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Development of a metamodel assisted sampling approach to aerodynamic shape optimization problems †

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Abstract

A new metamodel-assisted sampling search approach applied to the aerodynamic shape optimization of turbomachinery airfoils is presented in this paper. The proposed methodology integrates a non-uniform rational B-spline (NURBS) geometry representation, a two-dimensional flow analysis, and an improved metamodel driven optimization algorithm named approximated promising region identifier (APRI), which represents a momentous advancement of the existing space exploration techniques specifically for the high-dimensional expensive black-box (HEB) problems. The novel optimization method prospects the whole design space by generating sample points, reporting evaluating information using a surrogate model, and then focusing the search in the most promising region by deploying more agents. Using the integration of these adaptive tools and methods, the optimization results are considerably promising in terms of computational efficiency and performance enhancement of the turbomachinery blade airfoil shape in both design and off-design conditions.

Keywords: Global optimization; Metamodeling; Sampling search; Promising region approximation; Turbomachinery airfoil shape optimization

1. Introduction

Nowadays, the increasing demand for energy worldwide as well as its environmental effects has forced manufacturers to look for optimized component and system performances for more efficient energy conversion processes. Considering thermal power plants as one of the most important power generation systems, the heavy-duty gas turbine (GT) compressor blades' aerodynamic shape plays a key role in system performance enhancement. Thus, efficiency preservation/improvement of these blades' sections, i.e. the airfoils, is of interest to both end-users and original equipment manufacturers. That is actually why GT compressor designers have focused their efforts on the development of new automated techniques for blading process over the last years to improve the existing products or to establish completely new blade/airfoil designs.

At the same time, many robust and adaptive engineering optimization techniques are extensively used in recent years to optimize complex real life engineering applications. In this way, the optimization tends to require large computational power, especially when high fidelity analysis such as computational fluid dynamic (CFD) or finite element analysis (FEA) is used in the design evaluation. Among these, shape optimization of complex mechanical systems represents one of the challenges that require more efficient and robust optimization algorithms specifically because of its expensive and high-dimensional objective functions. To achieve the computational efficiency and accuracy for these problems, a fast and effective solution approach is in demand. This important requirement is considered as the main purpose of the present study where an efficient integration of new optimization methods and adaptive tools is employed to optimally reshape the existing airfoil geometry.

Metamodel-based or surrogate-assisted optimization, which uses efficient computational models ─normally known as meta-models or surrogates─ for approximating the fitness function value of any expensive optimization problems, is an attractive methodology recently applied by many researchers. As a matter of fact, metamodeling based optimization algorithms are of interest for complex/multidiscipline design optimization, which is normally limited by the massive computational effort. Metamodeling based optimization algorithms enable designers to conduct and perform the design process with less computation resources. Therefore, it has also found a huge interest and success in the aerodynamic shape optimization (ASO) filed and specifically in the optimization of compressor/turbine airfoils [1-10]. On the other hand, because of

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the considerable success and popularity of evolutionary algorithms in practice in recent years, many approaches have been introduced and are being developed to combine these algorithms with the metamodeling concept to establish hybrid optimization methodologies, which are called surrogate assisted evolutionary algorithms [11, 12]. In fact, as evolutionary algorithms are more often applied to solve black-box complex engineering problems, research interests in surrogate assisted evolutionary algorithms and their development have increased as well in recent years. The same for the ASO problems, most of references mentioned above have combined evolutionary algorithms with metamodels to decrease computation efforts and/or increase the quality of simulations. This approach dramatically decreases the number of expensive function evaluations, which is crucial specifically for the expensive high-dimensional problems where the optimization algorithm iteratively calls CFD evaluations.

One of the recent works in aerodynamic design area has been performed by Iuliano and Quagliarella [13] in which a computational methodology is proposed for CFD-based wing section design to draw out a reduced order model as surrogate evaluator. The model is based on the proper orthogonal decomposition (POD) and integrated in an evolutionary optimization framework to enhance the aerodynamic efficiency of an airfoil. Zhang et al. constructed a double-stage metamodel (DSM) combining merit of both interpolation and regression metamodels for wing aerodynamic design optimization using genetic algorithm [14]. It uses regression model as the first step to fit the original model, and then interpolation model is applied to improve accuracy of the metamodel.

In the field of turbomachinery, in particular, a good overview of single-/multi- objective problem formulations and optimization methods including zero order methods (Random search and random walk), simulated annealing (SA), genetic algorithm (GA), differential evolution (DE), particle swarm optimization (PSO), first order methods (Finite difference method, algorithmic differentiation, adjoint method), and second order methods are proposed by Verstraete [3]. The study focuses on using a metamodel assisted DE by employing artificial neural network (ANN) to optimally design blades of a radial compressor. Almost the same idea with a multi-layer ANN technique for estimation of GA expensive objective function was introduced by Shahrokhi and Jahangirian [4]. They have investigated the performance of this metamodel-based GA by two transonic airfoil design problems. In the same year, Asouti et al. presented a grid-enabled asynchronous metamodel-based evolutionary algorithm based on ANN metamodels, which was assessed with a number of ASO problems [5]. In 2010, an improved centrifugal compressor impeller optimization with a radial basis function (RBF) network and principle component analysis (PCA) is applied by Ma et al. to transform the training database of ANNs [15]. They have then compared the performances of the developed surrogate assisted optimization procedures using these different trained ANNs. Karakasis et al. [6] and Karakasis and Giannakoglou [8] examined performance of RBF-based metamodels to assist a multi-objective evolutionary algorithm by filtering the poorly performing individuals within each generation and subsequently by allowing only the most promising among them to be exactly evaluated using the expensive function. They also developed a technique for turbomachinery airfoil optimization, which relies upon the combined use of hierarchical, distributed, and metamodel assisted evolutionary algorithms.

