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Nearly optimal neural network stabilization of bipedal standing using genetic algorithm

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Abstract

In this work, stability control of bipedal standing is investigated. The biped is simplified as an inverted pendulum with a foot-link. The controller consists of a general regression neural network (GRNN) feedback control, which stabilizes the inverted pendulum in a region around the upright position, and a PID feedback control, which keeps the pendulum at the upright position. The GRNN controller is also designed to minimize an energy-related cost function while satisfying the constraints between the foot-link and the ground. The optimization has been carried out using the genetic algorithm (GA) and the GRNN is directly trained during optimization iteration process to provide the closed loop feedback optimal controller. The stability of the controlled system is analyzed using the concept of Lyapunov exponents, and a stability region is determined. Simulation results show that the controller can keep the inverted pendulum at the upright position while nearly minimizing an energy-related cost function and keeping the foot-link stationary on the ground. The work contributes to bipedal balancing control, which is important to the development of bipedal robots.

Keywords: Bipedal standing; Postural stability; Optimal control; Genetic algorithm; Neural network; Lyapunov exponents

1. Introduction

Balancing control is essential for bipedal locomotion. There are three requirements in designing balancing controllers: (1) maintaining postural stability, (2) improving energy efficiency and (3) satisfying the constraints between the foot-link and the ground. In spite of the attempts, there is little success in developing balancing control satisfying all three requirements. We believe that the lack of a constructive tool for stability analysis is one of the obstacles. For example, when an optimal control is designed, due to the complexity of the system, stability analysis based on Lyapunov's stability theory is challenging. On the other hand, when a balancing control is designed based on Lyapunov's stability theory, it has little flexibility to include the optimization criterion. Furthermore, it has often been assumed that the constraints between the feet and the ground are always satisfied once the feet contact the ground. However, the satisfaction of such constraints imposes bounds to the control torques (Yang and Wu, 2006a), which has significant effects on control design. Thus, it is challenging to develop balancing control considering all three requirements.

Due to the nonlinear behavior and inherent complexities of bipedal systems, nonlinear control becomes a valuable choice, and numerous control methods have been employed for regulating bipedal locomotion. The capability of neural networks for approximating generic functions makes them a valuable tool for the design of nonlinear controllers (see Plumer (1996) and the references therein). Generally, there are two methods for developing an optimal controller using neural networks. In the first method, a nominal optimal controller is designed first, which provides an open loop controller. The neural network is then trained to learn the nominal control law to produce a closed loop optimal controller. This method has been widely used in various applications, such as robotic manipulator control (Josin et al., 1988; Kuperstein

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and Wang, 1990), sensor/motor fusion (Kuprestien and Rubinstien, 1989), autonomous vehicle control (Pamerleau, 1991), and process control (Bhat et al., 1990). One limitation is that the approximation of the nominal control law inevitably introduces errors and may not completely match to the nominal optimal control law. In the second method, the neural network is directly trained during optimization to provide the closed loop feedback optimal controller. One advantage of this method is that it reduces the approximation error of the control law caused by direct derivation of the neural network's parameters. However, this method has not been used widely.

In this work, using the second method, a closed loop nearly optimal controller is developed using the general regression neural network (GRNN). A GRNN is chosen due to its advantages, such that, it can provide an adequate approximation of the systems with the outstanding characteristic of fast training that reduces the optimization workload during iterative process. GRNNs have the advantage that it is unnecessary to define the number of hidden layers or the number of neurons per layer in advance. In addition, a GRNN could be implemented on small microchips by using low memory that is a beneficial factor in engineering applications. The optimization is carried out using the genetic algorithm (GA), which is an optimization algorithm inspired by the mechanics of natural evolution to guide their exploration in a search space. The basic concepts of GAs were developed by Holland (1975) and a comprehensive overview has been provided by Goldenberg (1989) and Michalewicz (1996). GAs have been used in various problems associated with bipedal locomotion. For example, Capi et al. (2002) have applied a GA to generate an optimal trajectory for a bipedal robot walking. Cabodevila and Abba (1997) designed optimal gait of biped robot based on expansion of the joint trajectories by Fourier's series using a GA. Arakawa and Fukuda (1996) used a GA to generate natural motion of biped locomotion with energy optimization.

