Employing Fractals and FEM for Detailed Variation Analysis of Non-rigid Assemblies

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Abstract

Many studies on non-rigid assemblies, or assemblies of non-rigid components, suggest that the component variation affects the assembly dimensional quality. However, little is known about how the variation of surface micro-geometry of assembly components influences the assembly dimensional quality. In this paper, a new method based on the fractal geometry and finite element method (FEM) is proposed to study such an influence. In the new method, a special fractal function, named the Weierstrass-Mandelbrot (W-M) function, is used to extract and represent the characteristics of the variation of surface micro-geometry of assembly components. FEM is applied to analyze the deformation of non-rigid assemblies by integrating the variation of component micro-geometry. The sensitivity matrix between the component variation and assembly variation is obtained by using the existing influence coefficients method. It is found that contributions of the variation of surface micro-geometry of assembly components to the final variation of non-rigid assemblies could be substantial under certain conditions. The proposed method is illustrated through a case study on an assembly of two flat sheet metal components under different fixture releasing conditions.

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1. Introduction

Dimensional quality is one of the most important issues in the assembly of non-rigid components, which is widely seen in aerospace and automobile industries such as the assembly of auto bodies and airfoils. A lot of factors in the assembly process, such as the component variation, tool variation, fixture layout, and assembly sequence, have impact on the assembly dimension variation [1, 2]. For example, an auto body is often composed of hundreds of non-rigid sheet metal panel parts. All types of variation accumulate and propagate along with the assembly process [3]. Such accumulated variations would affect the final quality of the auto body. Unsatisfactory dimensional quality decreases product performance, increases warranty costs, and creates many problems, such as rework, rejects, and engineering changes. It is thus an important and interesting task to predict the dimensional variations of a final assembly during the design and process planning stage [1~3].

Currently, the variation analysis of non-rigid assemblies has attracted many researchers [1~9]. Liu and Hu [4] considered the compliant nature of sheet metal parts and proposed an influence coefficients method to analyze the effect of component variation and assembly spring-back on the assembly variation by applying linear mechanics and statistics. The influence coefficients method was a key technique to get the component stiffness matrix. Camelio et al. [5] successfully extended this approach to model the product variation in multi-station assembly systems. Hu [3] set up the “stream of variation” theory for the automotive body assembly variation analysis. Ceglarek and Shi [6] proposed a new variation analysis methodology for the sheet metal assembly based on physical / functional modeling of the fabricated error using a beam-based model. Hu et al. [7] developed a numerical simulation method for the assembly process incorporating compliant non-ideal components. The effects of various variation sources were analyzed. In addition, Heieh and Oh [8] represented a procedure for simulating the combined effects of deformation and dimensional variation in the elastic assembly. Cai et al. [9] discussed the fixture schemes and demonstrated that the N-2-1 fixture scheme was better than the 3-2-1 scheme for non-rigid assemblies.

In general, the component variation is recognized as a major problem in elastic assembly processes. A number of methods and tools have been developed to simulate the assembly processes and to analyze the assembly variation. However, little is known about how the variation of surface micro-geometry of assembly components affects the assembly dimensional quality.

In this paper, a novel method is proposed to investigate the influence of the variation of component surface micro-geometry on the assembly dimensional variation by applying the finite element method and fractal geometry. A fractal function, named Weierstrass-Mandelbrot (W-M) function [10~14], is used to extract and represent the characteristics of the variation of surface micro-geometry of assembly components. The W-M function is then used as an input for the finite element analysis to calculate the deformation of the final assembly [4, 8]. The contribution of the variation of surface micro-geometry of assembly components to the final assembly deformation is obtained by the influence coefficients method. The proposed method is implemented by using ANSYS [16, 17] and Matlab [18, 19], and is illustrated through a case study on the assembly of two flat sheet metal components.

In the next section, details of the assembly process modeling for non-rigid components will be discussed first. Then, Section 3 introduces the fractal geometry, and the modeling of component variations of surface micro-geometry by the W-M function. The systematic simulation flowchart and a case study on an assembly of two flat sheet metal components will be given in Section 4. Finally, conclusions are given in Section 5.

2. Non-rigid assembly process modeling

In order to analyze the non-rigid assembly variation in a typical assembly station, it is necessary to model the "real" complex assembly process. One of the most widely used approaches to model an assembly process is the mechanistic simulation methodology developed by Liu and Hu [4]. This methodology is based on the following assumptions on the assembly procedure [3, 4, 5, 7]:

1) all of the process operations occur simultaneously;

2) the component deformation is linear and elastic;

3) the component material is isotropic;

4) fixtures and tools are rigid;

5) no or negligible thermal deformation occurs during the assembly process; and

6) the stiffness matrix remains constant for deformed component shapes.

The assembly processes of components and subassemblies in a typical assembly station can be illustrated by Figure 1, and represented by the following steps [4, 5, 7]:

i) Placing components (Fig.1a)

Components are loaded and placed on work-holding fixtures using a locating scheme (Fig.1a). Since the fabrication error of components is a natural phenomenon in component manufacturing, the component variation $\{\delta_u\}$ offset from the design nominal will inevitably cause the initial matching gap. Here, index *u* refers to un-joined components. Cai et al. (1996) suggest that it is better to use the N-2-1 (N>3) fixture scheme than the 3-2-1 scheme for non-rigid assembly to assure the assembly quality because of the assembly deformation. That means, constraining N(>3) DOF (degree of freedom) in the first plane, 2 DOF in the second plane, and 1 DOF in the third plane.

