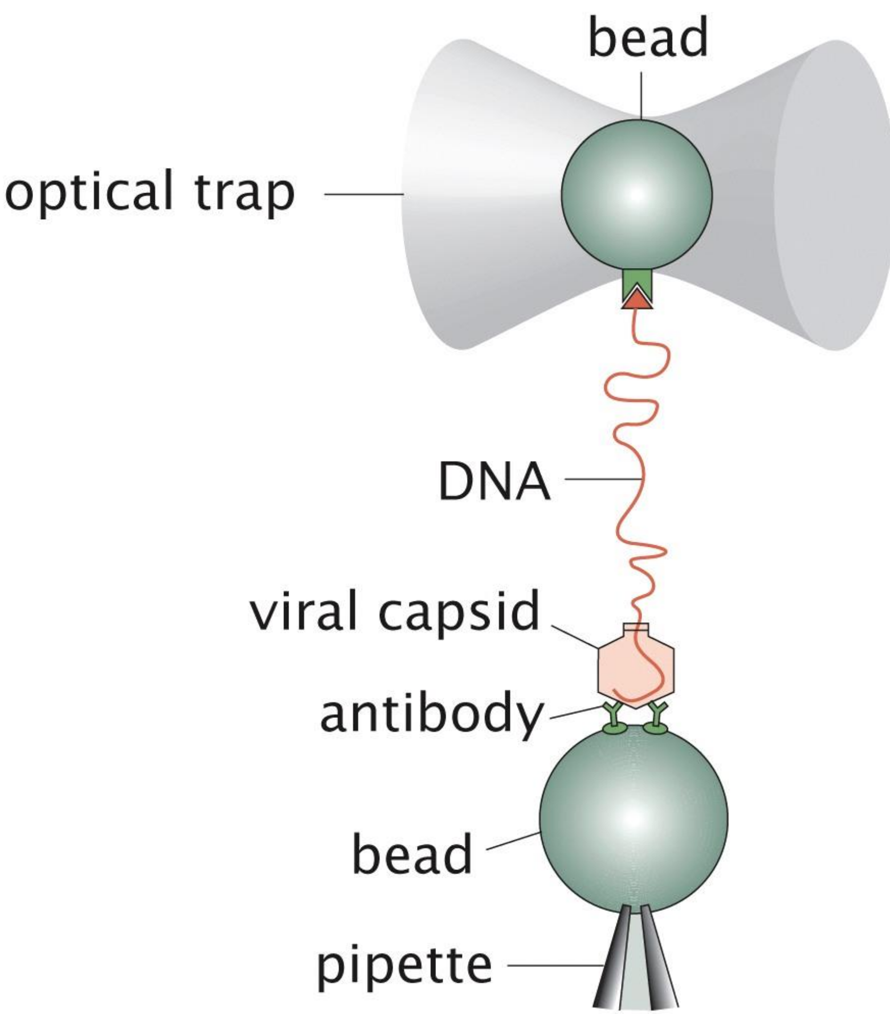


Figure 10.18 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

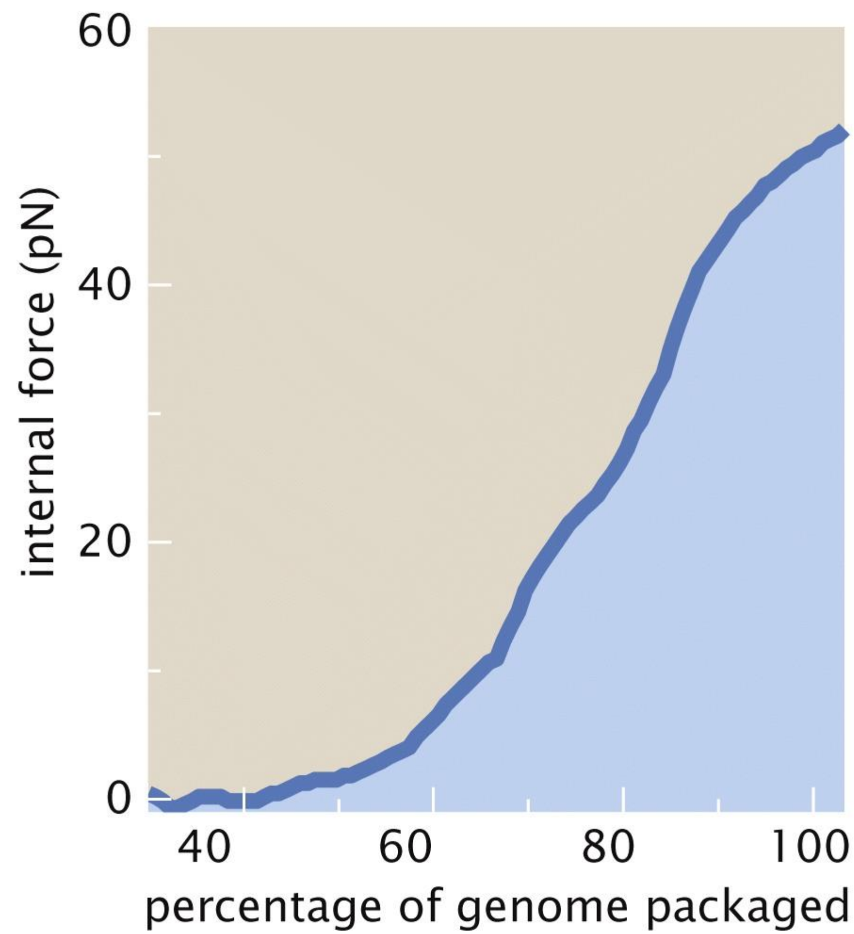
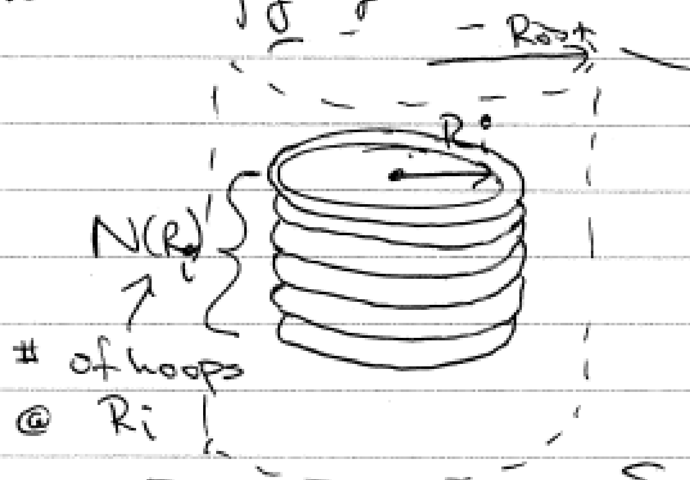


Figure 10.19b Physical Biology of the Cell, 2ed. (© Garland Science 2013)

Using an optical trap, can measure the force
the motor is exerting against the DNA inside
as a function of the amount packed into the capsid

~~Wet~~ Energy of bent DNA in capsid:



• DNA bent into $N(R_i)$ hoops of radius R_i

so $$E_{bend} = \sum_{R_i} E_{hoop}(R_i) \cdot N(R_i)$$

now

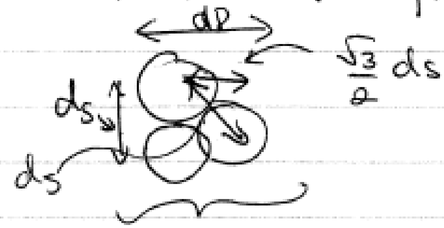
$$E_{hoop}(R_i) = \xi_p k_B T \frac{(2\pi R_i)}{2 R_i^2} = \pi \frac{\xi_p k_B T}{R_i}$$