Stability analysis of bipedal locomotion is important since it is the primary requirement. Wu et al. (1998) and Wu and Swain (2002) investigated the stability control of a human upper body during walking based on Lyapunov's stability theory. Grizzle et al. (2001) and Westervelt et al. (2003) studied stable walking using the method of Poincare mapping. Pai and Patton (1997) simulated the balancing of human standing with the consideration of the constraints between the feet and the ground. Huang et al. (2001) and Capi et al. (2002) investigated stable bipedal walking by keeping the zero moment point within the contact surface between the feet and the ground. Kolesnichenko and Shiriatev (2002) researched on stabilization of under actuated robotic locomotion systems. Although stability of bipedal locomotion has attracted much attention, stability analysis of bipedal standing based on Lyapunov's stability theory has been limited especially with the requirements of energy optimization and of the satisfaction of the constraints. This is due to the complexity of the systems and challenges in deriving a Lyapunov function.

The concept of Lyapunov exponents (Wolf et al., 1985) is a powerful tool in categorizing the system stability. Lyapunov exponents are defined as the average exponential rates of divergence or convergence of nearby orbits in the state-space. The signs of the exponents provide a qualitative picture of system's dynamics. Lyapunov exponents are independent of initial conditions, and are properties of the attactor geometry and dynamics. Since it is almost impossible to determine the exponents analytically, they are calculated based on either a mathematical model or a time series. Methods for calculating Lyapunov exponents based on a mathematic model have been developed (Wolf et al., 1985). The limitations are that the models are not always available, and even if they are available, due to their complexity and uncertainties, the calculation of Lyapunov exponents can be infeasible. The advantage of using a time series is that the data for only one state is needed, which can be measured experimentally. Three issues should be addressed when using a time series: the quantity, quality (measurement noise) of the data and the complexity of the system. These issues can have important effects on the calculated exponents. To remedy the problems, methods for calculating Lyapunov exponents from short and noisy time series have been developed (Zeng et al., 1990; Brown et al., 1991). Although much work is needed, we believe that the concept of Lyapunov exponents can be a constructive tool for stability analysis of nonlinear systems. However, to the best of our knowledge, it has not been used for the stability analysis of bipedal systems.

In this paper, the balancing control of bipedal standing is studied. The biped is assumed to move in a sagittal plane and is simplified as an inverted pendulum with one rigid link as feet. The controller is designed to move the inverted pendulum to the upright position and to satisfy the constraints between the foot-link and the ground. A GRNN feedback controller will be first designed to move the inverted pendulum in a region around the upright position while minimizing an energy-related cost function. A PID feedback controller is then used to keep the pendulum at the upright position more accurately. The foot-link is not fixed on the ground, but is required to be stationary. Three constraints are considered, i.e., no lifting, no slippage, and the center of pressure (COP) remaining within the contact region between the foot-link and the ground. To determine the stability of the control system, the largest Lyapunov exponent of the controlled bipedal system is calculated using the method by Wolf et al. (1985), and is used to determine a stability region. The time series is generated by the dynamic model, which is noise-free and can be of any amount. Thus, our approach will yield the true Lyapunov exponent by definition (Wolf et al., 1985).

The paper is organized as follows. Section 2 describes the dynamic model of a biped during standing, which includes the dynamic equations, constraint inequalities, the control algorithm and the cost function to be minimized. Section 3 gives the detailed design of the GRNN feedback controller and the GA optimization approach. Section 4 shows the stability analysis of the control system using the concept of Lyapunov exponents. Section 5 provides simulation results, followed by the conclusions.

2. Model description

An adequate bipedal model for investigating bipedal standing should be simple, but complex enough to capture the main dynamic characteristics of standing. It has been reported that standing human subjects, subject to small disturbance, typically respond by moving in a segittal plane, and they tend to keep the knees, hips and neck fairly straight, moving about the ankle (Kuo, 1995). Thus, it is reasonable to simplify a biped as an inverted pendulum. Inverted pendulum models have been used to study bipedal posture (Hemami and Stokes, 1983; Hemami and Katbab, 1982; Wu et al., 1998).

Based on the above discussion, we simplify the legs, trunk, arms and head as an inverted pendulum, and feet as one link, which provides a base of support on the ground. The feet position is assumed to be bilaterally symmetric and stationary, and the biped moves in the sagittal plane. The simplified bipedal model is shown in Fig. 1. The dynamic equations are developed using the