With application of metamodel-based evolutionary methods, a required trade-off has to be considered between accuracy of solution and CPU effort specifically for high-dimensional and expensive engineering problems. In other words, achieving an absolute satisfactory computational cost by means of evolution concept will lead to an unacceptable decrease in accuracy of global optima. Therefore, it would be favorable if other algorithms except the evolutionary techniques, which are still working based on metamodeling, can be applied for ASO so that global minima could be attained while computational time is considerably low.

In this paper, optimizing the geometry of a typical GT compressor blade airfoil is carried out using a guided random search technique with the objective of minimizing the total pressure loss coefficient for the design as well as off-design conditions. A metamodel-based global optimization (GO) algorithm called approximated promising region identifier (APRI) is developed to perform the design optimization process. Though this is a generic methodology, its performance is demonstrated in optimization of a compressor airfoil shape. Following the introduction, a short problem statement is briefly discussed to clarify the problem in-hand. The GO technique developed here is then introduced. Finally, implementation of airfoil shape optimization approach is explained. Last, the optimization results, discussion and conclusion are included.

2. Problem statement

While experimental activities remain decisive for ultimate assessment of compressor airfoils choices, numerical design optimization techniques, along with CFD simulation tools are assuming more and more importance for the detailed design and evaluation of designs. With the objective of minimizing the total pressure losses for the compressor design condition as well as maximizing the airfoils operating range in this study, design optimization is carried out by coupling an established MATLAB code for the geometry parameterization of the airfoils' shape, a blade-to-blade flow analysis in CFD module of COMSOL Multiphysics tool, and a developed APRI in MATLAB script as the optimization algorithm.

After measuring the in-hand blade point cloud, the geometry parameterization is implemented using non-uniform rational B-spline (NURBS) curve [16] within MATLAB script. For geometry parameterization, there are several mathematical techniques including Bezier and NURBS as two of the most popular ones. Considering all pros and cons of NURBS at the same time, its application in such geometry representation for optimization purposes seems to be more appropriate (more details can be found in other studies of the authors in Refs. [17] and [18]). In this implementation, accuracy and robustness of the optimization process become the core issues, where the NURBS control points' coordinates are considered as the design parameters in the optimization loop. To parametrically represent the airfoil geometry, a direct handling of airfoil shape is employed in this paper, which creates distinct curves for the suction side (SS), pressure side (PS), leading edge (LE) and trailing edge (TE) segments of the airfoil.

Once the geometry is generated, it goes to the flow solver software for a 2-D fluid flow analysis. Post-processed results of the acceptable solutions are then entered to the fitness calculation part. Lastly, since the CFD analysis for turbomachinery application is limited by the time-consuming computational effort specifically in an iterative optimization process, a modified space exploration and region elimination technique, called APRI, is developed as a generic surrogate assisted optimization strategy for the optimization task. In other words, because of the fact that most of the computational time is spent in the evaluation of the objective function, a faster solution approach would be more appropriate. APRI works by exploring the whole design space by sending (Sampling points) to explore the design space and report some information on it. Based on the information obtained from all agents, the algorithm focuses the search in the most promising region and explores it more by deploying more agents (Generating more sample points). Latin hypercube design (LHD) [19], which is a well-known sampling technique, is used as a sampling technique to generate sample points. Once a promising region is identified, this region is refined with more sample points. A surrogate model or metamodel is then constructed to mimic the expensive objective function and help in searching for the optimum solution. In the next section APRI will be described in more detail. Fig. 1 illustrates the integration of tools and methods for the blade geometry optimization.

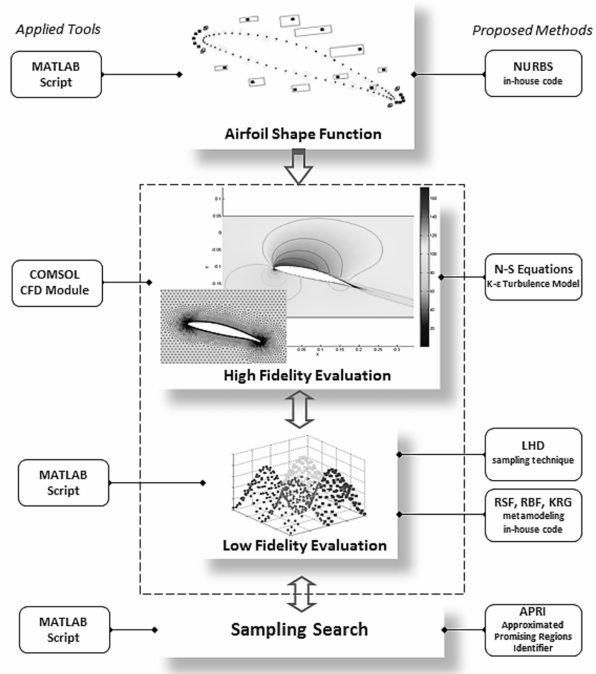

Fig. 1. Integration of tools and methods in airfoil shape optimization.

3. Approximated promising region identifier optimization algorithm

Surrogate assisted search, space exploration, and region reduction techniques are among the most effective optimization schemes for computationally demanding global design optimization problems. In this way, an effective approach to escape from the repeated evaluations of such expensive functions is to explore the design space in an optimal design problem by removing the less promising and previously searched regions. Indeed, identification of promising regions where there is a higher potential for design solutions will accelerate the global search process. In this section, APRI search algorithm used for airfoil shape optimization is introduced and the steps are explained.