(Insert here: Fig. 1 The non-rigid assembly process)

ii) Clamping components (Fig.1b)

The initial matching gap between components and subassemblies is forced to close by deforming components to the nominal position. Considering the component stiffness matrix $[K_u]$ that could be built through the finite element method, the relationship of the required clamping forces $\{F_u\}$ to the closed gap $\{\delta_u\}$ can be given by Eq.(1)

$$\{F_u\} = [K_u]\ \{\delta_u\} \qquad (1)$$

iii) Joining components (Fig.1c)

When using a joining method, such as spot welding, riveting, or gluing, to join two components, deformation occurs at each joint point as the gap between components is closed. The assembly force $\{F_u\}$ is still being applied.

iv) Releasing clamps/fixtures and subassembly spring-back (Fig.1d)

After assembling the two components, the clamps/fixtures are removed. The joined components will spring back to release the stored strain energy during the assembly operation. It is reasonable to assume that the spring-back force $\{F_w\}$ is equal to the clamping force $\{F_u\}$. Therefore, applying FEM to get the component and assembly stiffness matrix, the value of spring-back variation $\{\delta_w\}$ can be calculated by removing displacement boundaries both at clamping points and the releasing fixture locations to simulate clamps/fixtures release, as described in the following Eqs. (2)~(5):

$$\{F_w\} = [K_w]\ \{\delta_w\} \qquad (2)$$

$$\{F_w\} = \{F_u\} \qquad (3)$$

$$\{\delta_w\} = [K_w]^{-1}\ [K_u]\ \{\delta_u\} \qquad (4)$$

$$\{\delta_w\} = \{S_{uw}\}\{\delta_u\} \qquad (5)$$

Where, $\{S_{uw}\}$ is the sensitivity matrix. Index *u* represents the input source of variation and *w* the output measurement points. $\{S_{uw}\}$ represents the linear mapping relationship between the assembly variation and the component variation.

For a given specific assembly process and station, getting the stiffness matrix $[K_u]$ and $[K_w]$ by using commercial FEM software is the key issue to the assembly variation analysis procedure, because most software provides no direct means for users to access and operate the FEM stiffness matrix. The influence coefficients method, which is developed by

Liu and Hu [4], could be used to indirectly construct the sensitivity matrix $\{S_{uw}\}$ if the commercial FEM software embeds an application-oriented development language. In fact, this method uses FEM to compute the stiffness matrix $[K_u]$ and $[K_w]$, and obtains the sensitivity matrix $\{S_{uw}\}$ by Eq. $\{S_{uw}\}= [K_w]^{-1} [K_u]$. The procedure to achieve the stiffness matrix of assembly and/or component can be described as follows: a unit force is applied at each source of variation with the same direction of the deviation; FEM is then used to calculate the response at some specific points; after such response computation for all sources of variation, a response matrix can be constructed; the stiffness matrix can be obtained by inverting the response matrix since it is symmetric. Details about the influence coefficients method are in the reference [4].

3. Component variation modeling using fractals

3.1 Introduction of fractal geometry

It was the Polish mathematician Benoit B. Mandelbrot who first introduced the term 'fractal' (from the latin *fractus*, meaning 'broken') in 1975 to characterize spatial or temporal phenomena that are continuous but not differentiable[13]. Unlike more familiar Euclidean constructs, splitting a fractal into smaller pieces shall result in the resolution of more structures [14, 15]. Self-similarity is the property that fractal objects and processes inherit [14].

Fractal properties include scale independence, self-similarity, complexity, and infinite length / detail. It is well known that fractal structures do not have a single length scale, while a single time scale cannot characterize fractal processes (time series).

Nonetheless, the necessary and sufficient conditions for an object (or process) to possess fractal properties have not been formally defined [15].

Fractal theory provides methods to describe the inherent irregularity of natural objects [14, 15]. In fractal analysis, a constant parameter D, known as the fractal (or fractional) dimension, is treated as a relative measure of complexity, or as an index of the scale-dependency of a pattern. Excellent summaries of basic concepts of fractal geometry can be found in references [14, 15].

The fractal dimension is a statistical overall 'complexity' measurement. A mathematical fractal is formally defined as any series for which the Hausdorff dimension (a continuous function) exceeds the discrete topological dimension [14]. Currently there are several kinds of methods, such as box counting, pair counting, and power spectrum method to compute the fractal dimension for a given data set [15]. Topologically, a line is one-dimensional, that is D=1; the fractal dimension of a plane is D=2; and the dimension of a fractal curve is $1 < D < 2$, shown in Fig. 2.

(Insert here: Fig. 2 Fractal dimension of typical geometry entities)

Nowadays, fractal geometry has been widely applied to study the non-linearity and complexity of physical, chemical, biological, and/or engineering systems. For example, the property of “seashore” can be modeled using fractals. On the other hand, some complex patterns can be constructed by using iterative procedures. Fig. 3 shows one example of the process for the construction of the Koch Curve [15].

Currently, the fractal Brownian motion and fractal Weierstrass-Mandelbrot (W-M) function are used extensively in engineering application because their simple forms are easily understandable [10~12, 14, 15]. In the next sub-section, the Weierstrass-Mandelbrot (W-M) function and its application in the component variation modeling will be discussed.

(Insert here: Fig.3 An example of the Koch Curve iterated twice [15]. (a) A line of unit length. (b) The line increases in length by 4/3. (c) The length is again increased by 4/3, so it is now 16/9 of the initial unit length)

3.2 Component variation micro-geometry modeling using W-M function

It is inevitable that any manufactured component has fabrication variations due to uncertainties in manufacturing systems [10, 12]. The maximum and minimum of deviation should be identified under strict measurement and control so that the final product can satisfy the design requirements. Recent studies show that not only the amount of manufacturing variations but also the variation's micro-geometry influences a component's friction [10, 11]. In this paper, we will model the variation of surface micro-geometry of non-rigid components by using the fractal function W-M function in order to numerically analyze the effect of the variation of component surface micro-geometry on the final assembly dimensional quality.

The variation of surface micro-geometry of assembly components is very complex. Experiments show that most engineering surfaces / profiles appear to be irregular, and the portion of surfaces / profiles looks similar to the whole as it is amplified [10~12]. Even on a very small scale, the surfaces / profiles are obviously irregular. Self-affinity and self-similarity are the main characteristics of the topography of most engineering surfaces / profiles [10]. Therefore, such topography characteristics of a component profile can be used to analyze the variation of surface micro-geometry of assembly components.

