Topic 9: Fluids and Swimming Low Reynold’s Number

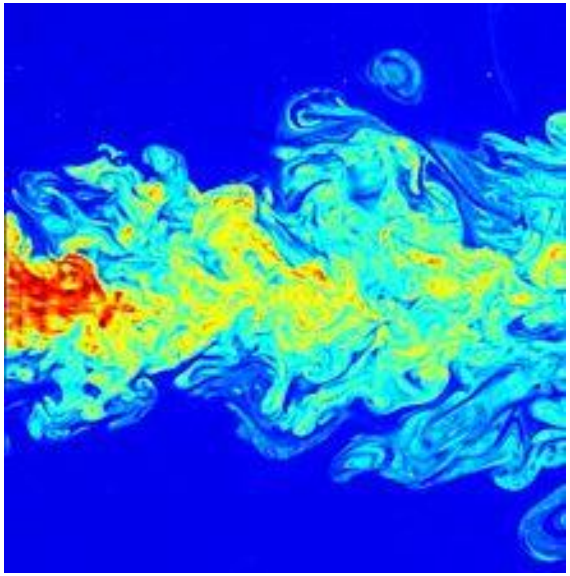
(Chapter 12 in book – and Random Walks in Biology by H. Berg)

Overview

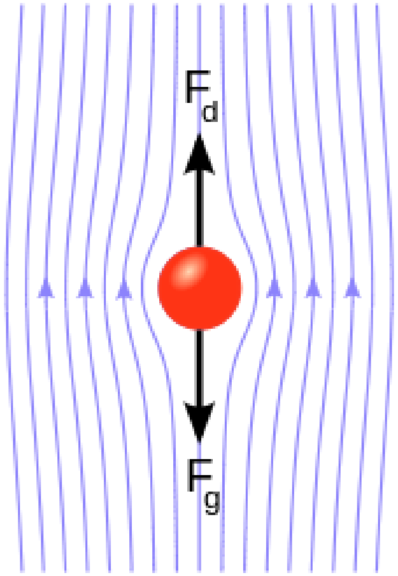
- What about non-diffusive transport in biology?
- Does water at the nano/micron scale behave the same way we experience it?
- What is viscosity?
- How do bacteria swim?

Turbulent or not?

Turbulent



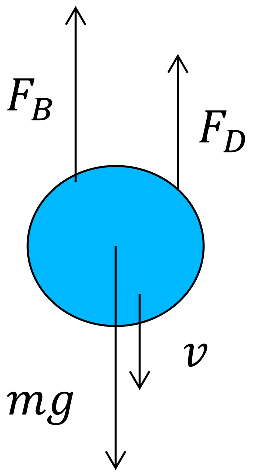
Laminar



What determines whether flow is turbulent or laminar?

A: it’s not only the ‘thickness/stickiness’ of the fluid, but also the particles size and speed. We’ll see that a bacteria swimming in water is like us trying to swim in honey.

Drag Force:



Consider a particle drifting down in
a fluid under the influence of gravity

Because of drag, the force eventually balance
and the particle attains a constant velocity

The three forces are:

$F_g = mg$ = weight, $F_B = m_w g$ = buoyant force

and for laminar fluids the drag force is,

$F_D = c\ v$ = drag force, where c is the 'drag coefficient'

@ equilibrium: F = 0, so

$$mg - m_w g - cv = 0$$

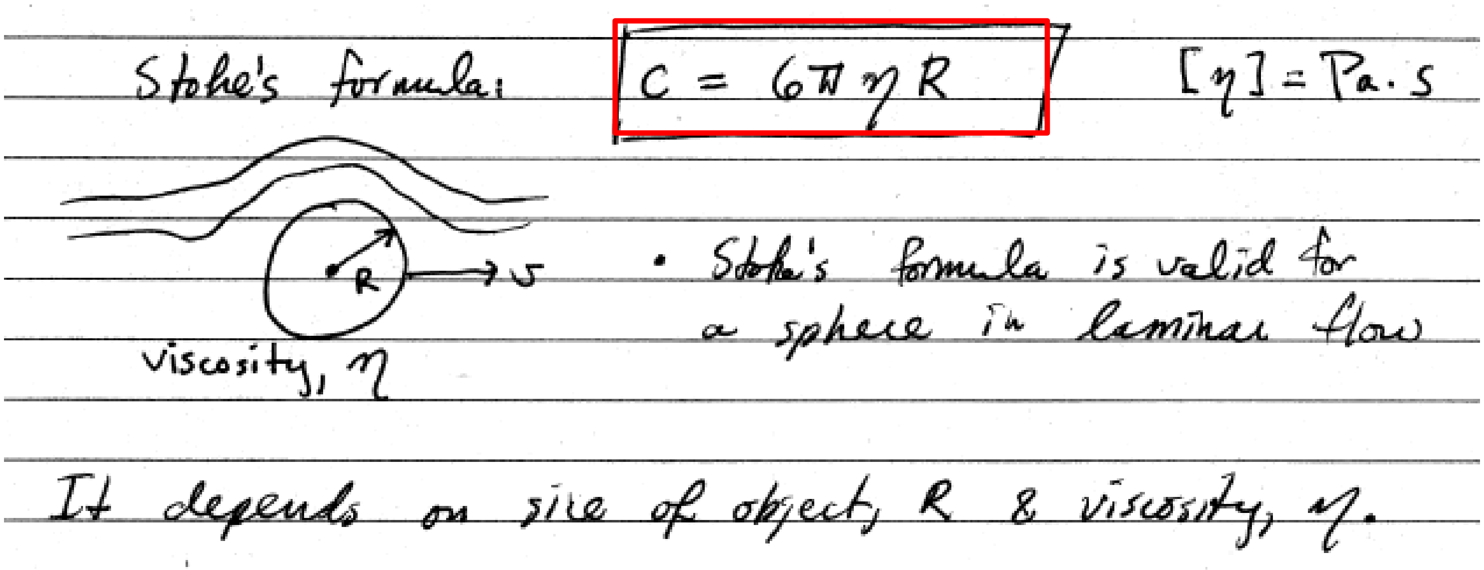
Drift velocity:

The particle achieves a constant drift velocity: $v = \frac{(m - m_w)}{c} g$

What is the nature of the drag coefficient? what does it depend on?

Clearly it depends on the particle size and on the properties of the liquid

For a spherical particle moving in a viscous fluid, we have a famous result:



Stoke's formula: $\boxed{C = 6\pi \eta R}$ $[\eta] = \text{Pa} \cdot \text{s}$

viscosity, η

- Stoke's formula is valid for a sphere in laminar flow

It depends on size of object, R & viscosity, η.