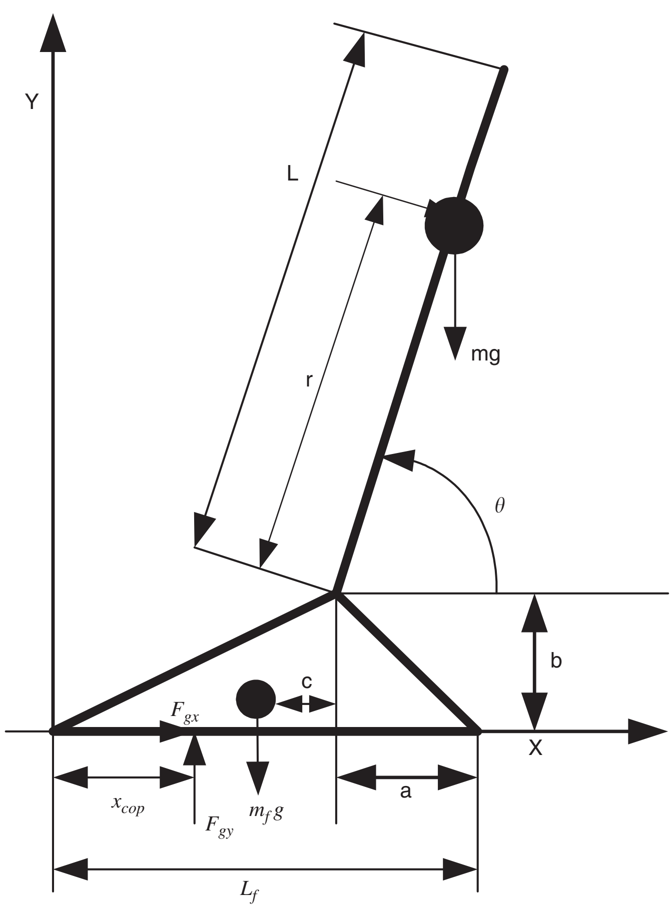

Fig. 1. Simplified bipedal model.

Euler–Lagrangian method shown below:

$$\tau = (I + mr^2)\ddot{\theta} + mgr\cos\theta, \tag{1}$$

$$F_{gx} = mr\ddot{\theta}\sin\theta + mr\dot{\theta}^2\cos\theta, \tag{2}$$

$$F_{gy} = (m_f + m)g + mr\ddot{\theta}\cos\theta - mr\dot{\theta}^2\sin\theta, \tag{3}$$

$$x_{COP} = L_f - \left[\frac{bF_{gx} - \tau + cm_f g}{F_{gy}} + a\right], \tag{4}$$

where F_{gx} and F_{gy} are the horizontal and vertical ground reaction forces. τ, θ, $\dot{\theta}$ and $\ddot{\theta}$ are the ankle torque, angular displacement, velocity and acceleration of the inverted pendulum (counter clockwise as "+"), respectively. The parameters a, b, c, x_{COP}, r, L, L_f, m_f and m are the horizontal distance between the ankle and the heel, ankle height, horizontal distance between the mass center of the foot-link and the ankle, horizontal distance between the COP and the toe, distance between the mass center of the inverted pendulum and the ankle, length of the pendulum, length of the foot-link, mass of the foot-link and mass of the pendulum, respectively.

Unlike most of previous work that the feet were assumed to be fixed on the ground, in this work, the foot-link is not fixed on the ground, but is required to be stationary during balancing. Three constraints are considered:

Gravity constraint. The vertical ground reaction force, F_{gy}, must be positive, indicating that the foot-link does not lift from the ground, i.e.,

$$F_{gy} \geqslant 0. \tag{5}$$

Friction constraint. The horizontal ground reaction force, F_{gx}, must not exceed the maximum static friction dictated by the coefficient of friction, μ, indicating that the foot does not slip:

$$|F_{gx}| \leqslant \mu F_{gy}. \tag{6}$$

COP constraint. The COP must reside within the boundary of the support, i.e.,

$$0 \leqslant x_{COP} \leqslant L_f. \tag{7}$$

Substituting Eqs. (1)–(4) into the above inequalities, the upper and lower bounds of the ankle torque, τ, can be determined, which are functions of the states, θ and $\dot{\theta}$. It was also found that the satisfaction of these constraints imposes an upper bound to the value of the angular velocity, as discussed in details by Yang and Wu (2006a).

A feedback controller is designed to stabilize the inverted pendulum at the upright position while keeping the constraints satisfied. The input to the controller is the error states $e = \theta - \theta_d$ and $\dot{e} = \dot{\theta} - \dot{\theta}_d$, where θ_d and $\dot{\theta}_d$ are the desired angle and angular velocity, and $\theta_d = 90^\circ$ and $\dot{\theta}_d = 0$ (rad/sec). The output is the torque, τ applied at the ankle joint. The state feedback controller consists of a GRNN and a PID controller. The GRNN stabilizes the disturbed biped to a small region around the upright position while minimizing the control torque.