The Weierstrass-Mandelbrot (W-M) function is often applied to study those profiles that appear to have self- affinity and self-similarity. The W-M function can be written as Eq.(6) [11, 13]

$$X(t) = G^{(D-1)} \sum_{n=n_1}^{\infty} \frac{\cos 2\pi r^n t}{r^{(2-D)n}} \tag{6}$$

Where

D: fractal dimension of the profile

G: scaling constant,

r^n: frequency modes, which correspond to the reciprocal of the wavelength λ

$$r^n = 1/\lambda^n \tag{7}$$

n_1: corresponds to the low cut-off frequency of the profile under measurement

$$r^{n1} = 1/L \quad (\ L\text{: profile length}) \tag{8}$$

r : =1.5 (it is suitable and practicable for general fractal cases [10~12])

The power spectrum density of the W-M function is very useful for the computation of the parameters D and G., and it can be statistically represented as:

$$S(\omega) = \frac{G^{2(D-1)}}{2\log r} \frac{1}{\omega^{(5-2D)}} \tag{9}$$

Eq.(9) indicates that the W-M function power spectrum density follows the power law, namely the linear relation between log(S) and log(ω) in a double logarithm co-ordination. Since most engineering profiles are fractal, the fractal dimension D and scaling constant G are determined by the power spectrum, and parameters D and G are independent of frequency ω, that means they are scale-independent. This is a typical characteristic of engineering fractal profiles.

When given the measured data of variation for a profile, the power spectrum density analysis can be applied, and then the logarithmic transformation can be made. On the log-log power law plot, the average slope (k) and y-intercept S_y are obtained though linear regression algorithms. The fractal dimension D and scaling constant G are most commonly estimated from Eqs.(10)~((11):

$$D = \frac{5-k}{2} \tag{10}$$

$$G = e^{\frac{S_y + \log(2\log r)}{2(D-1)}} \tag{11}$$

The fractal dimension D reflects the degree of variation complexity of the component surface micro-geometry. The W-M function can be used easily to analyze the degree of fractal complexity of the component variation, and to synthesize the component variations. The procedure is illustrated in Fig. 4. The synthesized component variation, since it is represented by the W-M function, can be easily applied for further analysis of the assembly variation.

(Inset here: Fig.4 Procedure of component variation modeling by W-M function)

From the viewpoint of manufacturing, different fractal dimension D corresponds to different manufacturing conditions. For example, a grinding profile generally has a smaller fractal dimension D than a milling profile [10, 12], while as we know, in general the quality of a grinding profile is better than that of a milling profile. Therefore, it is possible to make a good manufacturing plan by analyzing the variation of surface micro-geometry of assembly components.

4. Assembly variation simulation procedure and case studies

4.1 Assembly variation simulation procedure

Based on the four steps of the assembly process of components and subassemblies in a typical assembly station (shown in Fig.1) and the method on the component variation modeling by using the W-M function, the assembly variation simulation flowchart is summarized in Fig.5.

The entire analysis procedure shown in Fig.5 consists mainly of two portions. One is the variation of surface micro-geometry of assembly components by using the W-M function; the other is the four-step assembly process simulation based on the finite element analysis method.

In fact, the W-M function statistically represents the component variation, and it can be one of the displacement boundaries in FEM; thus, the deformation due to component variation can be computed through Eq. (5) derived in Section 2.

(Insert here: Fig.5 Flowchart of the assembly variation simulation procedure)

Generally, the FEM model can be created by "map-mesh" with structural elements so that the jointed spots are definitely together. The minimum clamping force is dependent on the material property and dimensions of the components. Since focus is on the variation of surface micro-geometry of assembly components and its contribution to the final product dimension variation, it is apparent that the more flexible the component material and the smaller the component dimensional size, the more prominent the influence will be. Therefore, it is important to study the assembly variation for high flexible assemblies and / or mini-machines (for example, MEMS systems).

The component joining process is simulated through coupled nodes in the FEM model, while the tool releasing process is simulated by removing the displacement boundaries at the released clamp / fixture points. The whole assembly process is assumed to be non-frictional and linear.

For non-rigid assembly, it is often needed to determine a set of points on components that should be critical points (CPs) to assure the assembly dimensional quality [5~8]. The characteristics of the CPs usually significantly affect the target value of the controlled variation, the performance of component function, and customer satisfaction. However, it is difficult to decide on the locations of CPs. The determination of CPs relies on such factors as the component shape, assembly process, component or subassembly performance, and assembly variation requirements [7].

The proposed assembly variation simulation procedure shown in Fig. 5 provides a method to analyze the variation of surface micro-geometry of assembly components and its influence on the product assembly variation. It can be implemented by using the software ANSYS and Matlab. ANSYS is used to generate the FEM model, to compute component deformation and the clamping force, to simulate the joining and releasing process, to calculate the spring back, and to get the assembly variation; while the Matlab can be applied to develop the program for the component variation analysis and synthesis procedures. It is very efficient and fast to obtain the variation of surface micro-geometry of assembly components by using the W-M function with Matlab.

4.2 Case study: assembly of two flat sheet metal components

An assembly of two identical flat sheet metal components by lap joints is selected as an example to verify the proposed approach. Assuming that these two components are manufactured under the same conditions, their fabrication variations are expected to be the same. The task then is to find the variation at each point in the assembly that corresponds to the variation of surface micro-geometry of assembly components.

1) Component geometry and material

The size of the flat sheet metal components used in this case study is 100×100×1mm, Young's modulus $E = 2.62e+9$ N/mm^2 and Poison's ratio $\nu = 0.3$.

2) Fixture and joining scheme

Due to the flexibility of the sheet metal components, the N-2-1 ($N > 3$) style of fixture [9] is adopted for each component in this example (shown in Fig.6). The positions of symbol ‘Δ’ indicate the fixture locations. All pair joint spots (indicated by symbol ‘x’) are simultaneously assembled together.

(Insert here: Fig.6 Assembly of two flat sheet metal components)

3) Component variation modeling

A variation signal from the component profile (shown in Fig. 7) is sampled by using a Coordinate Measurement Machine (CMM). During the measurement process, 200 points along the profile line AB showed in Fig. 6 are selected to be CMM measurement points. For the measured data of the component variation, the mean variation is computed first, and then the detailed variation is modeled by using the W-M function. The mean variation is found to be 0.5 mm. The log-log power spectrum density of the detailed variation is obtained in Fig.8, and the fractal parameters computed from Fig.8 are given in Table 1. The variation synthesized by using the W-M function is shown in Fig. 9. The analysis and synthesis programs are developed using Matlab.