Drag on vesicles:

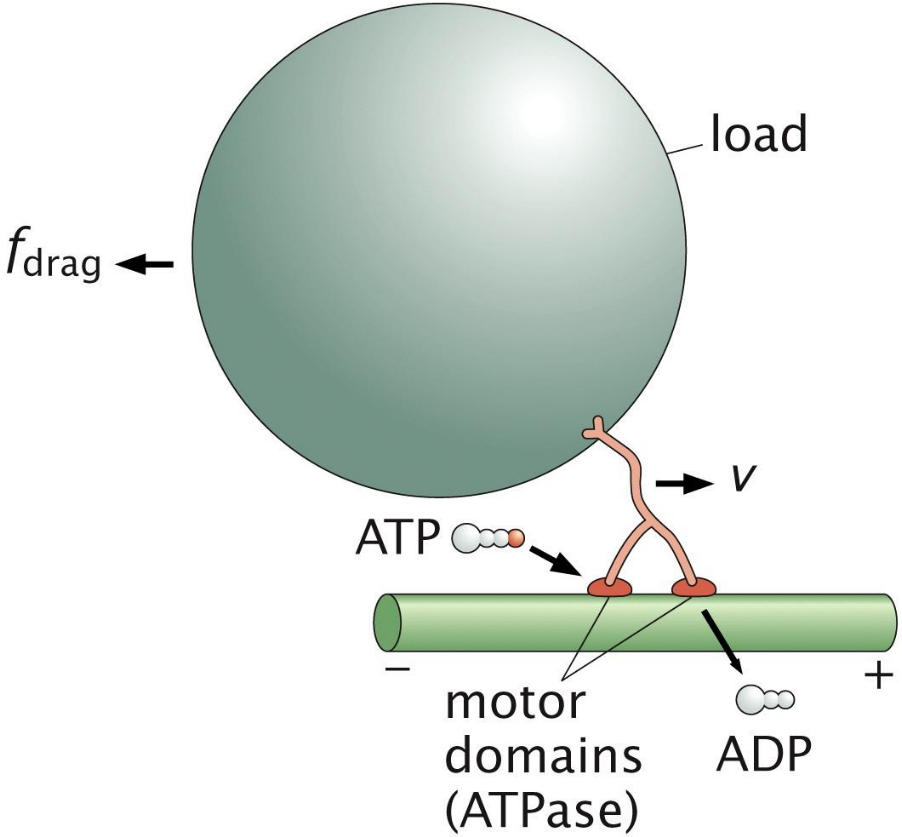
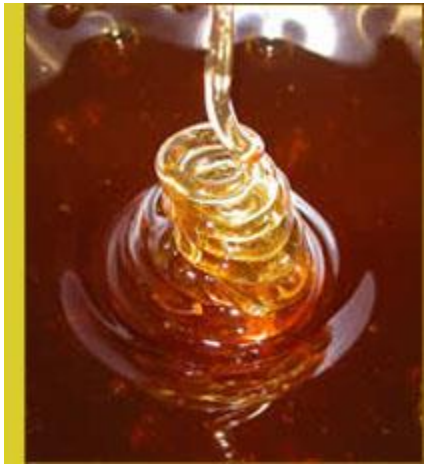


Figure 12.13 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

Vesicles experience a drag force

since flow is laminar, you can use Stoke’s equation to make a reasonable estimate of the drag force

Viscosity:



Viscosity is a measure of how sticky layers of fluid are to each other when a shear force is applied

We find it's hard to mix viscous fluids

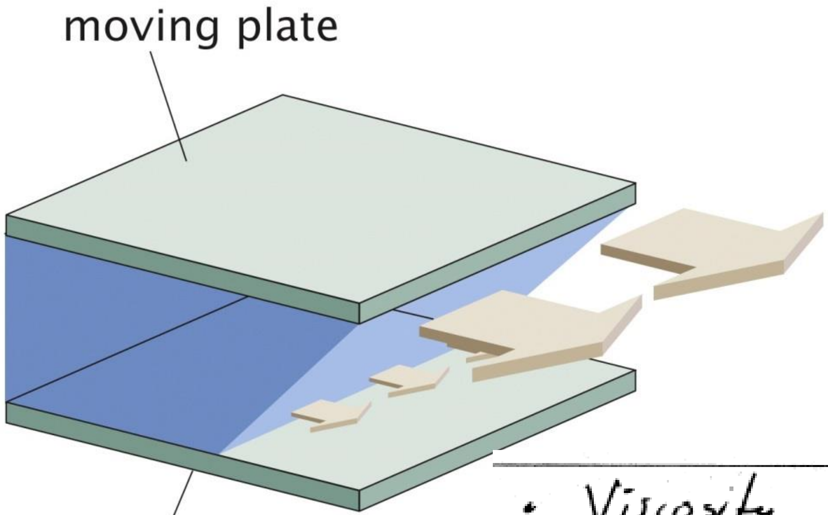
2 regimes: laminar flow and turbulent

laminar flow is reversible

turbulent flow is chaotic and not reversible

Experiment: mixing a dye in glycerin

Viscosity:



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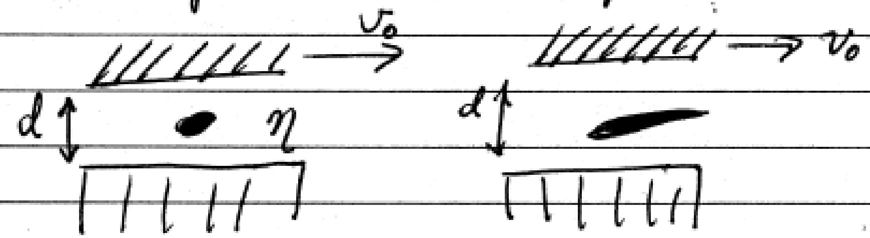
Consider the experiment on the left.

Q: how does the drag force of the moving plate depend on the parameters?