The developed algorithm consists of the following key elements:

- Exploring the design space by generating sample points using the Latin Hypercube sampling method;
- Identifying the most promising region based on evaluated sample points, and defining the new boundaries of the promising region. It is most likely or expected that the global optimum should be in that region;
- Fitting a response surface function (RSF), Kriging model (KRG), or radial basis function (RBF) with additional design experiments using latin hypercube designs over the identified promising region and identifying its minimum; and,
- Using the metamodel and the evaluated cheap points using the constructed metamodel RSF/KRG/RBF, the global optimum is obtained from the identified promising region.

The algorithm is simple and efficient as will be proven on the airfoil optimization. Notwithstanding the three above-mentioned metamodeling techniques used in APRI have widely known theory and been explained in literature, a brief mention of the formulation and key points may be helpful to compare and interpret the results.

3.1 Metamodeling techniques for approximation

RBF, which is specifically useful for representing irregular surfaces, use linear combinations of a radially symmetric function based on Euclidean distance or similar metric to build approximation models [20]. Beside the interpolating scheme as the first application, Dyn et al. made RBFs more practical by enabling them to smooth experiment data as well as interpolating it [21]. The form of these meta-models is a basis function dependent on the Euclidean distance between the sample points and the point to be predicted, as its name represents. Mathematically, the model can be expressed by the following Equation:

$$\hat{y}(x) = \sum_{i=1}^{N} c_i x - x_{0i} \tag{1}$$

where the approximated function $\hat{y}(x)$ is represented as a sum of N radial basis functions, c_i is a real valued weight, and x_{0i} is the input vector.

From the late twentieth century, however, RSFs have been used effectively as metamodels [22], while originally was developed for the analysis of physical experiments [23]. RSFs approximate expensive functions by using the least squares method on a series of points in the design variable space. Low order polynomials, such as the first and second order polynomials in Eqs. (2) and (3) are widely used as the response surface approximating functions like in this study:

$$\hat{y}(x) = \beta_0 + \sum_{i=1}^{k} \beta_i x_i \tag{2}$$

$$\hat{y}(x) = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \sum_{j=1}^{k} \beta_{ij} x_i x_j \tag{3}$$

where parameters, $\hat{\beta}$, are computed using least squares regression by minimizing the sum of the squares of the deviations of predicted function values, $\hat{y}(x)$, from the actual function values, $y(x)$, using Eq. (4):

$$\hat{\beta} = \left(F^T F\right)^{-1} F^T \hat{y} \tag{4}$$

where F is the design matrix of sample data points, and $\hat{y}$ contains the values of the response at each sample point. Polynomial response surface models can be easily constructed, but the over-simplification may be troublesome for modelling highly nonlinear or irregular behaviours [24].

On the other hand, KRG which was firstly developed for application in geostatistics is a stochastic technique based on spatial correlation functions that treat the deterministic computer response as a realization of a random function, with respect to the actual system response [25]. Because of its ability to mimic the behavior of the expensive functions it has gained popularity in different applications [26]. A Kriging model postulates a combination of a polynomial model and the minor departure in the form:

$$y(x) = f(x) + Z(x) \tag{5}$$

where $y(x)$ is the unknown function of interest, $f(x)$ is a known polynomial function often taken as constant, and $Z(x)$ is the correlation function which represents a stochastic process with mean at 0, variance σ^2, and nonzero covariance [27].

Table 1. Accuracy results of the used surrogate models.

Metamodels	Accuracy value			
	R^2		RRMSE	
	N = 500	N = 1000	m = 100	m = 500
RSF	0.6952	0.8618	0.7064	0.4982
KRG	0.6602	0. 9063	0.7385	0.4136
RBF	1.0000	1.0000	0.8294	0.6803
Mixed metamodeling	0.4070	0.5131	0.9001	0.8491

To measure the accuracy of theses surrogate models, the coefficient of determination (R-squared) and the relative root mean squared error (RRMSE) are taken as the indicators [28]:

$$R^2 = 1 - \frac{\sum_{i=1}^{N} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{N} (y_i - \bar{y}_i)^2} \tag{6}$$

$$RRMSE = \sqrt{\frac{(m-1)\sum_{i=1}^{m} (y_i - \hat{y}_i)^2}{m \sum_{i=1}^{m} (y_i - \bar{y}_i)^2}} \tag{7}$$

where N is the number of sampling points for metamodel construction, m is the number of test points in the data set used for validating, y_i is the value of the response from the original CFD model, $\hat{y}_i$ is the predicted response from the metamodels, and $\bar{y}_i$ is the mean value of the observed responses from the expensive CFD model.

Table 1 compares the accuracy of the above-mentioned metamodels for the nonlinear problem in hand based on different sample/test sizes. While it is desirable that R-squared values should be close to 1 indicating that the metamodel can be used, the relative RMSE should be as small as possible to demonstrate a more acceptable fitting. The table shows that the quality metrics of the surrogate models improve with increasing number of fitting designs or test points, although this improvement is more considerable for the quality metrics of KRG rather than those for RSF/RBF. Generally speaking, it can be seen that all the metamodels can indeed show promising error as well as R-squared values, which in turn, provide approximations with adequate accuracy. Among those, however, KRG and RSF yield better approximations than RBF and mixed metamodeling.

Focusing on the metamodel driven proposed optimization algorithm, the following section explains how it exactly works.

3.2 Steps of the proposed algorithm

The APRI algorithm starts searching by checking the values of the function near the candidate point and moves first to the right to compare the function value of the nearest sampled point with the candidate point (Also known as the center point). If the function value of the nearest point is greater than the center point, then the algorithm keeps moving in that direction until the function value of the new point becomes less than the function value of the point that was checked before it. The obtained point represents the edge of the unimodal region in that direction. The same happens in all directions, which represents the problem dimension.

As also briefly illustrated in the flowchart of Fig. 2, the optimization algorithm developed in this study involves the following steps:

(1) Generate a set of design data points, x_i, $i = 1,\cdots,n$ over the design space;

$$x_i = \{x_1, x_2, \cdots, x_n\} \qquad x_i \in S^p \tag{8}$$

where p represents the number of design variables, n is the number of initial design data points, and S stands for the entire design space.