(Insert here: Fig.7 The sampled component variation)

(Insert here: Fig.8 The log-log power spectrum density of detailed variation)

(Insert here: Table 1 Parameters in W-M function)

(Insert here: Fig. 9 The variation reconstructed by W-M function)

4) FEM modeling

The finite element computation model of the assembly of two flat sheets, shown in Fig.10, is created in ANSYS by assuming that the small elastic deformation does not significantly change the component geometry size. The element type is SHELL63. The number of elements and the number of nodes are 128 and 162, respectively. There are 9 pairs of nodes to be connected together in this model, corresponding to the 'x' symbols in Figure 10.

(Insert here: Fig.10 The FEM model for analyzing the assembly of two flat sheet metal components)

5) Computational results

After the FEM model for simulating the assembly process and the variation of surface micro-geometry of assembly components are obtained, the assembly variation that results from the detailed component variation can be computed by using Eq. (5) derived in Section 2. In this example, corresponding to the mean variation and the detailed variation in the component profile, the assembly variation distribution (Fig.11) is obtained under three different tool-releasing schemes respectively (see Table 2). The computational procedure is coded by APDL (ANSYS Parametric Design Language) in ANSYS.

(Insert here: Table 2 Tool releasing schemes)

(Insert here: Fig.11 Assembly variation corresponding to component variations and tool releasing schemes)

From Fig.11 a1)~ b3) we can see that the component variation propagation heavily relies on the assembly process. Different tool releasing schemes result in quite different assembly variation distributions. The complete fixture releasing scheme (Scheme 3 in Table 2) generates much larger assembly variation than the partially fixture releasing scheme (Scheme 2 in Table 2). Therefore, it is necessary to design the assembly process that meets the product dimensional tolerance. In addition, the assembly variation caused by the detailed component variation is considerable, which is also unsymmetrical even if the assembly condition is symmetrical. It is because the variation of surface micro-geometry of assembly components is complex and not symmetric, demonstrating fractal characteristics.

We can determine some CPs in components to check the influence of component variation on the assembly dimensional quality. In this example, we suppose that there are 3 CPs (shown in Fig.10). The assembly variations of these 3 CPs under 3 different tool-releasing schemes are extracted from computation results (see Fig.11), and are shown in Fig.12. It can be seen from Fig.12 that both assembly variations caused by the mean and the detailed component variation increase as more fixtures are released. The contribution of the variation of surface micro-geometry of assembly components to the final assembly variation is significant for Scheme 3. Thus, the incorporation of the analysis of micro-geometry of component variation can give a more accurate prediction of the final assembly quality.

(Insert here: Fig.12 Assembly variation of 3 CPs. (a) Assembly variation due to the mean component variation. (b) Assembly variation due to the detailed component variation)

5. Summary and conclusion

Non-rigid assembly is quite different from rigid assembly. Due to the deformation that occurs in the assembly process, the product dimensional quality will be affected by many factors. One of these factors is the component variation. In this paper, fractal geometry is applied to model the variation of surface micro-geometry of assembly components. The influence of the variation of surface micro-geometry of assembly components on the final assembly variation is then studied. It is found that different tool releasing schemes will produce quite different assembly variation distributions. With more fixtures released, the contribution of the variation of component surface micro-geometry to the final assembly variation is getting more significant. Moreover, the final assembly variation could be asymmetrical even under a fairly symmetric assembly condition, if the variation of surface micro-geometry of assembly components is taken into consideration. Therefore, the assembly variation caused from the variation of surface micro-geometry of assembly components should not be neglected in an assembly process plan for high precision assemblies. Given the developed method, quality of non-rigid assemblies can be more accurately determined.

Since the proposed methodology and related tools particularly focus on the investigation on the variation of surface micro-geometry of components and its influence on final product dimensional quality, it is more applicable for the assembly variation analysis of mini-machines that have compliant components. Furthermore, because different manufacturing process plans will result in different component variation patterns, the proposed approach can be integrated with process plan methods to optimize the manufacturing process plans to meet the product quality requirements.

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Tables:

Table 1 Parameters in W-M function

D (fractal dimension)	*G* (scaling constant)	*r* (constant)	n_1 (the lowest cut-off frequency mode)	*L* (sample length)
1.55	1.67e-8	1.5	-11.36	100 mm

Table 2 Tool releasing schemes

	Scheme 1	Scheme 2	Scheme 3
	Releasing all clamps	Releasing clamps + partial fixtures (A, C and D, see Fig.10) on part1	Releasing clamps + all fixtures (A, B, C and D, see Fig.10) on part1
Assembly variation due to mean component variation	Assembly variation distribution shown in Fig.11 a1)	Assembly variation distribution shown in Fig.11 a2)	Assembly variation distribution shown in Fig.11 a3)
Assembly variation due to detailed component variation	Assembly variation distribution shown in Fig.11 b1)	Assembly variation distribution shown in Fig.11 b2)	Assembly variation distribution shown in Fig.11 b3)

Legends for the figures:

Fig.1 The non-rigid assembly process

Fig.2 Fractal dimension of typical geometry entities

Fig.3 An example of the Koch Curve iterated twice [15]. (a) A line of unit length. (b) The line increases in length by 4/3. (c) The length is again increased by 4/3, so it is now 16/9 of the initial unit length

Fig.4 Procedure of component variation modeling by W-M function

Fig.5 Flowchart of the assembly variation simulation procedure

Fig.6 Assembly of two flat sheet metal components

Fig.7 The sampled component variation

Fig.8 The log-log power spectrum density of detailed variation

Fig. 9 The variation reconstructed by W-M function

Fig.10 The FEM model for analyzing the assembly of two flat sheet metal components

Fig.11 Assembly variation corresponding to component variations and tool releasing schemes

Fig.12 Assembly variation of 3 CPs. (a) Assembly variation due to the mean component variation. (b) Assembly variation due to the detailed component variation