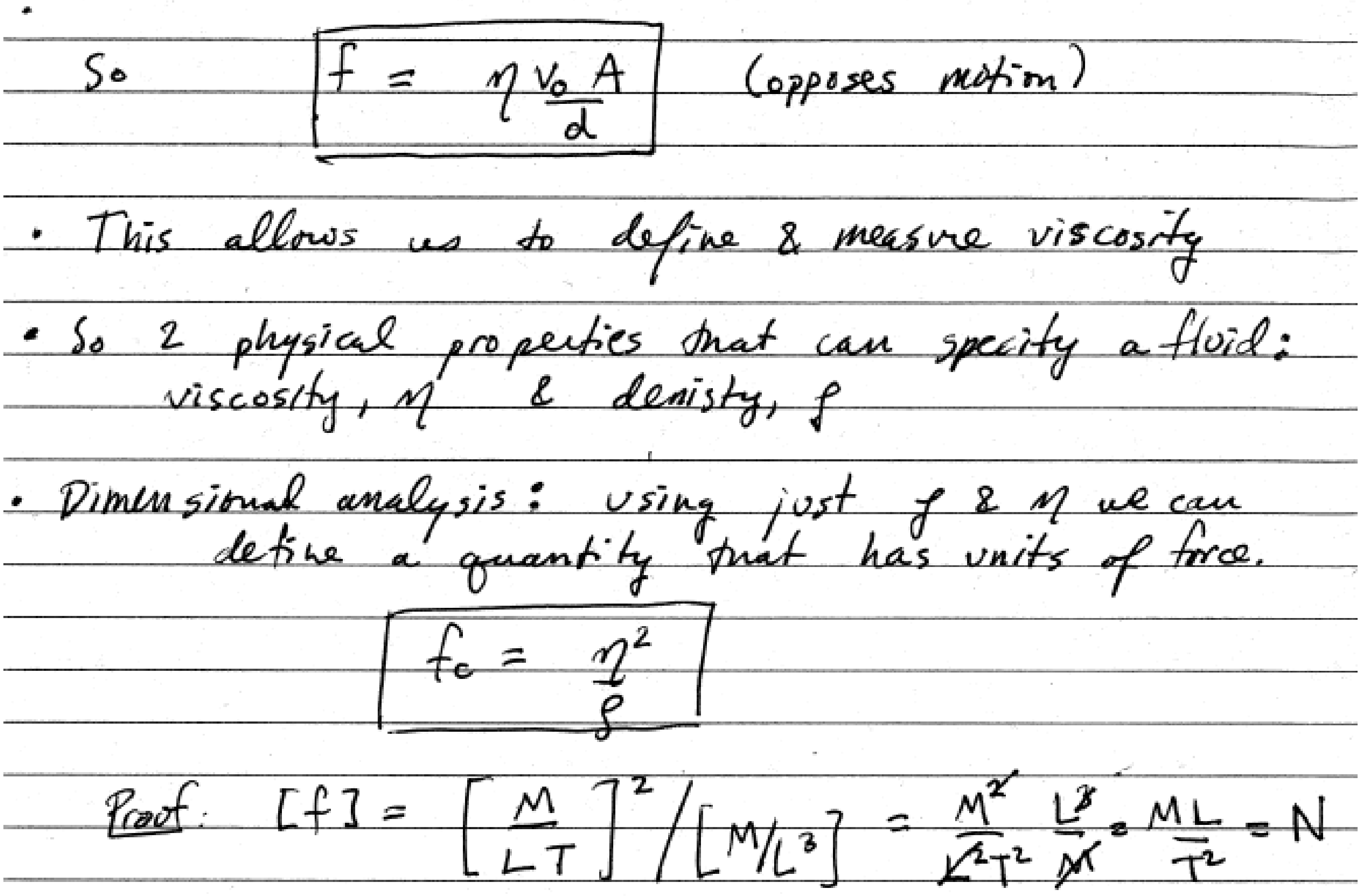
- Viscosity ≡ shear drag force



- Consider the drag force experienced by sliding a plate of area A @ speed v_0 over a liquid with viscosity η.
- What does the drag force, f depend on?

$f \propto A$; $f \propto v_0$; $f \propto \frac{1}{d}$ ← bigger d → less drag

Viscosity: Critical force



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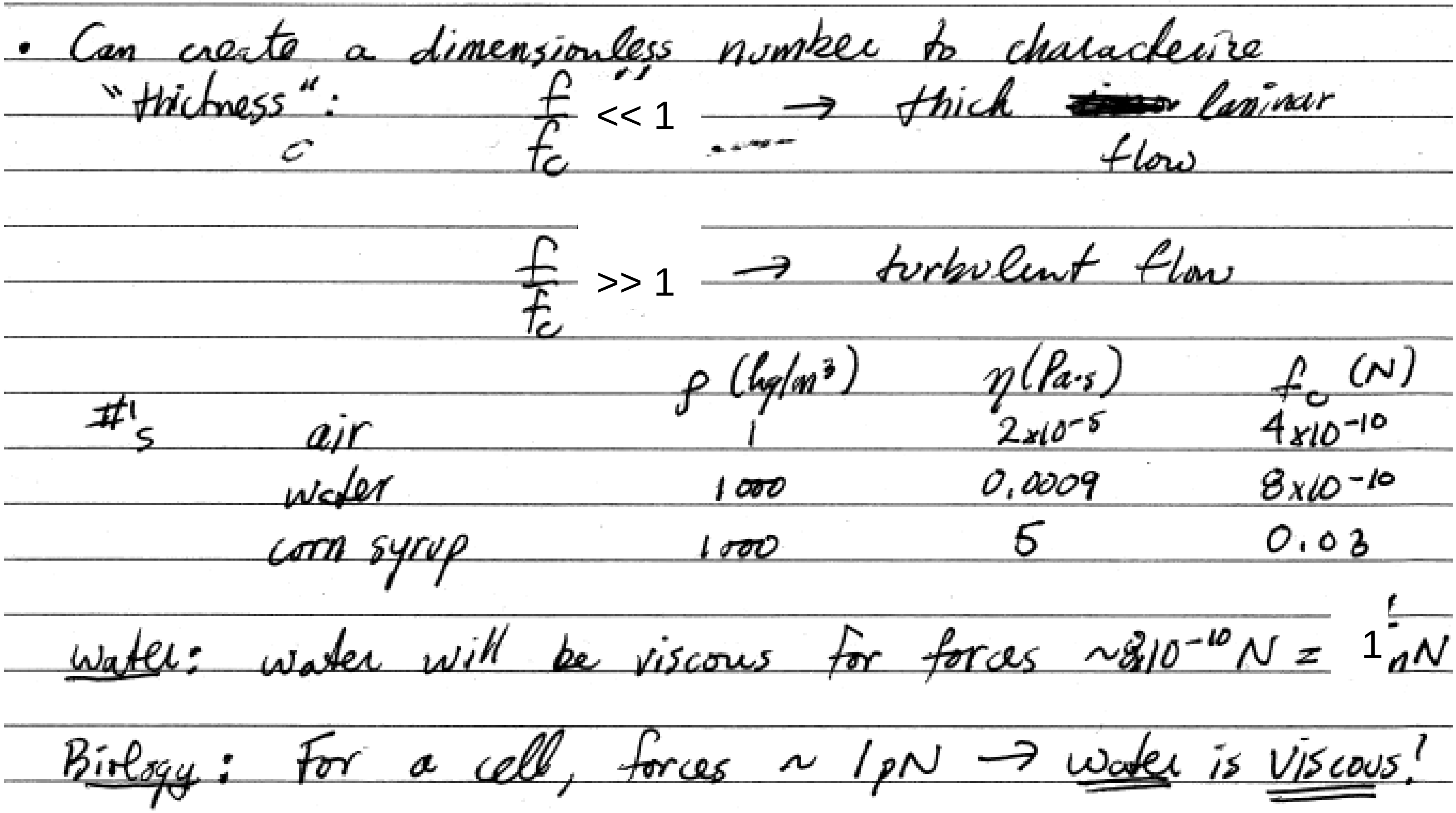
So $\boxed{f = \eta \frac{v_0 A}{d}}$ (opposes motion)