For increasing the accuracy of stabilization and due to limited number of sampling points for decreasing the optimization workload, a PID controller is then activated to keep the biped close to upright positions. The parameters of the PID controller are determined by trial and error to keep the pendulum at upright position. The control torque is always within the control bounds determined from inequlities (5)–(7). The block diagram of the controller is shown in Fig. 2. In addition to stabilizing the biped and satisfying the constraints, the GRNN controller is also designed to minimize the following cost function:

$$J = \frac{1}{2}\int_0^{t_f} \sqrt{\tau^{\mathrm{T}}\tau}\,\mathrm{d}t + C_1|\theta_d - \theta_f| + C_2|\dot{\theta}_d - \dot{\theta}_f| + \int_0^{t_f} C_3\,\mathrm{d}t, \tag{8a}$$

$$C_3 = \begin{cases} 0, & \text{if the constraints are satisfied,} \\ C, & \text{if the constraints are not satisfied,} \end{cases} \tag{8b}$$

where t_f is the final time instant. τ is the ankle torque. θ_d and $\dot{\theta}_d$ are the desired states, and θ_f and $\dot{\theta}_f$ are the actual states at the final time instant. C_1 and C_2 are the weighting coefficients. C_3 is the constraint function for satisfying the control bounds determined by the constraints between the foot-link and the ground (Yang and Wu, 2006a). The first term of the cost function is an integral of the control effort, $\int_0^{t_f} \sqrt{\tau^{\mathrm{T}}\tau}\,\mathrm{d}t$. Based on the discussion by Chevallereau and Aoustin (2001), reducing the control effort indicates the low energy consumption during stabilization. The integral of the control effort and the term associated with C_3, shown in Eq. (8a), are the cost representation in the transitory regime. The remaining two terms of the cost function, $C_1|\theta_d - \theta_f|$ and $C_2|\dot{\theta}_d - \dot{\theta}_f|$, are the errors of the final angular position and angular velocity of the inverted pendulum with respect to the desired ones, which represent the cost function in permanent regime. Note that the satisfaction of the constraints between the foot-link and the ground also imposes conditions on the angular velocity (Yang and Wu, 2006a), which is not considered in the control design. However, in the simulation study, the simulated angular velocity was compared to the velocity bound, and if the angular velocity is higher than the velocity bound, the constraints are violated, stabilization is out of the question and the simulation is terminated.

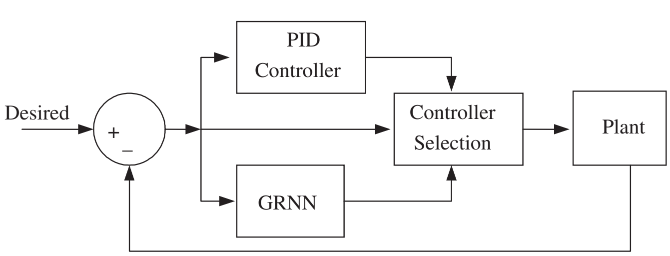

Fig. 2. Block diagram of the controller.

3. Development of general regression neural network using genetic algorithm

In this section, the design of a GRNN controller based on a GA for balancing of constrained bipedal standing is presented. The GRNN is a three-layer network that can be used to estimate the nonlinear functions. It is assumed that the vectors X and Y are the input and the output of the neural network, respectively. Estimation of Y according to an independent input variable X is the most probable value for the output Y with respect to the input X, which is denoted as $E(Y|X)$, the best estimation of Y. GRNN uses the joint probability density function (JPDF) to find $E(Y|X)$ as shown below:

$$E(Y|X) = \frac{\int_{-\infty}^{\infty} Yf(X,Y)\,\mathrm{d}y}{\int_{-\infty}^{\infty} f(X,Y)\,\mathrm{d}y}, \tag{9}$$

where $f(X, Y)$ denotes the JPDF of X and Y. Assuming that the sampling points, (X_i, Y_i) $(i = 1, 2,\ldots,n)$, are related through a Gaussian JPDF $f(X, Y)$, thus, $f_n(X, Y)$ can be defined as follows:

$$f_n(X,Y) = \frac{1}{(2\Pi)^{(P+1)/2}\sigma_n^{P+1}} \sum_{i=1}^{n} C_i(X,Y), \tag{10}$$

where

$$C_i(X,Y) = \exp\left(-\frac{(X-X_i)^{\mathrm{T}}(X-X_i) + (Y-Y_i)^{\mathrm{T}}(Y-Y_i)}{2\sigma^2}\right). \tag{11}$$

In Eq. (10), P is the dimension of vector X. It has been proven that, if $\lim_{n\to\infty} \sigma(n) = 0$ and $\lim_{n\to\infty} (\sigma^P(n)) = \infty$, we have that $\lim_{n\to\infty} f_n(X,Y) = f(X,Y)$. So using above considerations for n sampling points, Eq. (9) can be converted to:

$$E(Y|X) = \frac{\sum_{i=1}^{n} Y_i \exp\left(-D_i^2/2\sigma^2\right)}{\sum_{i=1}^{n} \exp\left(-D_i^2/2\sigma^2\right)}, \tag{12}$$

where

$$D_i^2 = (X - X_i)^{\mathrm{T}}(X - X_i). \tag{13}$$

The details of the derivation of Eq. (12), the training procedure and the parameter determination are explained in Holland (1975) and Specht (1991).