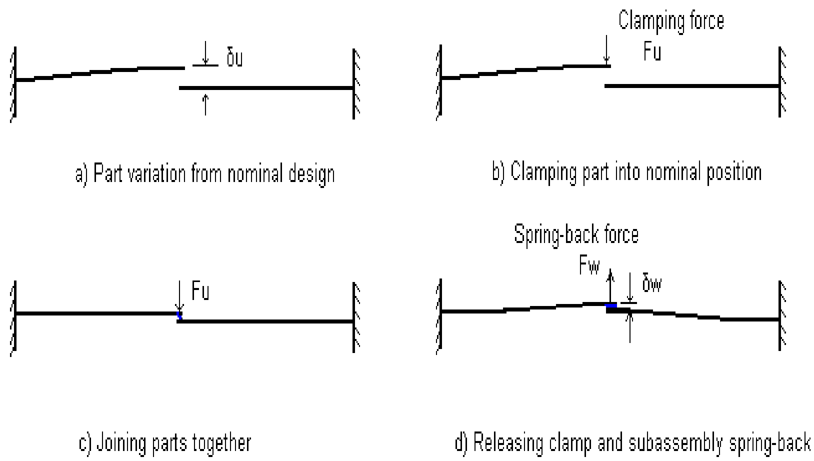


Fig. 1 The non-rigid assembly process

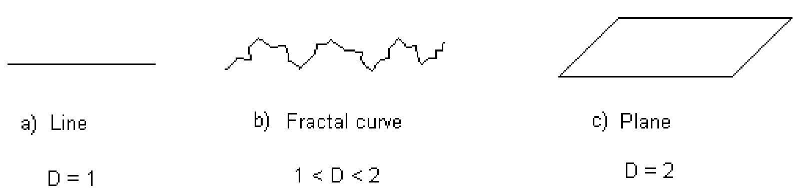
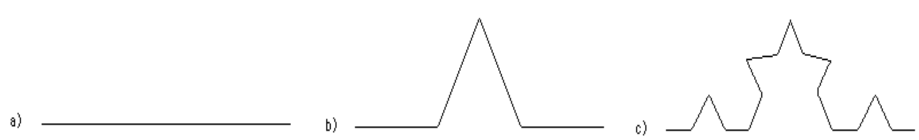


Fig. 2 Fractal dimension of typical geometry entities



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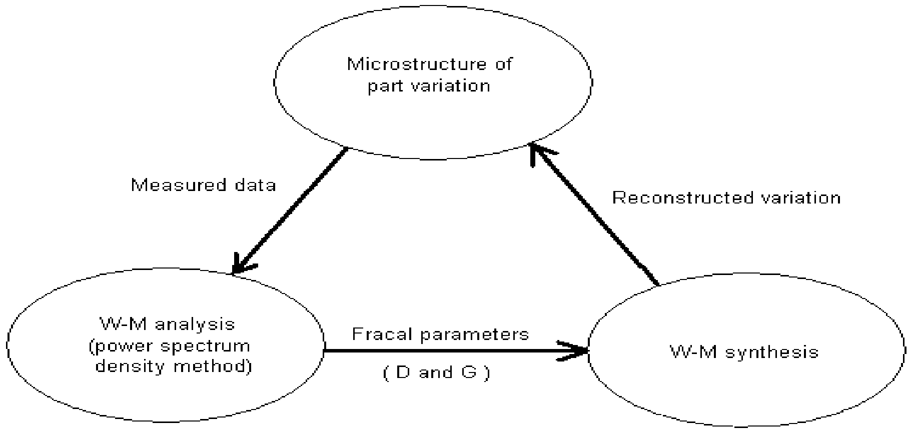

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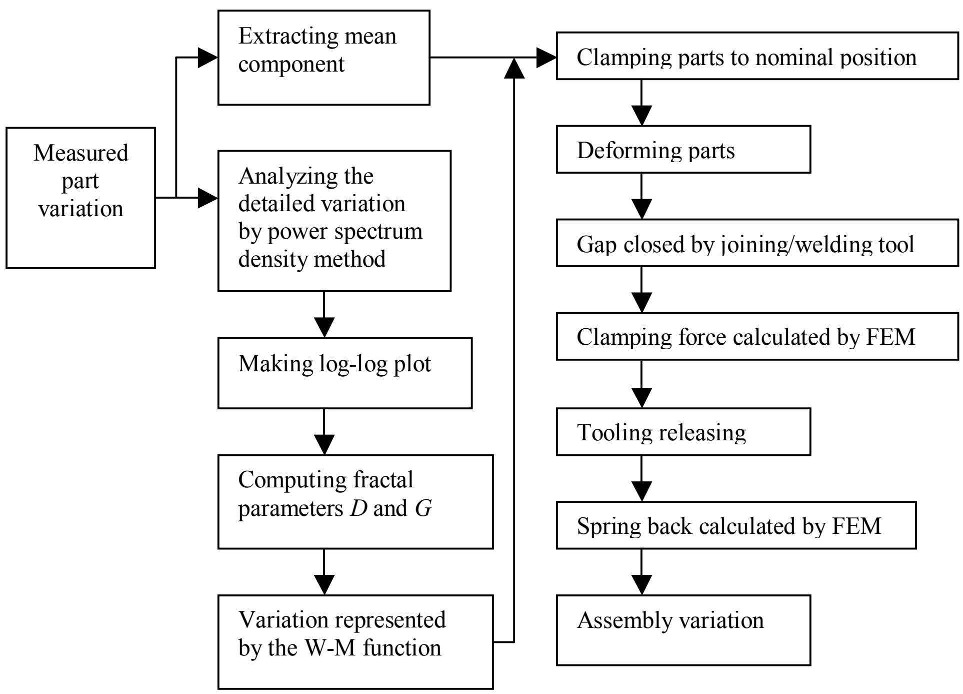