(2) Evaluate the values of the objective function and constraints using the selected design points;

$$y = \min\{f(x) : (x_i, f(x_i)) \in S\} \tag{9}$$

where y represents the minimum of expensive function values.

(3) Identify the new upper and lower boundaries of the promising unimodal region based on the obtained objective function values, by checking the function values of the neighboring points and comparing it with the previous evaluated points. If there is a sudden change in the function value (Values go up or down- maximization or minimization problem) that is considered as a boundary. Then check the other directions and the same procedure is followed. This will be the same in all directions (Depending on problem dimensionality). More details on how to identify the promising region boundaries and the neighboring design points that are seized between these boundaries have been discussed by Younis and Dong as space exploration and unimodal region elimination (SEUMRE) methodology [29].

(4) Once the new boundaries of the approximated promising region has been identified, refine the data set of this most promising region by adding more experiments or *expensive* points (Points evaluated using the objective function) into the region (Approximately 30-40% of the initial sample points) using LHD, and then introduce RSF/KRG/RBF approximation models over the region.

(5) Add more *cheap* design points that can be obtained using the easy-to-calculate approximation models. In present approach, around 10000 (Optional and depending on the complexity of the problem at hand) cheap points are generated using the meta-model over the promising unimodal region to identify the optimum point. A local optimizer can be used instead of generating these cheap points on the metamodel (Optional).

(6) Carry out the optimization search and identify the optimum point.

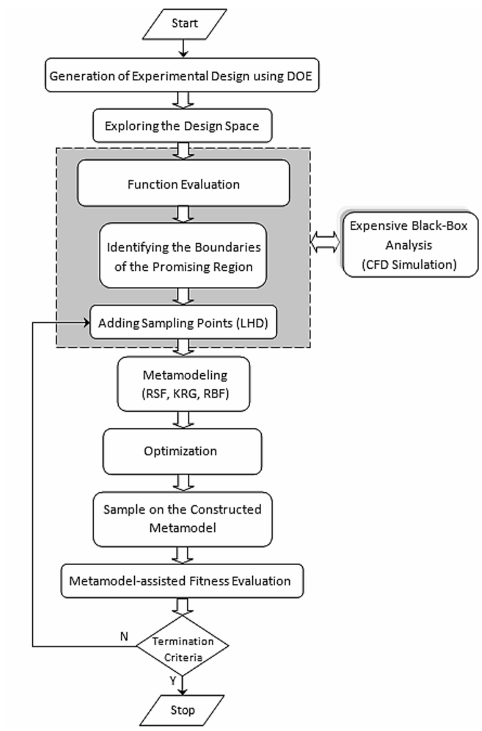

Fig. 2. Approximated promising regions identifier (APRI) flowchart.

This iterative process will continue until the termination criterion is satisfied.

In APRI, we just explore the whole space once and focus on one design space, in which case a metamodel is constructed and the global optimum value can be reached. Therefore, the way in which the algorithm searches for the global optima is totally different from that of its predecessors. Generally, this is the first metamodel-based exploration algorithm where the algorithm identifies and focuses on one region instead of wandering in the whole design space, checking regions one by one. This algorithm can identify the region in which the optimum might exist. Also this algorithm constructs one metamodel in the region of interest and not all over the design space, which significantly reduces the computations.

4. Airfoil shape optimization

Aerodynamic design optimization techniques for turbomachinery application have dramatically changed in the last years. While traditional 1-D and 2-D design procedures are consolidated for preliminary calculations, emerging techniques have been developed and are being used almost routinely within industries and academia. To this respect, designer experience plays a major and a decisive role; however, the complexity evidenced above claims for a more structured

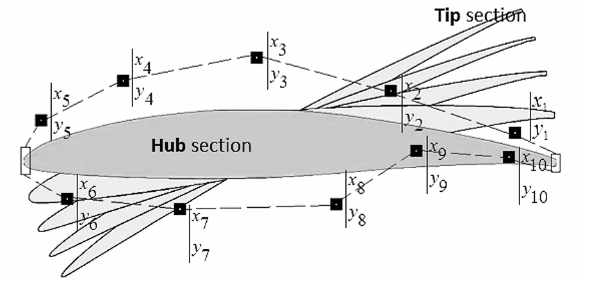

Fig. 3. Schematic diagram of twenty shape design variables.

and organized way of handling the problem, where mathematical and analytical tools are implemented and used as a decisive support in the decision making process. In addition to the optimization technique applied for an airfoil shape optimization framework, there are three main parts including shape parameterization, fitness function formulation, and airfoil flow analysis, which are briefly discussed in the following sections.

4.1 Geometry parameterization

The blade airfoil shape parameterization addresses 2-D profile construction from either the direct handling of curves of airfoil shapes or the superposition of the camber line and thickness distribution around it. From these two different approaches of geometry definition, however, the method of distinct parameterized curves has been employed for the SS, PS, LE, and TE of a blade section. Furthermore, NURBS curves [30, 31] have been used to parameterize the geometry fed to the optimization process. In this way, the locations of NURBS control points are considered as design variables.

The compressor airfoil considered in this study consists of four NURBS curves for four segments, and nine control points identify each segment. Each control point has two coordinates *x* and *y*, and so there are a total number of 72 design variables. However, 16 parameters, which are the intersection points of the segments are known and fixed. Other 16 parameters are also determined by enforcing C^2-continuity at the intersections of the segments. As a result, 40 variables will remain for the optimization algorithms as the design variables. To reduce the CPU effort, however, the geometry of LE and TE are considered constant to keep the number of design parameters to a minimum while attaining a high degree of geometric flexibility.