By considering a finite number of nodes, n, at pattern unit of the GRNN, the output of the GRNN is designed to be the best estimation of:

$$\tau(E) = \frac{\sum_{i=1}^{n} \tau_i \exp\left(-D^2/2\sigma^2\right)}{\sum_{i=1}^{n} \exp\left(-D^2/2\sigma^2\right)}, \tag{14}$$

where

$$D_i^2 = (E - E_i)^{\mathrm{T}}(E - E_i). \tag{15}$$

Note that vector E is the actual error of the system and E_i is the sample input of ith node in the GRNN. The input and output of GRNN are $E = \{e, \dot{e}\}^{\mathrm{T}}$ and τ, respectively. σ is the spreading factor, and it can be tuned through the algorithm (Breiman et al., 1977). In this work, $\theta_{\mathrm{d}} = 90^\circ$, $\dot{\theta}_{\mathrm{d}} = 0$ (rad/sec). $e_{\min} < e < e_{\max}$ and $\dot{e}_{\min} < \dot{e} < \dot{e}_{\max}$ where $e_{\min}, e_{\max}, \dot{e}_{\min}$ and $\dot{e}_{\max}$ are the minimum and maximum values of error in the angular displacement and angular velocity, respectively. They have been selected to satisfy three requirements: (1) to achieve an acceptable range of error in stabilizing the inverted pendulum around the upright position, (2) to have good accuracy for GRNN for approximating the optimal control, and (3) to have a reasonable number of sampling points to reduce the computing load.

The sampling points for training the GRNN, (E_i, τ_i) ($i = 1, 2, \ldots, n$), are determined such that the bounds of the control torque determined from inequalities (5)–(7) are satisfied and the cost function, shown in Eq (8), is minimized. Thus, searching the n sampling points is an optimization problem, which is carried out using a GA. GAs are optimization algorithms based on the principle of natural evolution and population genetics. A GA comprises a set of individuals (the population) and a set of biologically inspired operators (the genetic operators). The individuals have genes, which are the potential solutions for the problem. A GA has three operators: (1) Selection, in which individuals are chosen to participate in the reproduction of new individuals. The method of the tournament selection is used here, which selects two or more individuals randomly and makes the next generation by choosing the highest fitness one. (2) Crossover, which combines the characteristics of two parent individuals to form two offspring. In this work, the arithmetical crossover is used. (3) Mutation, which alters one or more genes of selected individuals by a random change. In the current study, a "real" scheme is used to form chromosomes (GAlib: Matthew's C++ Genetic Algorithms Library). A chromosome is defined as an ordered list of sample nodes as shown in Fig. 3. Each gene of a chromosome represents a sample node consisting of the error states e_i, $\dot{e}_i$ and the torque, τ_i, which denote the optimum point in error phase plane and the torque value of the corresponding point.

Based on the initial conditions and constraints on the control torque, the initial population is generated. Every population introduces a separate form of a GRNN controller because every population includes a separate combination of sample points. Simulation of the bipedal stabilization is conducted using the controller as shown in Fig. 2. Next, the cost function is calculated based on the

e_1 $\dot{e}_1$ τ_1	e_2 $\dot{e}_2$ τ_2	……………………	e_n $\dot{e}_n$ τ_n

Fig. 3. A chromosome representing the sample nodes of GRNN.