so

$$E_{bend} = \pi \xi_p k_B T \sum_{R_i} \frac{N(R_i)}{R_i}$$

Bending energy calculation continued …

Convert Σ into integral over radius via



so $\sum_i = \int \frac{2}{\sqrt{3}\, ds}\, dR$

\# of hoops in dR

$$= \frac{dR}{\left(\frac{\sqrt{3}\, ds}{2}\right)}$$

radius

So

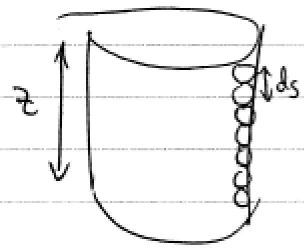
$$E_{bend} = \frac{2\pi\, \xi_p k_B T}{\sqrt{3}\, d_s} \int_R^{R_{out}} \frac{N(R')}{R'}\, dR' \qquad (1)$$

- This gives the bending energy as a function of R, the extent of the DNA that has been packed into the capsid.
- Would rather express it in terms of L, the length of DNA inside the capsid.

$$L = \frac{2}{\sqrt{3} d_s} \int_R^{R_{out}} 2\pi R' N(R')\, dR$$

Bending Energy continued: Packing into a cylinder

• Consider a cylindrical capsid



so $N(R) = \frac{z}{d_s}$ (indep of R)

So Eqn (1) gives

$$E_{bend}(R) = \frac{2\pi \xi_p k_B T z}{\sqrt{3}\, d_s^2} \ln\left(\frac{R_{out}}{R}\right) \qquad (2)$$

and

$$L(R) = \frac{2\pi z}{\sqrt{3}\, d_s^2}\left(R_{out}^2 - R^2\right)$$

or

$$R = R_{out}\sqrt{1 - (\sqrt{3}\, d_s^2 L)/(2\pi z R_{out}^2)} \qquad (3)$$

Equations 2 and 3 can then be combined to give the bending energy as a function of the amount of genome, L that is packed into the virus