Hence, as illustrated in Fig. 3, the final total number of design variables is twenty (The control points of suction and pressure surfaces). Based on that, the APRI optimization technique has been applied to locate the optimal position of these twenty control points according to the fitness function formulated in the next section.

4.2 Objective function

Besides the optimization technique and parameterization policy, formulation of the objective function has also a key effect on the results of the airfoil optimization process. The parameterized profiles are made by the geometry code, and then fed to the COMSOL CFD for a 2-D fluid flow analysis. Following the convergence check, post-processed results of the accepted profiles are then entered to the fitness calculation part where the airfoils' loss values, *L*, should be minimized with respect to any geometry. Seeing Fig. 3, the single objective function is determined as follows:

$$\begin{aligned}
&Min \;\; L\% = \left(a_1.L_s + a_2.L_d + a_3.L_c + PF\right)\times 100\,.\\
&\text{Subject to:}\\
&\qquad \left(\left|y_3 - y_7\right| \text{ and } \left|y_3 - y_8\right|\right) \ge 15\% \quad \text{of the chord}\\
&\qquad 0 \le x_i \,/\, chord \le 100\% \qquad\qquad (10)\\
&\qquad 40\% \le y_i \,/\, chord \le 55\%
\end{aligned}$$

where
$L_s = \dfrac{(P_{01}-P_{02})_{stall}}{0.5\rho V_1^2}$, $L_d = \dfrac{(P_{01}-P_{02})_{des}}{0.5\rho V_1^2}$, $L_c = \dfrac{(P_{01}-P_{02})_{choke}}{0.5\rho V_1^2}$ are total pressure losses for stall, design and choke conditions respectively, y_i are design variables as in Fig. 3, a_i are the weighting factors, *PF* is the penalty function of geometry constraint, P_{01} is inlet total pressure, P_{02} is outlet total pressure, and V_1 is inlet velocity. The minimization of total pressure loss at the right side (L_c) and left side of the design point (L_s) effectively widens the range of operation. Therefore, this formulation considers both total pressure loss, and the operation range through a weighted sum approach. In addition to the profile limitations considered in the geometric modeling, the minimum acceptable thickness of airfoils from structural issues point of view is taken into account. The weighting factors a_i can be defined based on the significance of the defined terms in *L* which may be different based on the design optimization priorities. For example, stall margin increase is assumed to be more important than the choke margin rise in this specific case. In the interim, the results have high sensitivity to the defined weights; they should accurately be specified based on the existing experience as well as priorities of the optimization goals. Based on that, the weighting factors for the optimization process are specified as follows: $a_1 = 0.20$, $a_2 = 0.70$ and $a_3 = 0.10$. Generally speaking, sum of these coefficients should be unity and none of them has to be negative. More details have been presented by Safari et al. [32].

4.3 Flow analysis

In the computational domain for airfoil flow analysis, the air flows from the left to the right so that the left side is defined as the inlet containing the velocity information and the right boundary of the domain is the outlet specified by the expected static pressure difference. The upper and lower boundaries are outlined as the periodic flow conditions that are actually addressing a cascade of the rotor blades. Also, a wall function formulation is enforced along the airfoil surface. The simulation is carried out for a relatively high subsonic compressible flow ($Mach \sim 0.6$) which has a high Reynolds number as well ($Re \sim 2.8\times10^6$).

The preliminary grid generated for the mentioned domain consists of free triangular physics-controlled fine mesh. However, more appropriate mesh to produce the same level of accuracy for different designs would be constructed through the adaptive unstructured mesh refinement capability of the analysis tool. To save also the computational time, the virtual operations option is used to reduce the number of mesh elements without a major impact on the fitness evaluation [32].

Using stationary two-dimensional fluid flow analysis capability of COMSOL Multiphysics, a single-phase turbulent interface has been selected in which the conservation laws of the momentum, mass and energy is formulated. Considering predefined initial and boundary conditions, the partial differential equations of the conservation laws are solved using finite element formulations. COMSOL CFD module contains governing Navier-Stokes equations and the standard *k*-ε model [33] introducing two additional transport equations and two dependent variables which are the turbulent kinetic energy, κ, and the turbulent kinetic energy dissipation rate, ε. Since CPU time is an important criterion for any aerodynamic design optimization procedure, the κ-ε model which is the most broadly used turbulence model with an acceptable computational cost as well as good accuracy is employed. In this way, the turbulent viscosity could be written as follows:

$$\mu_T = 0.09\bar{\rho}\, k^2 \big/ \varepsilon \ . \tag{11}$$

Transport equations for κ and ε are represented by Eqs. (12) and (13), respectively:

$$\bar{\rho}\tilde{\boldsymbol{u}}.\nabla k = \nabla.\left(\left(\mu + \mu_T\right)\nabla k\right) + P_k - \bar{\rho}\varepsilon \tag{12}$$

$$\bar{\rho}\tilde{\boldsymbol{u}}.\nabla \varepsilon = \nabla.\left(\left(\mu + 0.77\mu_T\right)\nabla \varepsilon\right) + 1.44\,\varepsilon \big/ k\, P_k - 1.92\bar{\rho}\,\varepsilon^2 \big/ k \tag{13}$$

and the production term is:

$$P_k = \mu_T\left(\nabla\tilde{\boldsymbol{u}} : \left(\nabla\tilde{\boldsymbol{u}} + \left(\nabla\tilde{\boldsymbol{u}}\right)^T\right) - 0.67\left(\nabla.\tilde{\boldsymbol{u}}\right)^2\right) - 0.67\bar{\rho}k\nabla.\tilde{\boldsymbol{u}} \tag{14}$$

where $\bar{\rho}$ is density, μ is dynamic viscosity, and $\tilde{\boldsymbol{u}}$ is density-based average of velocity vector so that $\nabla.\left(\bar{\rho}\tilde{\boldsymbol{u}}\right) = 0$.