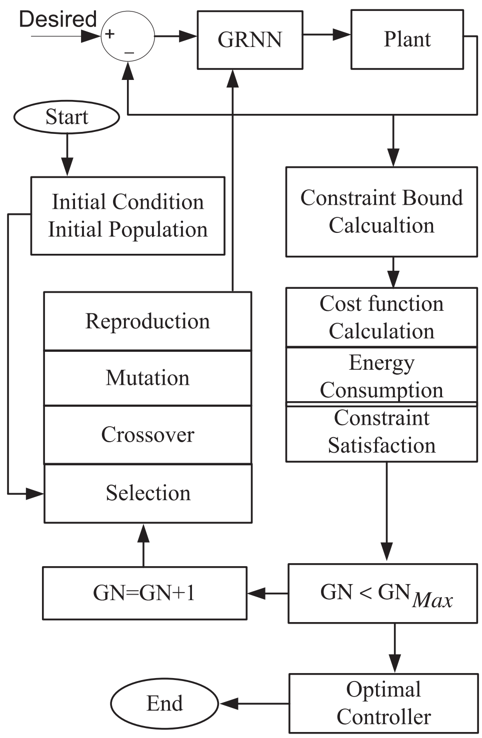

Fig. 4. Block diagram of algorithm.

simulation results with the consideration of the satisfaction of all constraints. Then the GA optimization routine will be started to find the next generation. Several evolutionary operators are set up which include reproduction, crossover operator and mutation. Searching is continued until the termination criterion (maximum number of generations G_{nmax}) is satisfied. The block diagram of the optimal GRNN feedback control based on a GA is shown in Fig. 4.

4. Stability analysis

The control is designed to stabilize the bipedal model at an upright position, while minimizing the cost function and satisfying the constraints between the foot-link and the ground. The stability of the control system is a crucial issue and required to be analyzed. Due to the complexity of the control systems, the concept of Lyapunov exponent is used to characterize the system stability.

Due to the complexity of the control algorithm, the method, developed by Wolf et al. (1985), to calculate the largest Lyapunov exponents based on a time series is used. The time series is generated by the dynamic systems, shown in Eq. (1), combined with the control law. In brief, for a given time series $x(t)$, an m-dimensional state space is reconstructed with delay coordinates, i.e., a point on the attractor is given by $\{x(t), x(t + \tilde{t}), \ldots, x(t + [m-1]\tilde{t})\}$, where $\tilde{t}$ is the delay time. By computing the vector distance $L(t_0)$ of two points at time t_0 and $L'(t_1)$ at later time t_1 and repeating the procedure until the fictitious trajectory has traversed the entire data file, the largest Lyapunov exponent is estimated as follows:

$$\lambda_1 = \frac{1}{t_M - t_0} \sum_{k=1}^{M} \log_2 \frac{L'(t_k)}{L(t_{k-1})}, \tag{16}$$

where M is the total number of replacement steps. Also during the calculation of the largest Lyapunov exponents, if the length of vector $L'(t_1)$ between the two points becomes too large, a new reference point capable of minimizing the replacement length and the orientation change is chosen.

The signs of Lyapunov exponents indicate the stability property of the dynamic systems. For example, when all Lyapunov exponents are negative, trajectories from all directions in the state-space converge to the equilibrium point. In this case, the system is exponentially stable about the equilibrium point. If one exponent is zero while others are negative, trajectories converge from all but one direction in the state-space and the attractor is a one-dimensional curve. If the trajectory is further bounded and forms a closed loop, the system performs a periodic motion and has a stable limit cycle. If at least one Lyapunov exponent is positive, two initially nearby trajectories separate at exponential rate and the system is chaotic.

It is known that although Lyapunov exponents are calculated along a single solution trajectory, they have the same values for all trajectories in the same stability region. Thus, the determination of a stability region becomes a crucial issue. The stability region is defined to include initial states, from which all of the largest Lyapunov exponents are convergent to the same negative value. The algorithm developed by Nusse and Yorke (1998) is adapted here. First, the region of interest is divided into a number of grid boxes where the grid box at the origin of the state-space (also called 'center box') contains the equilibrium point. Next, each neighboring box is tested such that if the largest Lyapunov exponent is negative, the neighboring grid box belongs to the stability region.

5. Simulation results

The developed control algorithm has been implemented with C++ in the PC environment. Parameters of the bipedal model are: $L = 50\,\text{cm}$, $L_f = 27.6\,\text{cm}$, $a = 25\,\text{cm}$ and $m = 1\,\text{kg}$, $m_f = 0.9\,\text{kg}$, $r = 0.287\,\text{m}$, $b = 2\,\text{cm}$. The initial states are $\theta_d - \theta = 0.26\,(\text{rad})$ and $\dot{\theta} = -0.2(\text{rad/sec})$. The values of $e_{\max} = 0.05\,\text{rad}$, $e_{\min} = -0.05\,\text{rad}$, $\dot{e}_{\max} = 0.4(\text{rad/sec})$ and $\dot{e}_{\min} = -0.4(\text{rad/sec})$ are used. The weighting coefficients, shown in Eq. (8a), were selected as $C_1 = 10$, $C_2 = 0.2$. The constant C_3 of the constraint function, shown in Eq. (8b), was taken as 2.0. In this work, GALIB software was used for GA optimization. For better results in terms of the solution quality and CPU time, a real-valued GA was employed. We considered $N = 40$ for the pattern unit of the GRNN, so each individual of the GA consists 40 genes of e, $\dot{e}$ and τ. The following GA parameters were used: crossover probability $p_c = 0.5$, mutation probability $p_m = 0.05$ and population size = 200. The maximum number of generations, $G_{n\max}$, is 120. As the biped entered the region of $|\theta_d - \theta| < 2.5^\circ$ with the angular velocity below 0.4 rad/s, the GRNN controller was switched to a PID controller to keep the biped at the upright position.