Fig.5 Flowchart of the assembly variation simulation procedure

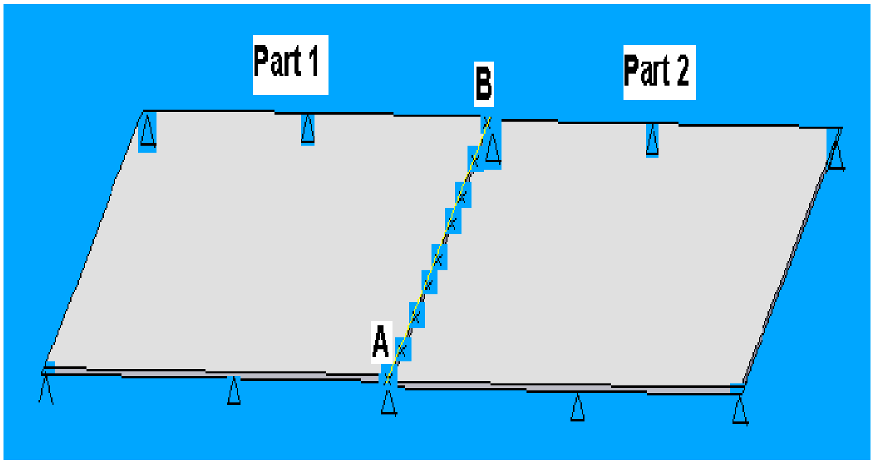


Fig.6 Assembly of two flat sheet metal components

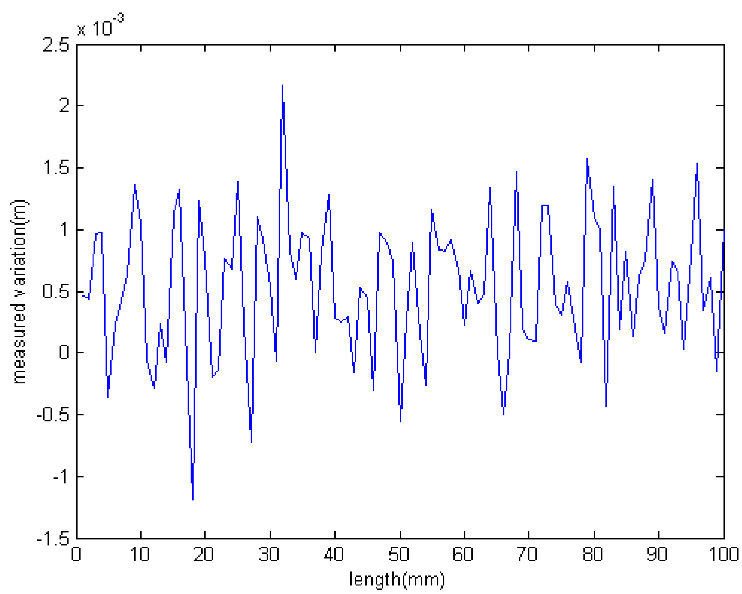


Fig.7 The sampled component variation

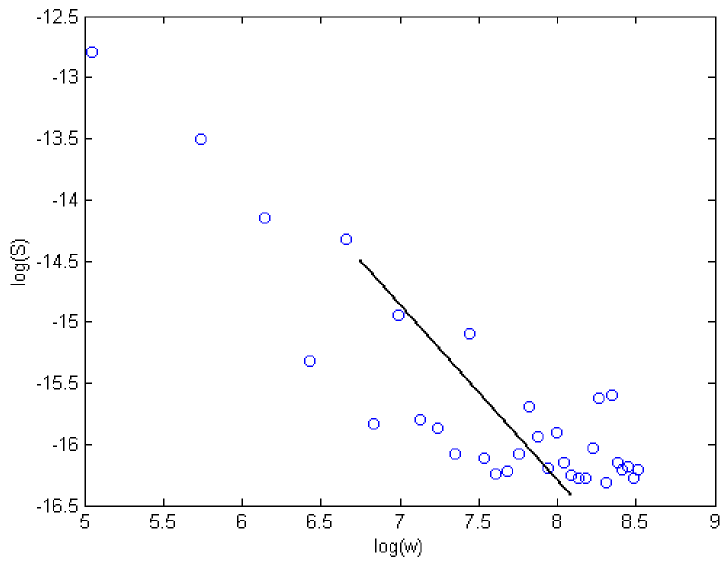


Fig.8 The log-log power spectrum density of detailed variation

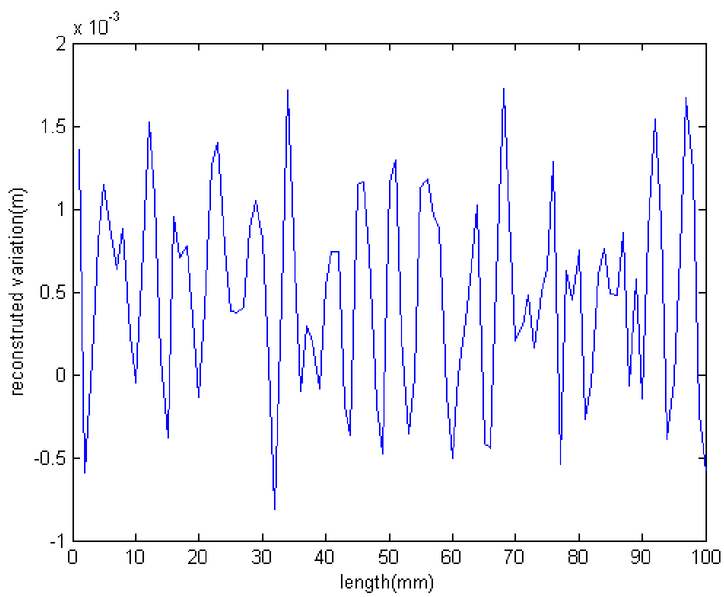