- This allows us to define & measure viscosity
- So 2 physical properties that can specity a fluid: viscosity, η & denisty, ρ
- Dimensional analysis: using just ρ & η we can define a quantity that has units of force.

$$\boxed{f_c = \frac{\eta^2}{\rho}}$$

Proof: $[f] = \left[\frac{M}{LT}\right]^2 \Big/ \left[M/L^3\right] = \frac{M^{\cancel{2}}}{\cancel{L^2}T^2}\frac{L^{\cancel{3}}}{\cancel{M}} = \frac{ML}{T^2} = N$

Viscosity: Critical force in biology



- Can create a dimensionless number to characterize "thickness":

$\frac{f}{f_c} << 1$ → thick laminar flow

$\frac{f}{f_c} >> 1$ → turbulent flow

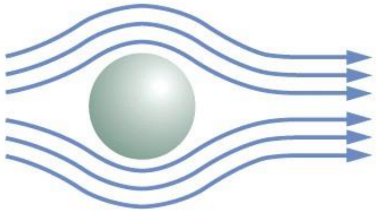
#'s	ρ (kg/m³)	η (Pa·s)	f_c (N)
air	1	2×10^{-5}	4×10^{-10}
water	1000	0.0009	8×10^{-10}
corn syrup	1000	5	0.03

<u>Water:</u> water will be viscous for forces $\sim 8\times10^{-10}$ N = 1 nN

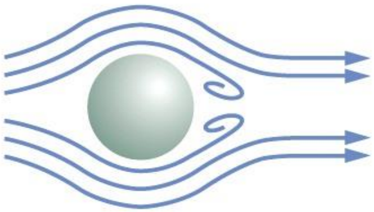
<u>Biology:</u> For a cell, forces ~ 1 pN → <u>water</u> is <u>viscous</u>!

so cells experience water in the laminar regime – clearly very different than how we experience H20

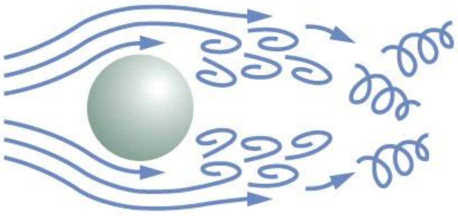
Reynold's Number:



Re < 10
Laminar flow



Re 10–40
Vortices form and are maintained



Re 40–20,000
Vortices form and are periodically shed

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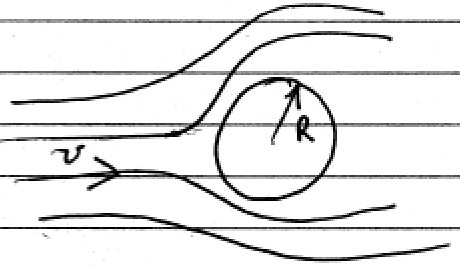
- Previous section showed that the character of a fluid depends on the size of the external force & changed @ $f_c = \eta^2/\rho$

- We can say more though. Consider an object in a fluid – will the flow be laminar or turbulent?



- The liquid experiences 2 forces

① inertial force due to colliding @ sphere
② viscous drag force

Idea: Ⓐ if inertial force > viscous force → turbulent

Ⓑ if viscous force > inertial force → laminar

$$\frac{\text{inertial force}}{\text{viscous force}} \equiv \boxed{\mathcal{R} = \frac{v R \rho}{\eta}}$$

≡ Reynold's number,

Some #'s:

$$\boxed{\mathcal{R} = \frac{v R \rho}{\eta}}$$

e.g.: Swimming whale: $R \sim 10\,m$ & $v \approx 10\,m/s$

so $$\mathcal{R} = \frac{(10\,m/s)(10\,m)(1000\,kg/m^3)}{(0.001\,Pa\cdot s)} = 10^8\,! \gg 1$$

→ turbulence

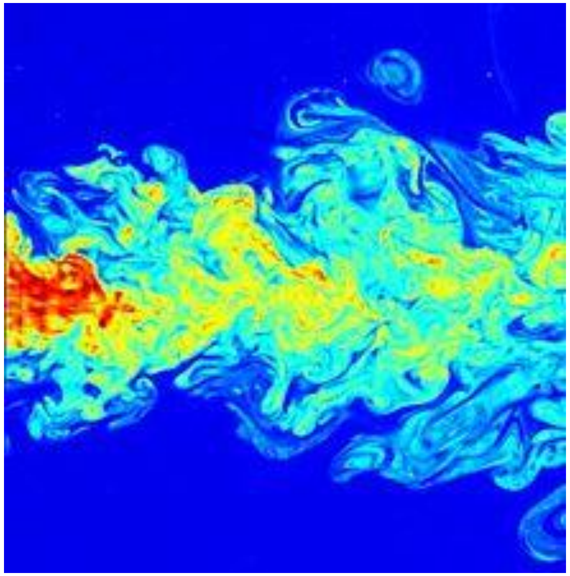
Swimming bacteria: $R \approx 10^{-6}\,m$ $\quad v \approx 30\times10^{-6}\,m/s$

$$\mathcal{R} = \frac{(30\times10^{-6})(10^{-6})(1000)}{(0.001)} \approx 3\times10^{-5} \ll 1$$

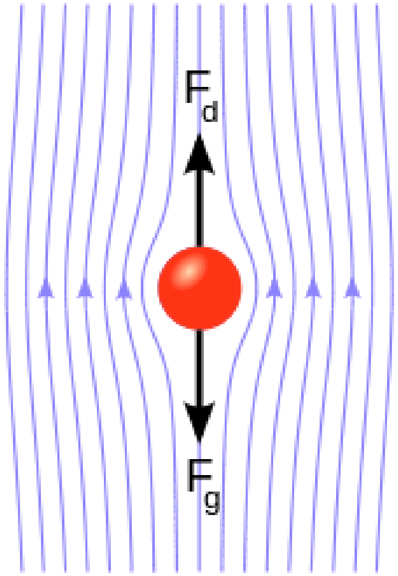
→ laminar & dominated by friction.

Can you swim in honey???

Turbulent



Laminar



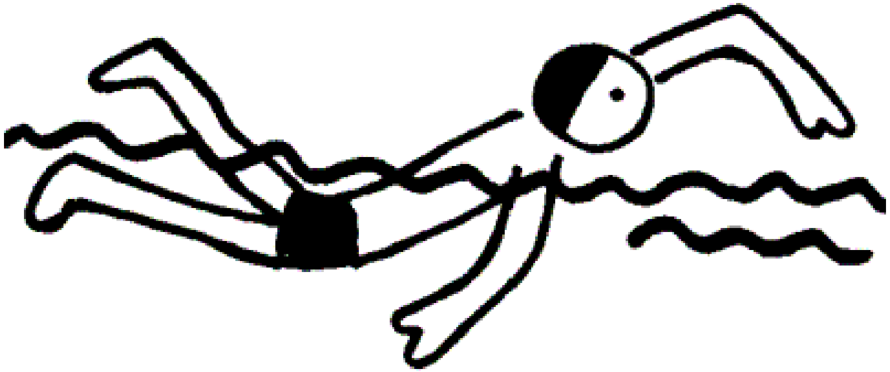
What determines whether flow is turbulent or laminar?

