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Customized dimensional analysis conceptual modelling framework for design optimization—a case study on the cross-flow micro turbine model

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ABSTRACT

Dimensional Analysis Conceptual Modelling (DACM) is a framework used for conceptual modelling and simulation in system and product designs. The framework is based on cause–effect analysis between variables and functions in a problem. This article presents an approach that mobilizes concepts from the DACM framework to assist solve high-dimensional expensive optimization problems with lower computational costs. The latter fundamentally utilizes theories and concepts from well-practised dimensional analysis, functional modelling and bond graphing. Statistical design-of-experiments theory is also utilized in the framework to measure impact levels of variables towards the objective. Simplifying as well as decomposing followed by optimization of expensive problems are the focuses of the article. To illustrate the approach, a case study on the performance optimization of a cross-flow micro hydro turbine is presented. The customized DACM framework assisted optimization approach converges faster and returns better results than the one without. A single-step simplification approach is employed in the case study and it returns a better average optimization result with about only one fifth of the function evaluations compared to optimization using the original model.

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Causality analysis; conceptual modelling; DACM assisted optimization; high-dimensional optimization; micro cross-flow turbine

1. Introduction

Dimensional Analysis Conceptual Modelling (DACM) is a framework developed essentially for conceptual modelling and simulation in engineering systems and product design aiming to simplify and solve engineering problems. It utilizes the well-practised theory and concepts of Dimensional Analysis (DA), functional modelling and bond graphing in conceptual modelling and model simplification (Coatanéa *et al.* 2016). Moreover, the framework employs the cause–effect logical analysis that the human mind uses in explaining the causalities between functions and factors affecting them as discussed by Kahneman (2011). The DACM framework's initial goal was to reduce the complexity of modelling problems and resolve issues associated with the system and product design as early as possible in the design process. DA theory, which has been used for more than a century in the mathematical modelling of various engineering and non-engineering problems (Islam and Lye 2009), is one of the useful tools in the framework. The Vashy–Buckingham Π theorem, among others, is one of

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the popular dimensional analysis tools (Barenblatt 1996; Islam and Lye 2009) utilized in the DACM framework. The theorem supports the creation of clusters of variables that are dimensionless. The DACM framework added to this initial theorem a supplementary benefit which allows selection of the meaningful dimensionless clusters corresponding to the objectives of the problem under consideration. This is obtained through generating causal graphs guiding the creation of dimensionless group. The causal graphs are generated using a combination of functional modelling and causal rules from bond graph theory (Paytner 2000). The qualitative objective of the problem under consideration is then propagated backward in the causal network employing different techniques and concepts in the DACM framework, such as the mathematical machinery developed by Bhashkar and Nigam (1990). This then allows contradictions between the variables in the problem to be detected, which in turn helps designers obtain a better understanding of the design weaknesses and find innovative solutions mitigating the contradictions. Moreover, the use of the statistical design-of-experiments concept, integrated into the causal graph generated, allows the DACM framework to compute the importance levels of different paths present in the causal graph. This, hence, provides an additional approach to decomposing and simplifying complex, high-dimensional and multi-disciplinary modelling and simulation problems.

On the other hand, the optimization of high-dimensional and multi-disciplinary engineering problems is becoming computationally intensive and expensive, which is the growing challenge that designers are facing regardless of advances in computational capabilities (Wang and Shan 2006). Therefore, new or customized approaches should be introduced in order to tackle such challenges. With their abilities to simplify and solve engineering problems, the techniques employed in the DACM framework could be customized to help decompose and simplify high-dimensional optimization problems. The customized framework could, hence, be employed to contribute in the computational cost reduction effort of the optimization of such computationally intensive high-dimensional problems.

In this study, optimization frameworks that utilize customized DACM frameworks are developed and presented. A customized framework decomposes and simplifies models of expensive high-dimensional optimization problems in order to help reduce the computational costs that would otherwise be incurred. In such frameworks, the Genetic Algorithm (GA) tool, the well-known metaheuristic global optimization tool, in MATLAB^{ő} version R2014b (Gen and Cheng 2000; The MathWorks 2014) is deployed to carry out the optimization. To illustrate the approach, a case study on the theoretical performance optimization of a cross-flow micro hydro turbine model is presented using its existing mathematical equations. Accordingly, the customized procedures in the DACM framework are presented in the next section (Section 2). The dimensional analysis theory and the causal network construction technique used in the framework for the backward propagation of qualitative objectives and for the variables' impact level designation are presented in Section 3, while Section 4 presents the proposed optimization frameworks. The details of the case study are presented in Section 5 followed by the results and discussions in Section 6. Finally, conclusions are drawn in Section 7.