Lastly, for each iteration of the shape optimization procedure, a single airfoil is sequentially analyzed at multiple angles of attack to simulate design and off-design conditions.

Table 2. Investigation of the effect of the number of design variables on the performance of APRI (10 independent runs of the test function with the same termination criteria).

# Design Variables	Fitness values			Average of No. of evaluations	Average of No. of iterations	Average of CPU time (min)
	Best	*Mean*	*Median*			
5	0.0229	0.0770	0.0423	276	60	1.5
10	0.0514	0.0994	0.0823	279	61	2
20	0.1104	0.1798	0.1576	553	152	38
25	0.1191	0.1693	0.1301	777	227	50
30	0.1398	0.1929	0.1928	824	242	156

5. Designs of experiment and simulation results

Using the integration of adaptive tools and methods, the preliminary results are considerably promising in terms of computation time, number of function evaluations and the airfoils' shape performance enhancement from aerodynamic efficiency point of view. In this section, the results obtained from the application of developed optimization framework are investigated in order to show the ability of the proposed technique to noticeably increase the complex shape optimization process efficiency compared to the evolutionary methods. All coding in this research has been implemented in the integrated COMSOL 4.3a and MATLAB® R2011b, on a PC with Intel(R) Xeon(R) CPU X5650 @ 2.66 GHz with 2 processors and 32.0 GB RAM.

Before a comparative analysis, the impact of the number of design variables, i.e. degree of freedom (DOF) of the geometric model, on APRI's performance is investigated. Table 2 represents the best, average and median of the optima found by APRI as well as the number of evaluations for an exponential test function with different numbers of variables ($f(\boldsymbol{x}) = 1 - exp\left[-\frac{1}{60}\sum_{1}^{n} x_n^2\right]$, $0 \le x_n \le 2$). The results are the average of 10 independent runs to decrease the randomness effect. As can be seen, the proposed algorithm is working robust in a variety of dimensionality so that fitness values and/or number of evaluations increase almost linearly with exponential growth of the search space due to increasing problem dimensions. As a result, APRI is adequately robust to the design variable count and still presents good numerical performance without significant drop-off for the high-dimensional problems.

Tables 3 and 4 show the results obtained from APRI using RSF, Kriging, and RBF metamodeling separately. The obtained results are also compared with the mixed metamodels driven SEUMRE (as another recently developed metamodeling-based search and space exploration method by the authors

Table 3. Performance comparison and optimization results.

Optimization algorithms	Runs	f^*	NOE	# Iterations	Optimization algorithms	Runs	f^*	NOE	# Iterations
RSF driven APRI	#1	6.1259	31	7	*Mixed metamodeling driven SEUMRE*	#1	6.3371	83	3
	#2	5.7501	52	14		#2	6.5365	365	6
	#3	5.4434	499	163		#3	6.5268	87	3
	#4	5.5953	187	59		#4	6.6788	190	5
	#5	5.9008	100	30		#5	6.6677	63	3
	#6	5.5063	538	176		#6	6.5815	266	6
	#7	5.6189	160	50		#7	6.5833	202	5
	#8	5.7874	113	22		#8	6.9440	82	3
	#9	5.7005	131	28		#9	6.5815	266	6
	#10	5.5794	131	28		#10	7.0973	82	3
KRG driven APRI	#1	6.4540	120	31	*Real-coded GA*	#1	6.0119	3360	41
	#2	6.7335	57	10		#2	5.7665	5040	62
	#3	6.6370	57	10		#3	6.1693	3040	37
	#4	6.4700	120	31		#4	5.9118	4080	50
	#5	6.3006	111	28		#5	5.8120	3600	44
	#6	6.3754	120	31		#6	5.9246	2880	35
	#7	6.5617	141	38		#7	5.9555	3920	48
	#8	6.4839	54	9		#8	5.8697	2880	35
	#9	6.4110	120	31		#9	6.0015	4800	59
	#10	6.4609	69	14		#10	5.9417	3760	46
RBF driven APRI	#1	6.2117	601	197					
	#2	6.6380	37	9					
	#3	6.6542	64	18					
	#4	6.3095	235	75					
	#5	6.1181	172	54					
	#6	6.1121	631	207					
	#7	6.0378	655	215					
	#8	6.5854	66	13					
	#9	6.3868	68	7					
	#10	6.8791	56	3					

Table 4. Comparison of median and SD values.

Optimization algorithms	f^*			NOE		# Iterations		Average of CPU time (min)
	Best	Median	SD	Best	Median	Best	Median	
RSF APRI	5.4434	5.6597	0.2020	31	131	7	29	532
KRG APRI	6.3006	6.4654	0.1264	54	116	9	30	769
RBF APRI	6.0378	6.3482	0.2833	37	120	3	36	581
SEUMRE	6.3371	6.5824	0.2179	63	139	3	4	1850
RC-GA	5.7665	5.9331	0.1126	2880	3680	35	45	6465

[34]) and a real-coded genetic algorithm (RC-GA)'s results (as a well-known and widely used guided random algorithm) to highlight the advantages of the proposed approach for such high-dimensional shape optimization problems. The algorithms' parameters make a huge difference in the obtained optimum or the best obtained airfoil geometry and changing one parameter might easily affect the results.

Accordingly, the most favorable parameters for RC-GA have been selected based on existing experience [32] and the parameters of all the metamodeling-based techniques are also set correspondingly to have a conservative comparison. The results are shown at a small termination criteria, e, equal to 0.001 which represents the difference between the two function values. When e is small it becomes harder for the algorithm to coverage to a global solution in less computation time but the accuracy of the results will be high. Based on the experience, it can be seen that with low e value, APRI still yields results with acceptable accuracy which reflects the high performance of proposed optimization algorithm. All the results are for ten different runs; the number of runs considered being fair for such random sampling based algorithms.