Reynold’s number = *Re* = (liquid density) x (size) x (speed) / (viscosity)

So it also depends on how big you are and how fast you are swimming through the liquid!

Re > 1000 = Turbulent *Re* < 100 = laminar

Swimming: turbulent or laminar?



Swimming in water:

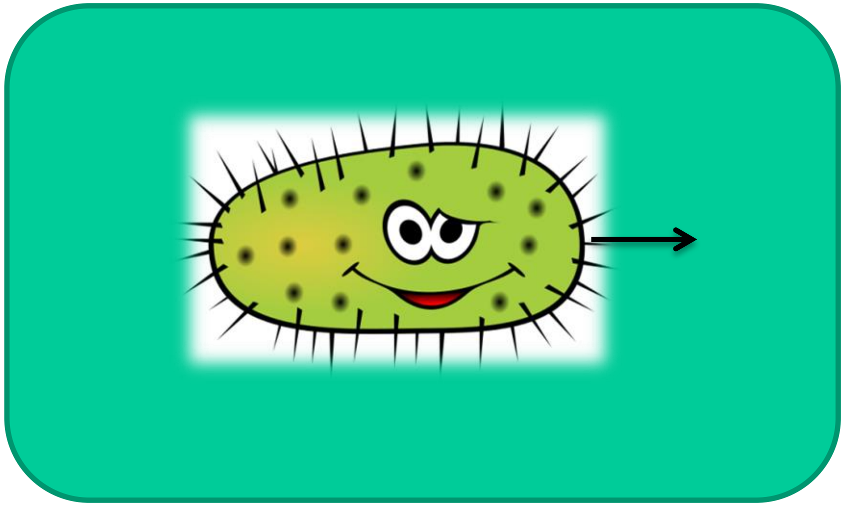
speed ~ 1 m /s ; size ~ 1 m

Re ~ 1×10^5 = turbulent

Swimming in honey:

Honey viscosity ~ 10000 x water

Re ~ 10 = laminar



Bacteria swimming in water:

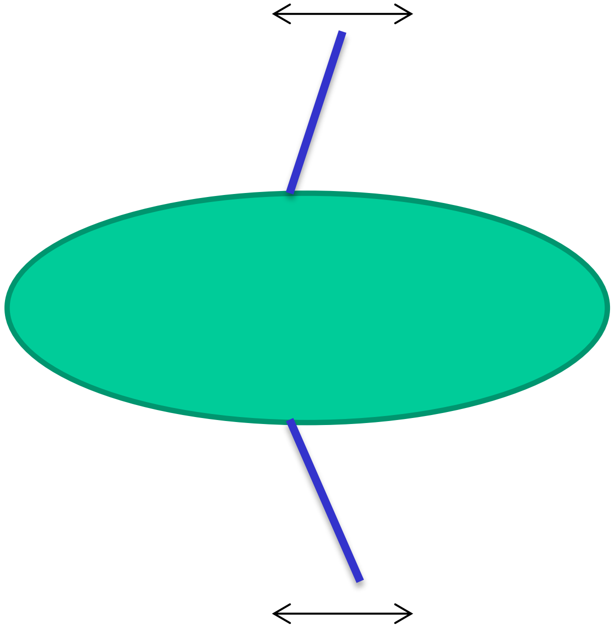
Speed ~ 30×10^{-6} m/s ; size ~ 1×10^{-6} m

Re ~ 1 x 10-5 = laminar

So bacteria swimming in water is like us in honey

Strategies for swimming I

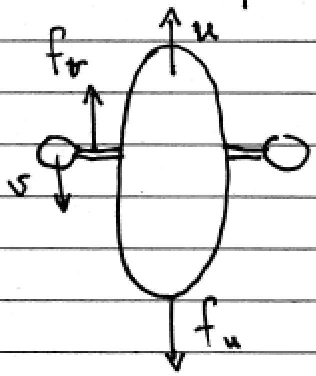
If you kick your feet in honey (as you would in water) will you move???



Symmetric, back-and-forth motion

Swimming strategies I: symmetric motion fails at low Re

- how do bacteria swim in such a frictional liquid when the flow is laminar?
- Consider pushing with paddle motions



  - move paddles relative to body with speed v
  - liquid pushes on paddle with force $f_v = c_v(u-v)$
  - drag on bacteria $f_u = c_u u$
- Since moving with constant velocity $\Rightarrow$ $f_v = f_u$

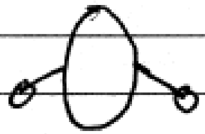
$$\Rightarrow \quad u = \frac{c_1 v}{(c_1 + c_2)}$$

- distance moved: $\Delta x = u \cdot \Delta t$

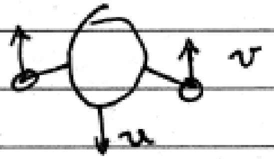
Swimming strategies I: symmetric motion fails at low Re

- However, need to return paddles on back stroke along the same path. Thus the bacteria ends up pushing back

after st



back stroke



Since motion is reciprocal → $\Delta x' = -\Delta x$ & the bacteria doesn't go anywhere!

To move/swim, motion can not be reciprocal.

Proof: Consider a backstroke with the paddle moving with v' for a time dt', so the backward velocity of the bacteria is $u' = c_1 v'/(c_1+c_2)$, but $v' dt' = v\,dt$, since the paddles have to return to the same spot.

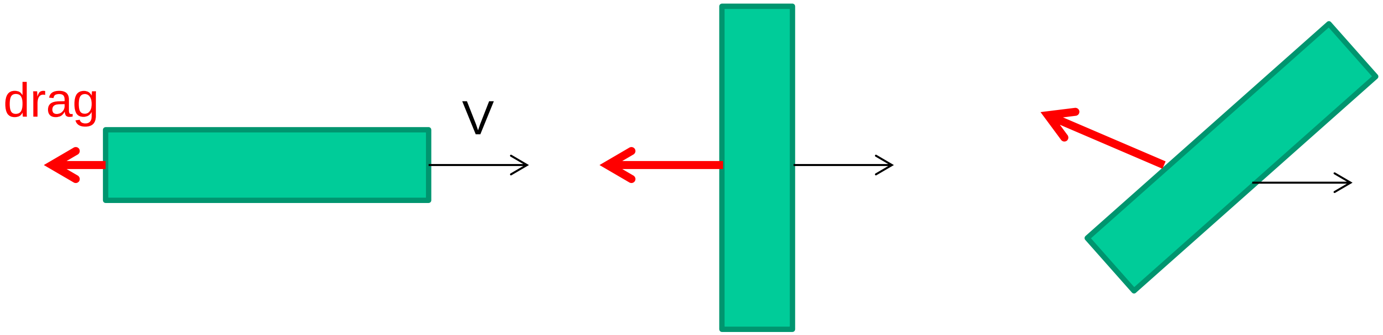
So the distance travelled is $dx' = -u'\,dt = -c_1 v\,dt/(c_1+c_2) = -u\,dt = -dx$

hence $dx' = -dx$ and you don't go anywhere

Strategies for swimming II

So performing symmetric swimming motion will get you no where in honey.