The convergence test was carried out first, and we found that with $G_{n\max} = 120$, the cost function converged as shown in Fig. 5. The angular displacement and the angular velocity of the ankle joint are shown in Fig. 6(a). The bold dash curve is the optimal trajectory with the initial states $\theta = 0.26\,(\text{rad})$ and $\dot{\theta} = -0.2\,(\text{rad/sec})$, and the solid curves are trajectories starting from all 24 grid boxes neighboring the optimal one, as will be discussed later. The bounds of the ankle torque determined based on the constraints (5)–(7) and the control torques are also shown in Fig. 6(b). Fig. 6 shows that the bipedal model was stabilized close to an upright position, and the control torque was always bounded within the desired limits indicating that the constraints between the foot-link and the ground were satisfied.

As shown in Eq. (8), the GRNN controller is designed to minimize a torque-cost function. The integral amount of torque for the proposed system is 1.78 (N m s). We compared the integral amount of torque from our control system with the one from previous published work (Yang and Wu, 2006b), where a state-switching PD controller was used to stabilize the same bipedal model and satisfying the same constraints, but without any considerations of optimization. The integral amount of torque of the state-switching PD control system with the same initial condition as optimal trajectory is equal to 4.27 (N m s). It is clear that the proposed controller reduces the required torques, which indicates the energy consumed is reduced significantly.

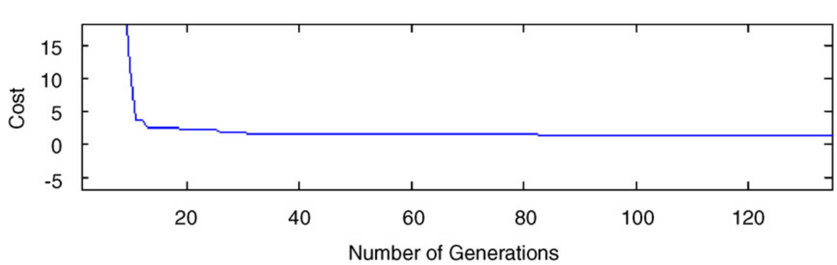

Fig. 5. Convergence of the cost function.