We can calculate the force that the DNA is exerting by, f = - dE/dL

Force exerted by bent DNA

Elastic energy increases as you pack more DNA into the virus

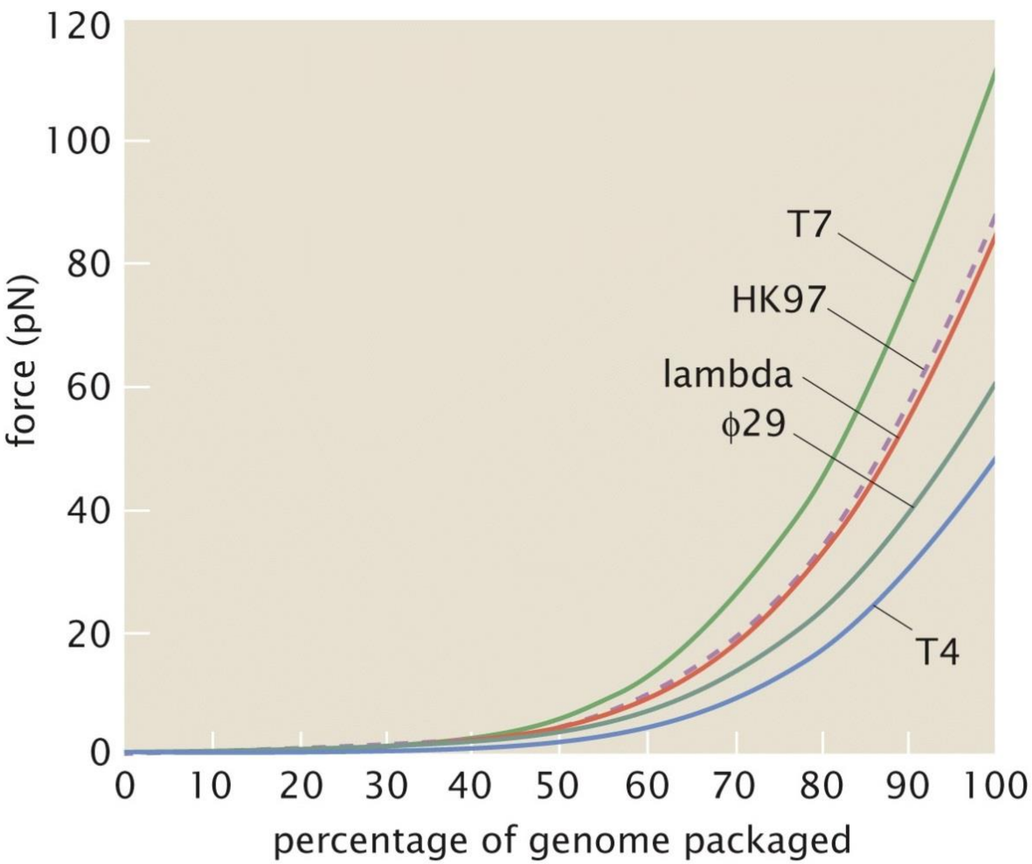


Figure 10.20 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

N.B. DNA has a –ve charge → strands want to repel, so besides bending energy there's also repulsive electrostatic energy that wants to keep the strands as separated as possible. It's a balancing act between minimizing the bending energy, and minimizing the repulsive energy.