2. Customize DACM framework

This section presents the customized procedures in the DACM framework that are utilized to simplify as well as decompose any existing models of optimization problems. The customized procedures of the process can be summarized in the following few fundamental steps.

Step 1: The first step of the framework is to list down and categorize the system variables based on the variable classification approach presented in the original DACM framework, such as: exogenous variables, independent design variables, dependent design variables, constant state variables, constraint variables, connecting variables, performance variables and objective variable(s) (Coatanéa *et al.* 2016).The independent design variables are those used as optimization parameters, while the exogenous variables are characteristic variables, which usually have constant values like the constant state variables. Dependent variables are intermediate variables in the model expressed in terms of the independent variables and any of the previous variable types. On the other hand, the performance variables are the crucial variables that have direct impacts on the objective variable(s) of the problem. Constraint variables, as the name implies, are variables that represent relations in constraint functions.

Step 2: All variables are assigned specific node shapes and/or colours based on the associated classifications of the system variables given in Step 1.

Step 3: Causal ordering of the variables is developed based on their causal relations.
This is the fundamental step in the DACM process. The cause–effect relations between variables presented in the form of a causal graph are defined during this phase. The various causal rules used in developing the causal graph are derived by integrating functional modelling, bond graph (Shim 2002) and dimensional analysis metrics (Shen and Peng 2006). Unlike the original intent of the DACM framework, which aimed to develop new models, existing models/empirical equations can be used and the variables' causal relations can be presented, as in the case study in this article.

Step 4: Associated qualitative objectives of the objective variable(s) is(are) determined and back propagated.
The qualitative objective(s) associated to the objective variable(s) are determined in this step. The qualitative objectives determined are then backward propagated to all variables in the causal graph built. These qualitative objectives are described by either one of the two terms, *i.e.* maximizing or minimizing.

Step 5: Contradictions are identified and located.
The process in Step 4 may generate contradictions between some of the independent variables in the causal graph (Ring 2014), which means that the resulting objective propagation, for instance, may lead to demanding that variables respond to contradictory objectives simultaneously (Warfield and Ring 2004). Those variables with or without contradictions are therefore identified at this stage.

Step 6: Independent variables with contradictions are identified.
Following the process in Step 5, this step dictates the separation of the independent variables with and without contradictions. Any variable without contradiction can now be eliminated from the variable list in the process. The first stage simplification is carried out in this step.

Step 7: The impact level of each variables towards the corresponding output objective variable(s) in the causal graph are computed.
This step is about computing the sensitivity of variables by carrying out a virtual design of experiment using the low and high order of magnitudes of the variables in the problem. The effects of the different variables impacting the performance variables, and hence the objective variable(s), are computed and the variables are ranked according to their impact level. It later helps to select potentially the most promising design directions. This is applied to qualitatively analysing and computing values to identify independent variables with relatively little effect on the objective function. Then the second stage simplification is carried out, which removes those optimization variables with least impact on the objective(s). Note again that removing means qualitatively determining the value from the given solution space of the particular variable. The model simplification process using the customized DACM framework is summarized using the process flowchart in Figure 1.

3. Dimensional analysis, causal network construction and impact level computation

Dimensional analysis, causal network and impact level computation are the essential theory and concepts employed in the frameworks. In the simplification process, the first two help realize the