How to swim in honey??? Need to perform asymmetric motion.

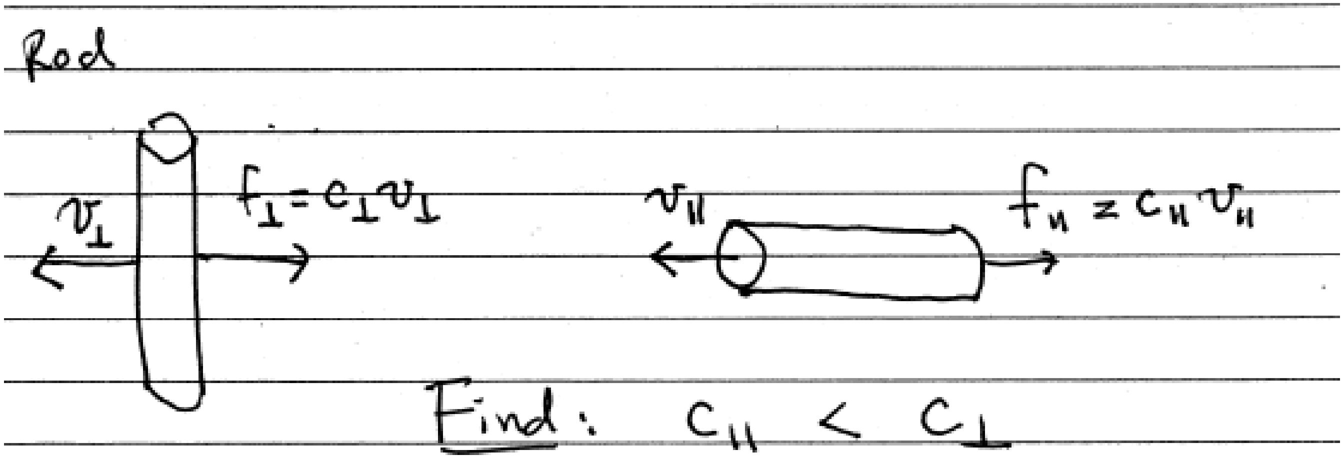


Many bacteria swim by using a helical propeller that exploits asymmetry in drag forces

Key: viscous drag force depends on shape.

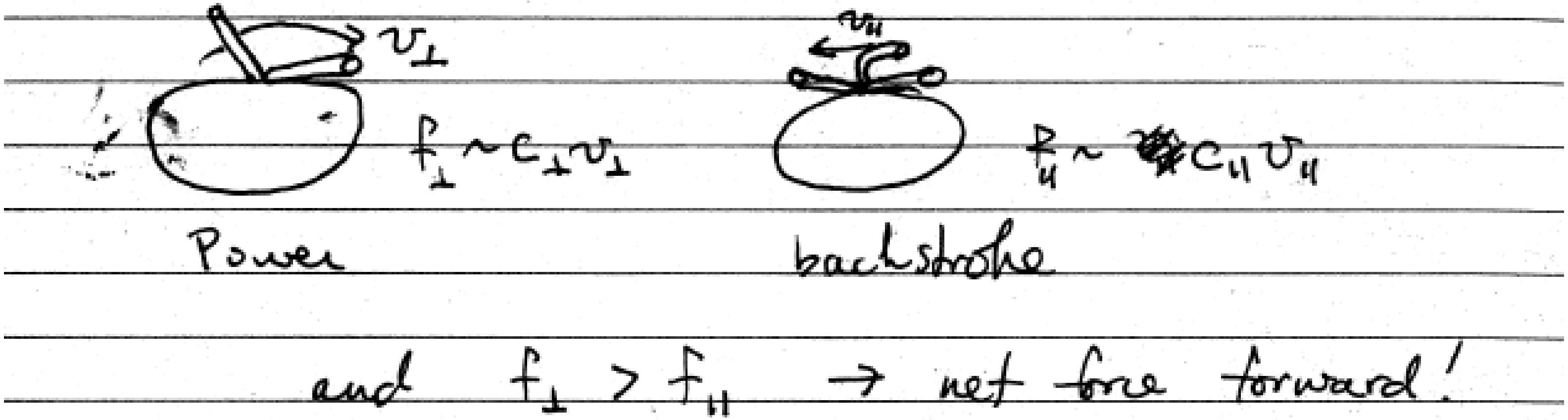
See Life at Low Reynold's Number, Berg and Purcell

Drag and the shape of the paddle:



Find: $c_\parallel < c_\perp$

Bacteria with cilia:

