Mathematics of Sums:

Notation:

$$x_1 + x_2 + x_3 + \cdots + x_N = \sum_{i=1}^{N} x_i$$

← dummy index

Adding or subtracting:

$$\sum_i x_i + \sum_j x_j = 2\sum_i x_i$$

$$(x_1 + x_2 + x_3 + \ldots) + (x_1 + x_2 + \ldots)$$

$$\sum_i x_i^2 - \sum_k x_k = \sum_i (x_i^2 - x_i)$$

Sum of constants:

$$\sum_{i=1}^{N} c = (c + c + \ldots) = Nc$$

c ↑ a constant

Product of sums:

$$\left(\sum_i x_i\right)\left(\sum_j x_j\right) = \sum_i \sum_j x_i x_j$$

IT'S NOT $\sum_i x_i^2$

$$(x_1 + x_2 + x_3 + \ldots) \times (x_1 + x_2 + \ldots) = (x_1^2 + x_1 x_2 + x_1 x_3 + \ldots x_2^2 + x_2 x_1 + \ldots)$$

$$= \sum_i x_i^2 + \sum_{i \neq j} x_i x_j$$

↑ sum over i & j with j not equal to i