Fig. 9 The variation reconstructed by W-M function

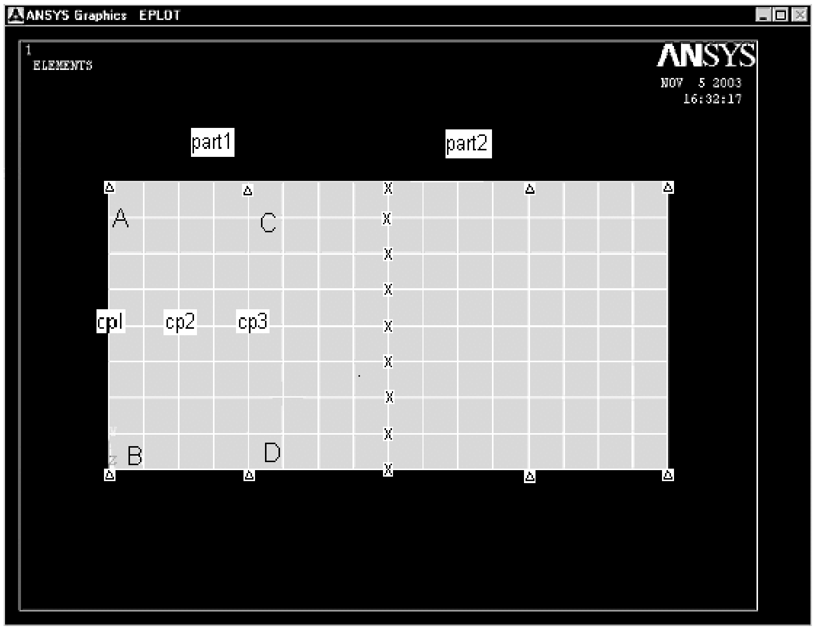
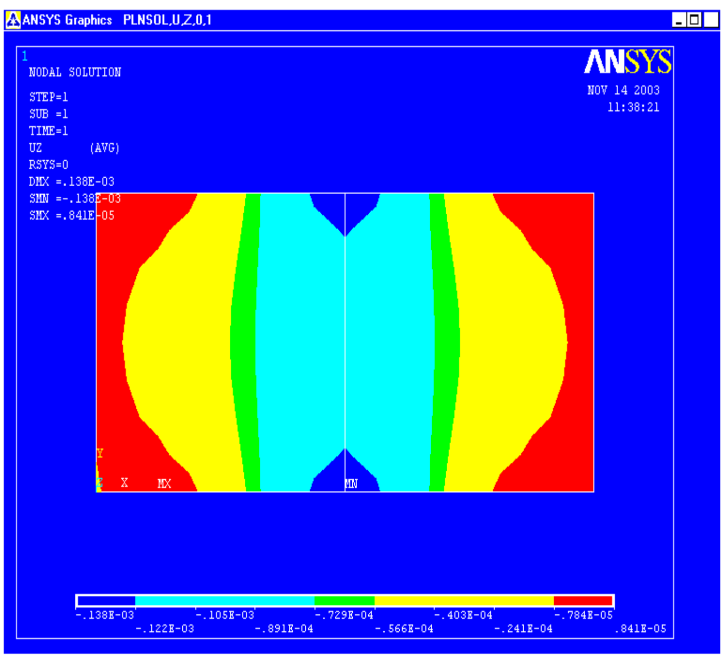
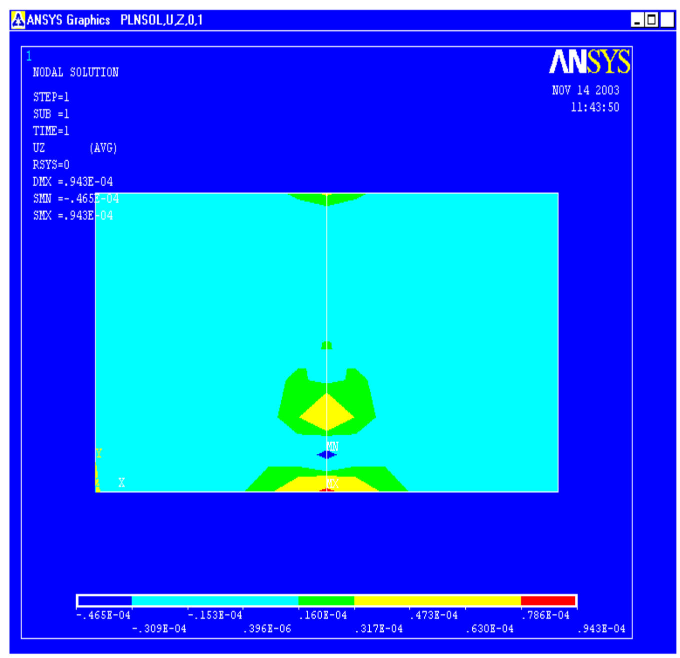


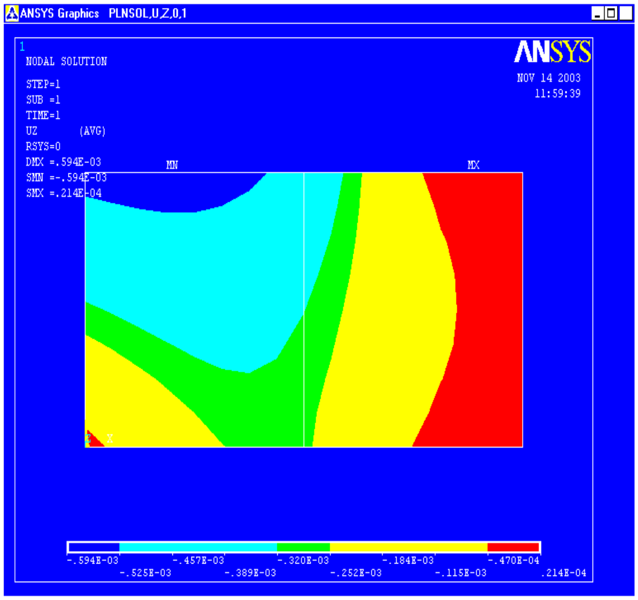
Fig.10 The FEM model for analyzing the assembly of two flat sheet metal components



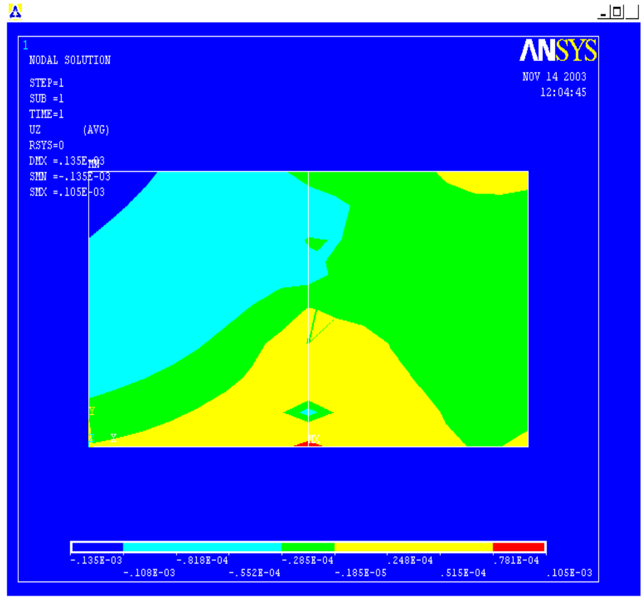
a1)