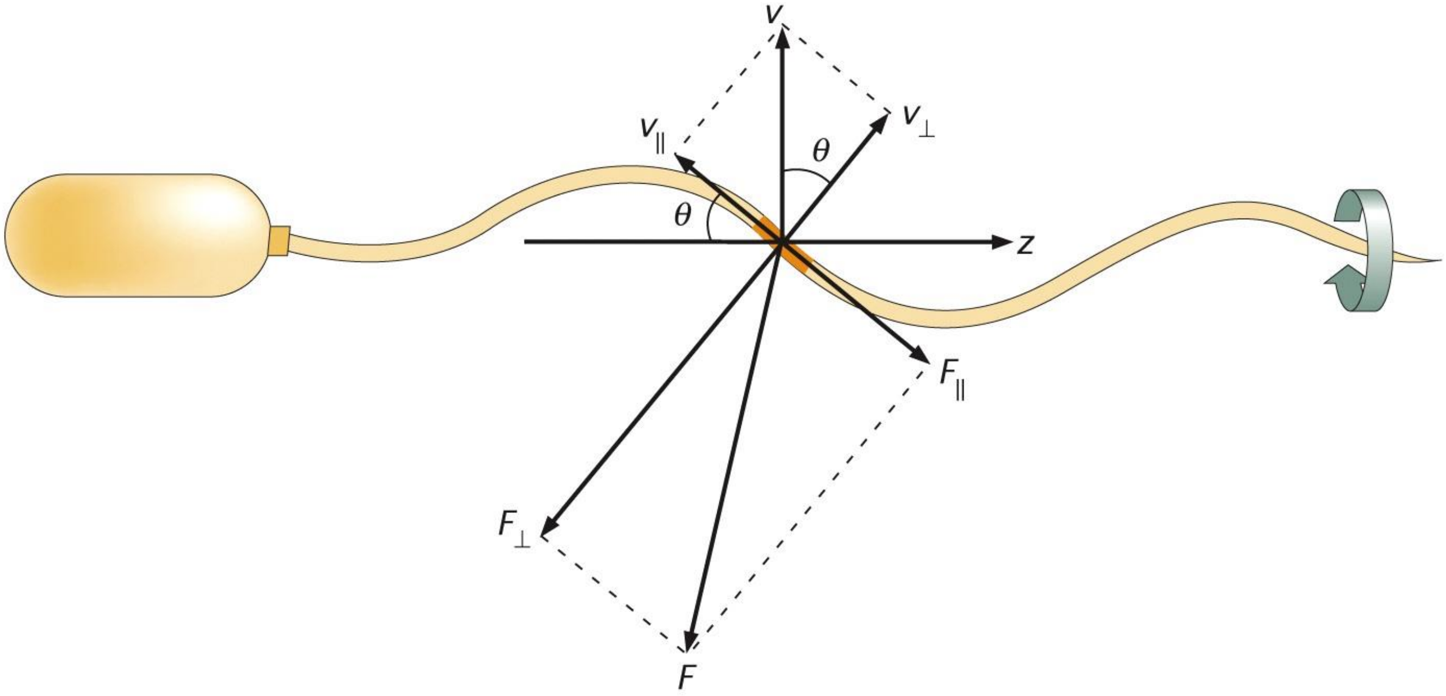


and $f_\perp > f_\parallel$ → net force forward!

E. coli swimming

E. coli have a helical flagellum.

Q: How does a helix allow it to swim at low Reynold’s number?



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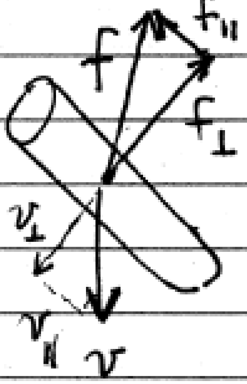
Forces on rotating helix:

First consider the drag on a rod which is being pulled

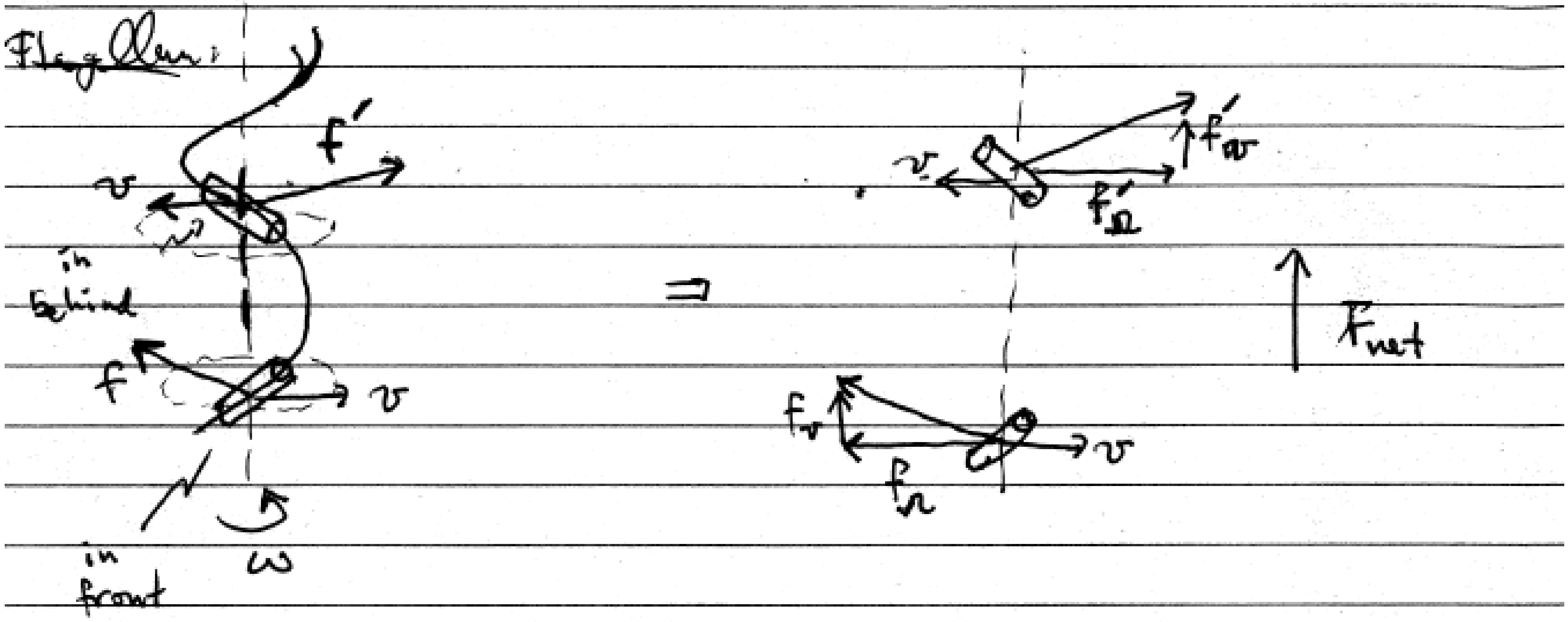


• Because $\frac{f_\perp}{f_\parallel} = \frac{c_\perp v_\perp}{c_\parallel v_\parallel} \neq \frac{v_\perp}{v_\parallel}$

Key: so $\vec{f}$ is not in the same direction as $\vec{v}$!

What implication does this have for a helical flagellum?

Forces on rotating helix:



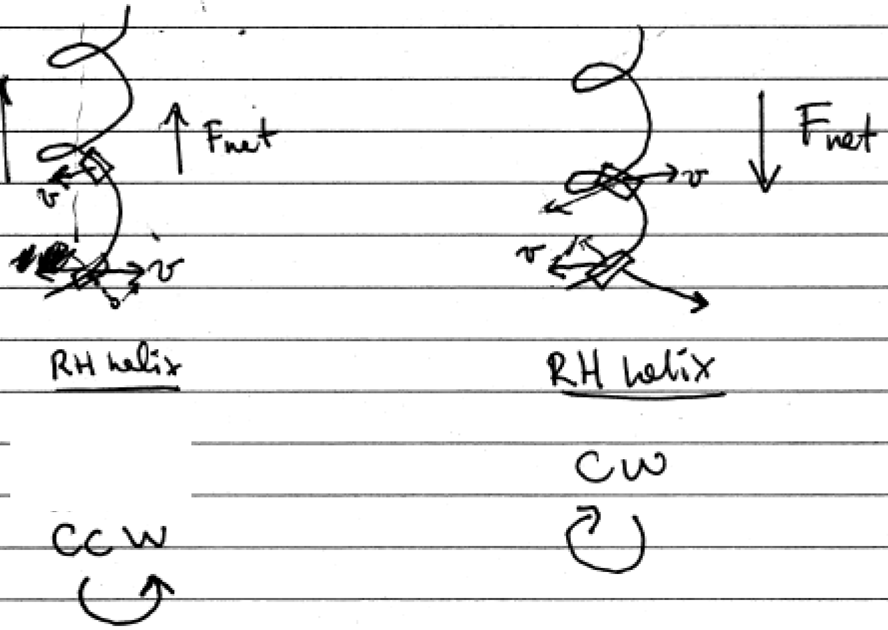
- Forces: ⊥ to axis: $\vec{f}_\Omega = -\vec{f}_{\Omega}'$ so forces cancel