Applications II: Forming a nucleosome

147 bp of DNA wrap around histone complex to form a nucleosome

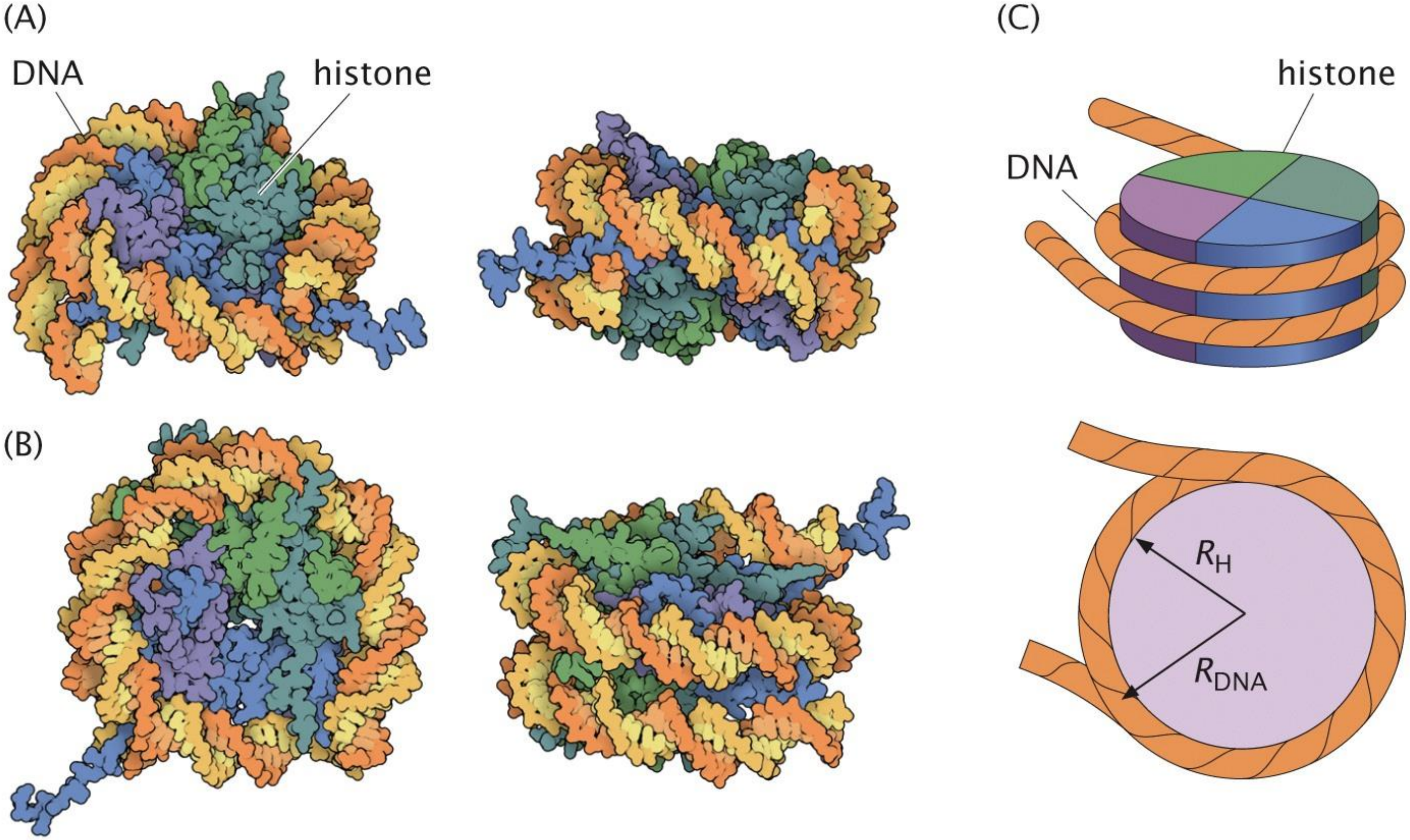
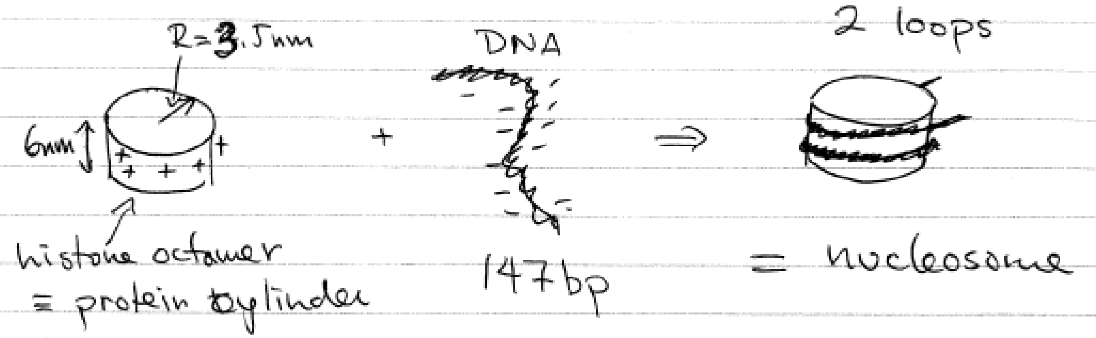


Figure 10.21 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

Recall, persistence length of DNA ~ 50 nm = 150 bp. So to form a nucleosome costs a lot of bending energy. Where does this energy come from?

Energy of formation for nucleosomes



- The free energy of nucleosome formation is

$$F_{tot} = F_{bend} + F_{charge}$$

Now,

$$F_{bend} = \underbrace{2}_{\text{2 loops}}\left(\pi \frac{\xi_p k_B T}{R}\right) \approx 70\, k_B T \quad \text{for } R = 4.5$$

- E_{charge} is an adhesive interaction.

$$F_{charge} = 2(\gamma L) = 2(2\pi R)\gamma$$

where $\gamma \equiv$ adhesive energy per length

and $\gamma < 0$

- So $G_{tot} = 70\, k_B T + 56\,\gamma$

Need $\gamma < -1.2\; k\,T$

in order to form nucleosome

Summary:

All biopolymers have some degree of internal rigidity

For small deformations they behave elastically

Bending energy depends on how curved the bend is

Applications:

looked at how to pack DNA in viruses
- is ther an optimal capsid shape?

looked at the energy of formation of nucleosomes – Electrostatic attraction betweeen DNA and histones overcomes the cost to bend the DNA

…. other applications, DNA loop formation in transcriptional regulation, cytoskeleton buckling, …