As shown, the proposed sampling method with all three metamodeling techniques outperforms SEUMRE and RC-GA in terms of computational efficiency. The average of CPU time for APRI optimization is 630 minutes, while SEUMRE requires much more amount of CPU effort and RC-GA takes about ten times more to reach even higher weighted pressure loss value equal to 5.9331. Computation time is an important factor that real world application pays much attention to. Table 4 shows the best, median, and standard deviation (SD) of the minimum loss function values, f^*, number of expensive evaluations, *NOE*, and number of iterations. Though RC-GA achieves better optimum results than KRG/RBF driven APRI, RSF-based APRI converges to considerably better minimum loss value than RC-GA and SEUMRE as well (The loss function value of 5.6597 is the best among all function values). This can be because of behavior of the airfoil loss function calculation results (Black-box CFD post-processing), which could be more accurately approximated by the polynomial functions.

The other important factor that should be noticed is the number of function evaluations, which reflects how many evaluation and how much computation time, is required to converge to the optimum airfoil shape with minimum total pressure losses. Obviously, the median of number of expensive function evaluations for surrogate assisted methods is intensely decreased in comparison with the RC-GA. The GAs, in general, need many CFD evaluations for convergence, which is not acceptable especially for expensive objective function evaluations in ASO. Hence because of the fact that most of the computational time is spent in the evaluation of the objective function, surrogate-assisted optimization procedures are promising search tools for real-world applications (Such as the optimization of turbomachinery blade geometries) in terms of the number of expensive function calls and consequently CPU effort. On the other hand, among the different features of the proposed metamodel-based approach, Kriging which is commonly expected to be more efficient shows the relatively better performance from the number of evaluations point of view. Nevertheless, it consumes more time to reach even higher function value due to its specific sampling rule. It can be also because of the particular way by which APRI searches for the global optima and sensitivity of KRG metamodels to the sample points' distribution and their distance. This might be even more crucial issue for such high-dimensional data where the search space is relatively large.

As evidenced in these tables, any feature of APRI presents better performance than the most efficient configuration of SEUMRE in which three surrogates including KRG, RBF and RSF metamodels are combined using a mechanism [34] to make a hybrid approximate model over the region of search. Considering all the criteria, however, it can be seen that RSF-assisted APRI performs well and outperformed all other methods for this high-dimensional application.

Lastly, with respect to the comparison shown in Table 4, performance of the proposed search method using RSF is compared with the results of the real coded genetic algorithm in terms of static pressure distribution as well as Mach number distribution around the optimized airfoil shapes. Illustrated results of one of 10 runs for each optimization algorithms are brought below just to show considerable difference as well as achieved improvement. Fig. 4 shows the profile of airfoil pressure surface and its design variables (NURBS polygons and control points) before and after aerodynamic shape optimization. The optimum curve has been generated after the optimization. It can be seen that the most important geometric changes are made on the second half of the airfoil chord toward the TE. Also, APRI and RC-GA propose two totally different profiles for the lower side of the existing airfoil.

Comparison of the whole airfoil geometry has been conducted as depicted in Fig. 5. The dotted line represents the un-optimized or starting (Datum) airfoil geometry. The dashed line represents the airfoil geometry obtained using RC-GA optimizer, while the continuous line shows the optimized airfoil geometry generated using APRI.

The most obvious conclusion that can be drawn from this figure is that optimization process has significantly changed the second half of the airfoil shape along the chord, i.e. from maximum thickness to TE.

Since the highest pick of velocity distribution occurs on the airfoil's SS, the influence of its geometry is prevailing in loss production. The effects of this geometric change can be seen in illustrations of pressure/velocity distribution. Figs. 6 and 7 show the static pressure distribution and Mach number distribution respectively where APRI, RC-GA and the un-optimized cases are compared.

As shown in Fig. 6, there are some significant differences in static pressure distribution. The optimum shape demonstrates a more moderate acceleration for the first half of the chord on the SS of the airfoil (Specifically from 25% to 50% of the chord), which leads to the considerable viscous loss reduction and prevention of boundary layer separation. This may result in sooner starting of diffusion compared with the datum as well as GA-based airfoils, due to the change of laminar-

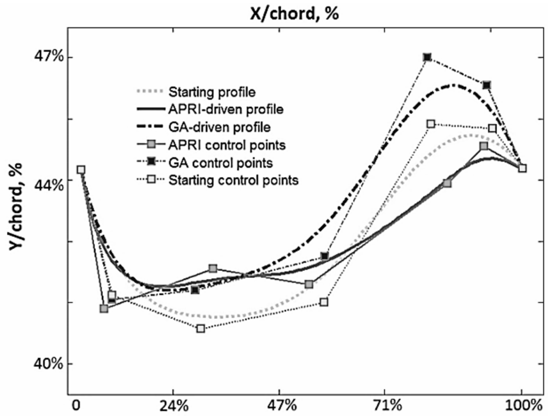

Fig. 4. Airfoil's PS profile and its design variables before and after aerodynamic shape optimization using APRI.

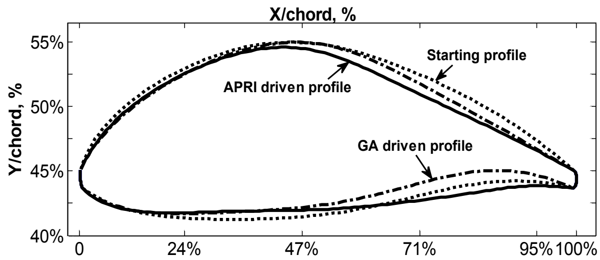

Fig. 5. Whole airfoil profile before and after optimization.