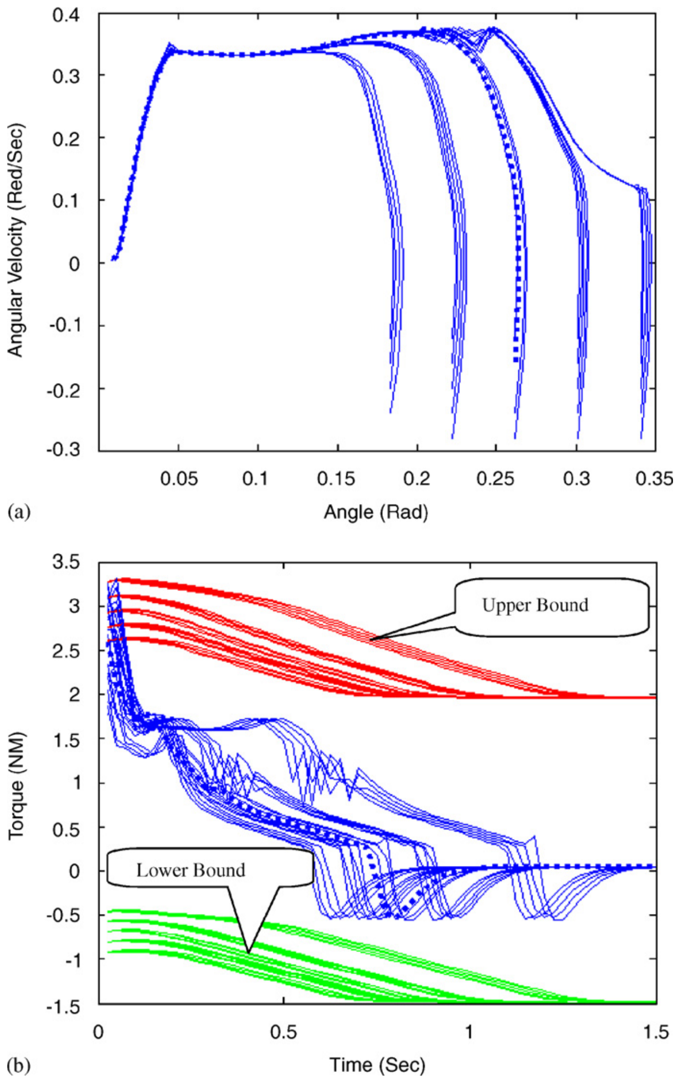

Fig. 6. Performance of the optimal controlled bipedal system with different initial states: (a) trajectories $\theta_d - \theta$ and (b) control torques.

The largest Lyapunov exponent was determined to be -3.3, indicating that the optimal control system is exponentially stable about the upright position. The region:

$$\Gamma := \{e, \dot{e}; 0.18\,\text{rad} \leqslant e < 0.34\,\text{rad} \text{ and } -0.28\,\text{rad/s} \leqslant \dot{e} \leqslant -0.12\,\text{rad/s}\}$$

in the phase plane is divided into 24 grid boxes with the size of 0.007 rad (0.4°) and 0.007 (rad/s) for θ and $\dot{\theta}$, respectively. The initial states for the nominal controller, used for optimization $\pi/2 - 0.26$ rad and -0.2(rad/s). We found that all the largest Lyapunov exponents are negative and the mean value is -3.5 with deviation of 0.3. The finding of negative Lyapunov exponents indicates that the above region, Γ, is a part of the stability region. This is confirmed by the simulations shown in Fig. 6(a). Note that region, Γ, is a part of the stability region, but not necessarily the entire stability region. Finding the largest stability region is important, but it is off the scope of this work.

The spreading factor σ in the GRNN has an important influence on the performance of the controller. We found that for a lower value of σ, the GRNN can stabilize the biped closer to the upright position. However, it is more sensitive to the initial states. For a larger value of σ, the GRNN is more robust to the disturbance of the initial states, but it is less accurate in term of keeping the biped close to the upright position. In this work, we found the best spreading factor σ as 0.7 by trial and error. However, it is highly desirable to determine the spreading factor σ by some optimization methods.

6. Conclusions

In this paper, a balancing control has been developed, which can stabilize the biped at the upright position and satisfy all constraints between the foot-link and the ground. The controller consists of a GRNN controller, which stabilizes the biped in a region around the upright position while reducing the energy consumption by partially minimizing the torque at the ankle joint, and a PID controller, which keeps the biped at the upright position. The optimization has been carried out based on a GA. The stability of the optimal controlled bipedal system is investigated using the concept of Lyapunov exponents, and a stability region is determined.

For much of the previous work, design of balancing control satisfying all three requirements has been extremely limited. We believe that the lack of an effective tool for stability analysis is one of the main obstacles. Thus, the first contribution is that we proposed to study the system stability using the concept of Lyapunov exponents and demonstrated that the concept of Lyapunov exponents is a constructive tool. This contribution is novel in that we set up a framework, which makes the control design satisfying all three criteria feasible. This is because the researchers can design neural network control or optimal control satisfying the constraints without being restricted by the lack of tools for proving the stability.

The second contribution is that we developed a partially optimal neural network controller, which is directly trained during optimization to provide the closed loop feedback control. One advantage of this method is that it reduces the approximation error of the control law caused by direct derivation of the neural network's parameters. We used a GRNN due to its outstanding characteristic of fast training and GA for optimization. We demonstrated that our controllers is effective and satisfies all three requirements.

Although the work presented here is theoretical and the time series is generated by the simulation model, the results can be extended to real robot balancing control. As discussed earlier, when the time series is generated by experiments, issues of quantity and quality (measurement noise) of the data must be considered. Since almost unlimited noise-free data was generated from the simulation model, the Wolf's method (Wolf et al., 1985) was used here for its simplicity. When the quantity and the quality of the experimentally measured time series are of concerns, the methods developed by Zeng et al. (1990) and Brown et al. (1991) are recommended since they were specifically developed for short and noise time series.

As for the future work, since the value of spreading factor σ in the GRNN has an important influence on the performance of the controller, it is desirable to develop a method for determining an optimal spreading factor σ. We are also considering to design a multi-parallel-GRNN trained from several initial points of state space to make the optimal controller more global. Another future work is to test the effectiveness of our controller and the method of stability analysis experimentally.

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