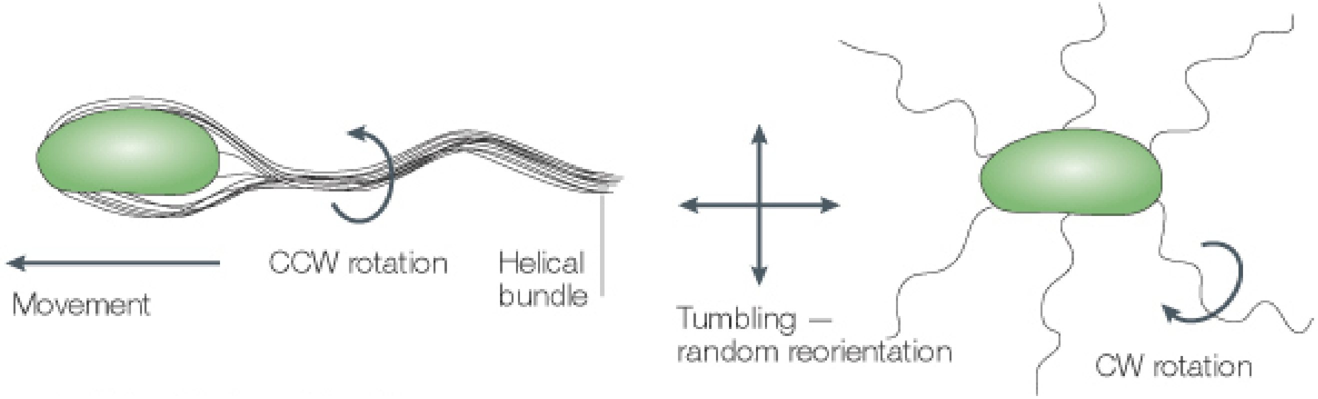
  ∥ to axis: $\vec{f}_v = \vec{f}_v'$ so forces ADD!

∴ there is a NET force ∥ to helix axis.

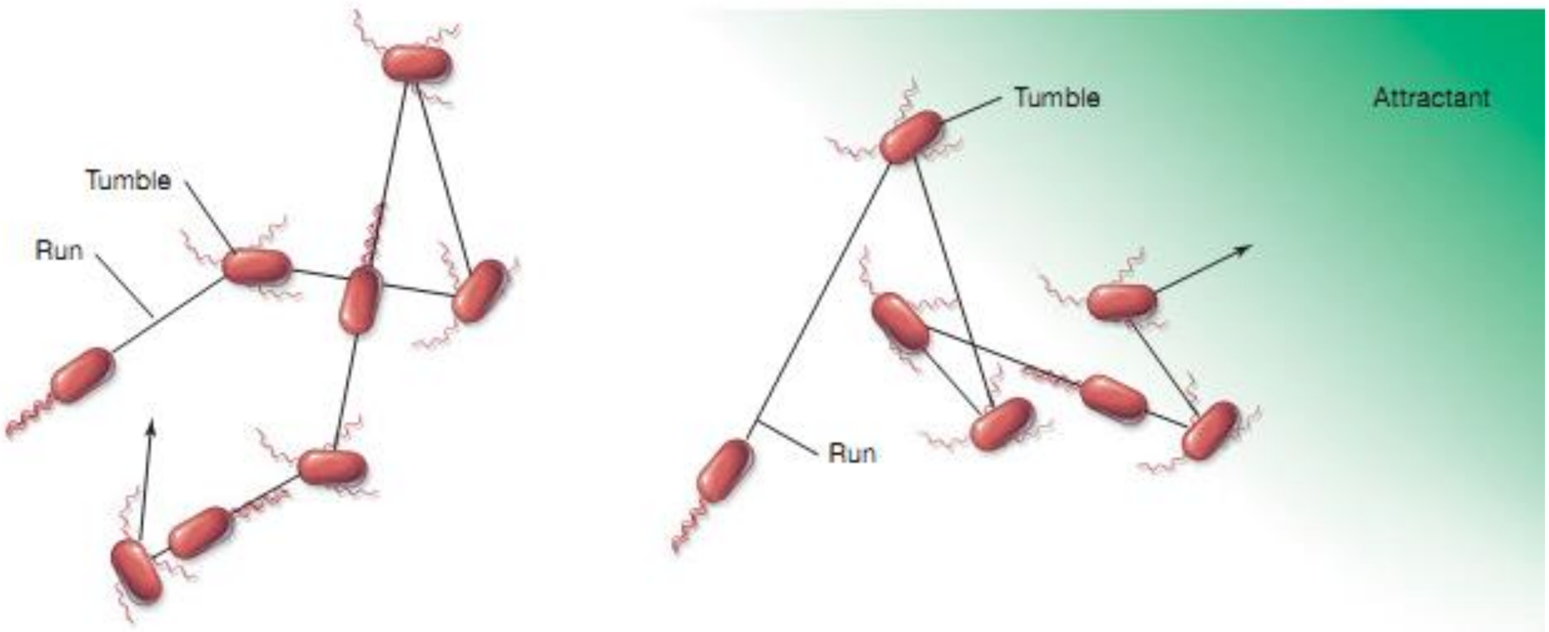
Force on bacterium:

- This net force propels bacteria





Chemotaxis:



E. coli switch the frequency of CCW (run) vs CW (tumble) rotation of their flagella based on their sensing of food in the environment

= a biased random walk (i.e. they step more frequently in the direction of the food)

Q: how can something the size of a micron where diffusing mixes things ~ milliseconds measure a spatial gradient?

A: they store a memory of the signal and take a derivative = spatial gradient using time

Summary:

Things moving in a fluid experience a drag force

This drag force depends on whether flow is turbulent of laminar

Viscosity describes a fluid’s resistance to shearing it’s layers

The Reynold’s number dictates the type of flow and depends on viscosity, the fluid’s density and the size and motion of the particle

Cells experience life at low Reynold’s number

They have to use asymmetric motion in order to swim

Looked at how a helix can generate the necessary asymmetry to generate a forward force in a low Reynold’s number environment