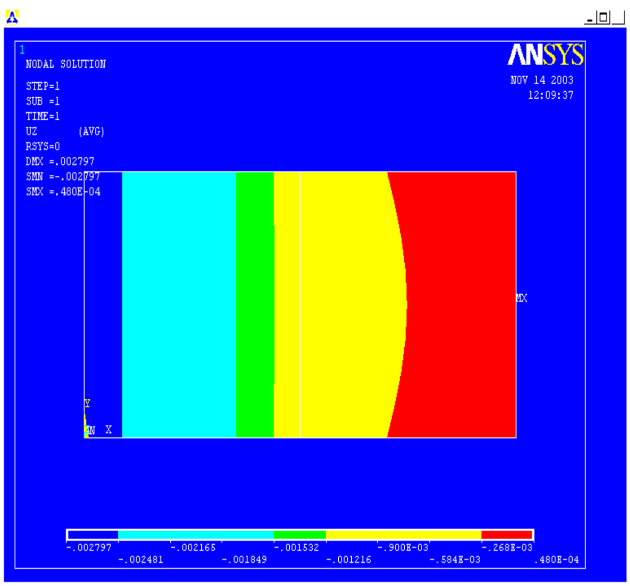
b1)



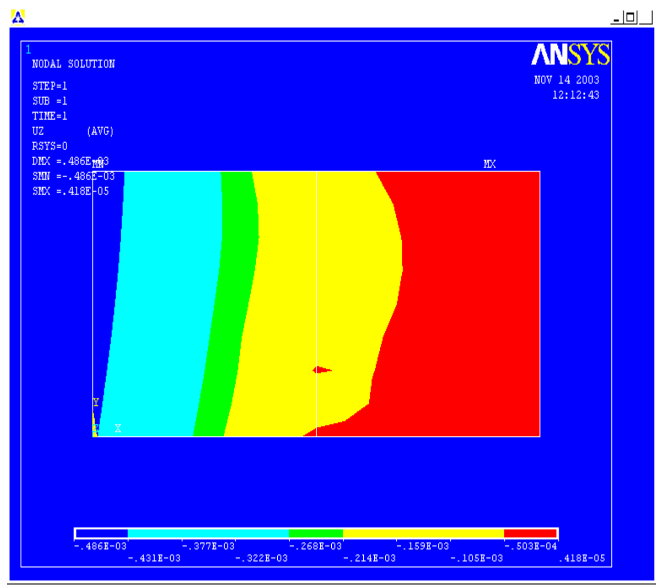
a2)



b2)

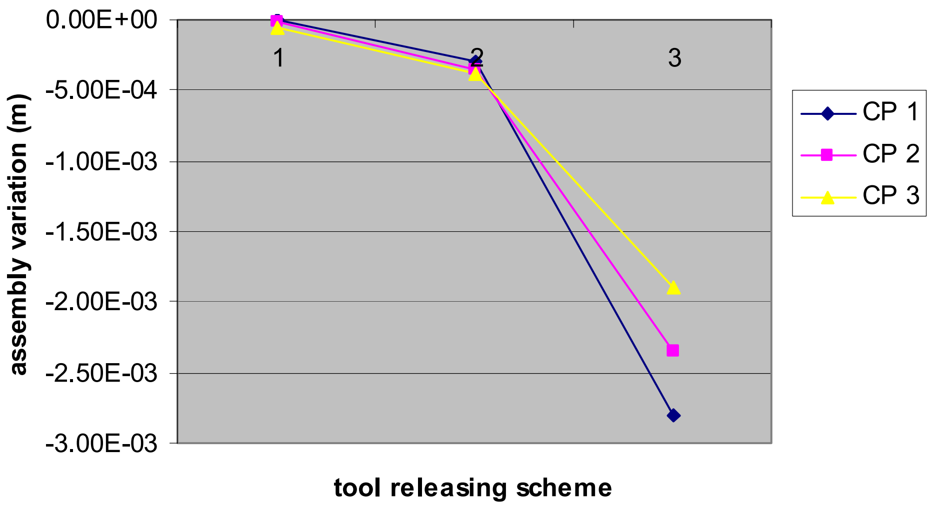


a3)

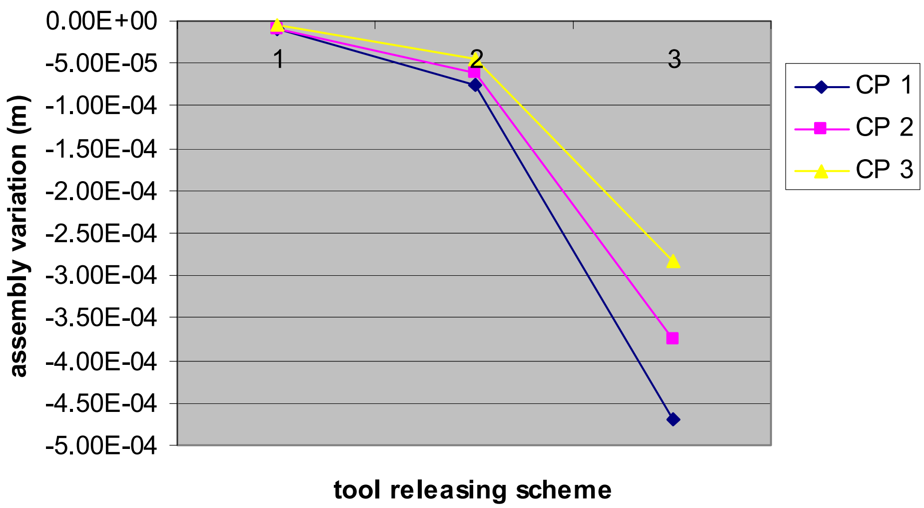


b3)

Fig.11 Assembly variation corresponding to component variations and tool releasing schemes



(a)



(b)

Fig.12 Assembly variation of 3 CPs. (a) Assembly variation due to the mean component variation. (b) Assembly variation due to the detailed component variation