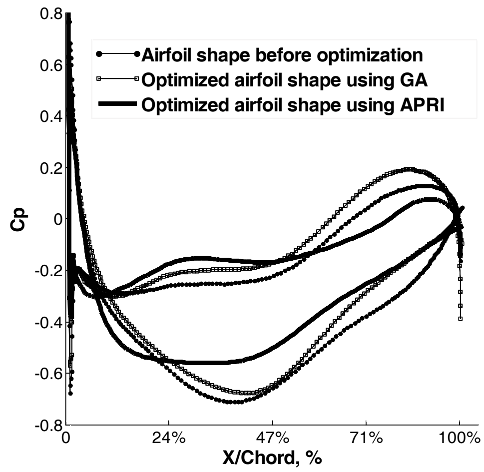

Fig. 6. Comparison of pressure distribution.

turbulent transition position. The location of this transition point is important because the dissipation rate sustains a faster increase after that point. At the same time, due to the movement of the maximum curvature position, Fig. 6 shows that shock on the SS (The minimum value on the bottom segment of the graph) significantly decreases within the APRI-driven optimum airfoil shape. Furthermore, the figure shows different patterns of the slope of the pressure curves, specifically for the pressure surface segment as a result of geometry changes.

As the entropy generation (Flow losses) can be supposed a

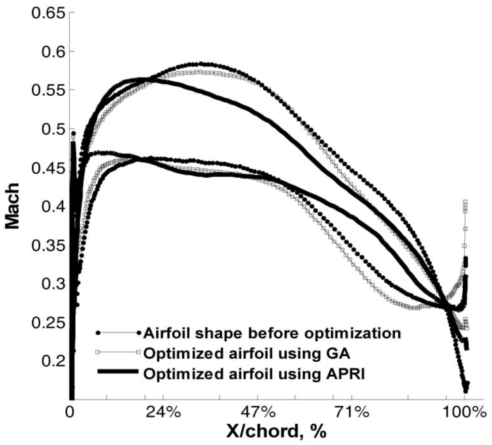

Fig. 7. Comparison of velocity distribution.

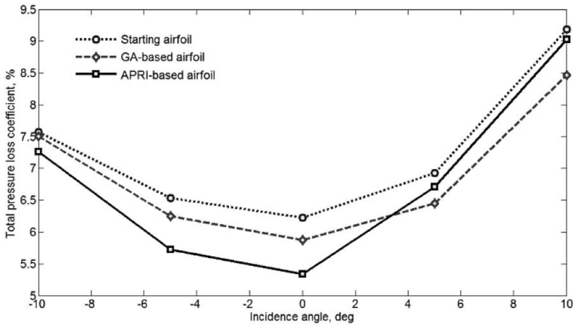

Fig. 8. Airfoils operating range; Datum design, GA optimized and APRI optimized shapes.

cubic function of the free-stream velocity, the Mach number distribution an issue of discussion.

It can be seen from Fig. 7 that the pick value of the Mach number distribution on the upper side of the airfoil is markedly lower than the pick value for the initial and GA-optimized airfoils. The fact that it takes place closer to the LE implies that the diffusion takes place sooner with respect to the GA-optimized profile. All these characteristics could reduce the possibility of flow separation results in rigorous justifications about flow quality improvement for the compressor stator.

The total pressure loss coefficient resulting from analysis of the initial airfoil as well as the optimum from incidence angle of -10 to +10 degrees have been shown in Fig. 8. As illustrated in this figure, the APRI-driven profile outperforms the current shape in both design and off-design performance (i.e. about 1% reduction in total pressure loss at design condition (0^o incidence angle) as well as increased operating range specifically for stall condition). While the airfoil's operating range for the stall condition is almost unchanged for profiles driven from RC-GA optimization, APRI noticeably shows better performance in this regard. Nevertheless, in terms of the goals to be achieved, as previously stated in the fitness function definition part, the figure addresses that RC-GA gives more improvement of the compressor airfoil off-design behavior in choke condition (Positive angles) comparing to APRI optimization.

As it was anticipated, APRI proves to be an efficient as well as robust algorithm even compared to the techniques which have been used in the optimization of aerodynamic shapes for a long time.

6. Conclusions

This study presents a contribution to the metamodel driven sampling optimization driven performance enhancement of turbomachinery airfoil targeting the minimization of the total pressure losses for the design condition as well as maximization of the airfoils operating range. The approach discussed in the paper introduces a modified metamodel-assisted optimization approach, namely APRI, applied to the shape optimization problem. The implementation involves coupling a computer code for the geometry parameterization of the airfoils' shape, a blade-to-blade flow-field analysis in a commercial CFD tool, and APRI optimization approach developed in this work. APRI explores the entire field of interest once and then focuses on one region instead of wondering in the whole design space checking regions one by one. In this way, the algorithm constructs one metamodel in the region of interest and not all over the design space which significantly reduces the computation effort of identifying the global optimum. The comparison of different features of search techniques shows that RSF driven APRI is found to be an efficient, robust and computationally affordable optimization algorithm for high-dimensional ASO problems that outperforms any other configuration of the algorithm, mixed metamodels-based SEUMRE method and real-coded GA.

The preliminary results, from the integrated application of these adaptive tools and methods, are considerably promising in terms of computation time, number of function evaluations, and finally the airfoil shape performance enhancement in both design and off-design conditions. In other words, the new profile derived by the proposed optimization algorithm represents extremely satisfactory performance over the whole operating range selected as a reference here ($\pm 10^{\circ}$) specifically for the stall condition. The whole proposed framework, consisting of geometric modeling, CFD analysis, and optimization algorithm, is extendable to the blade design optimization in a three-dimensional infrastructure.

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