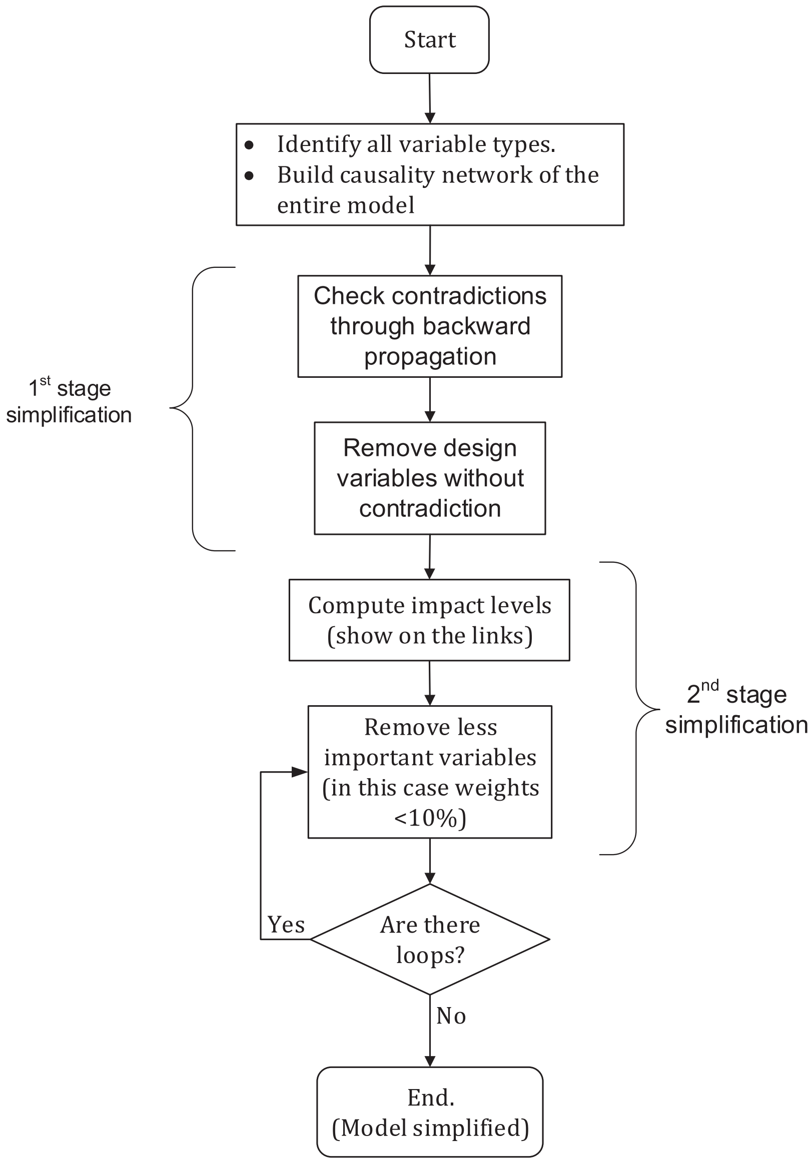

Figure 1. Process flowchart of DACM framework based simplification.

backward propagation of the qualitative objective of the objective function, while the causal network and impact level computation are utilized in the decomposition process later in the framework.

3.1. Dimensional analysis

The application of DA theory is wide spanning in physics and engineering (Barenblatt 1996; Matz and Krempff 1959; Maxwell [1873] 1892). Before carrying out any detailed modelling or simulation of engineering problems, the complexity can be reduced by employing DA (Bridgman 1963), which abides by the dimensional homogeneity principle. The latter principle is the most familiar principle in the theory of dimensional analysis. In the most familiar dimensional notation, for instance, force is represented as $M \cdot L \cdot T^{-2}$. Dimensional representations like this one are a combination of mass (M), length (L) and time (T). The famous expression of Newton's 2nd law, $F = m \cdot a$, with F (force), m (mass) and a (acceleration), is constrained by the dimensional homogeneity principle. By carrying out a dimensional check on both sides of Newton's 2nd law, the principle can be verified. Vashy–Buckingham's Π theorem is one of the widely used tools in dimensional analysis. It was first stated and proved by Buckingham in 1914 (Barenblatt 1996). The theorem is employed to identify the number of independent dimensionless variables that can characterize a given physical situation. By grouping the variables into dimensionless primitives, the complexity of the problem can then be simplified. Apart from that, the theorem is utilized in the framework to characterize the effect of the factors towards the objective of the problem under consideration. According to the theorem, every formulation that takes the form $y_0 = f(x_1, x_2, x_3 \ldots, x_n)$ can take the alternative form:

$$\pi_0 = f(\pi_1, \pi_2, \pi_3, \ldots, \pi_n), \tag{1}$$

where π_i are the dimensionless products. The form is obtained as a result of the Vashy–Buckingham theorem. For a typical function, dimensionless numbers basically take the product form shown in Equation (2):

$$\pi_k = y_i \cdot x_j^{\alpha_{ij}} \cdot x_l^{\alpha_{il}} \cdot x_m^{\alpha_{im}}, \tag{2}$$

where y_i represents the performance variables expressed as Equation (3), α_{ij} represents the exponents and x_i represents the repeating variables.

$$y_i = \pi_k \cdot x_j^{-\alpha_{ij}} \cdot x_l^{-\alpha_{il}} \cdot x_m^{-\alpha_{im}}. \tag{3}$$

The dimensionless form of the primitive presented in Equation (2) is utilized in the DACM framework. In their article, Coatanéa *et al.* (2016) discussed the concept of the DACM framework in detail. In the concept, the relation between the performance and repeating variables is shown by the sign of the exponents, α_{ij}. The signs are obtained from the partial differential equation relations, as shown in Equation (4). The equation relations, in turn, are obtained by employing the mathematical machinery developed by Bhashkar and Nigam (1990). This means that the performance variable (y_i) increases or decreases if α_{ij} is negative or positive, respectively, with respect to the corresponding repeating variable (x_i). This concept provides a powerful approach to propagating qualitative optimization objectives in the causal network and hence identifying any objective contradictions between the variables.

$$\frac{\delta y_i}{\delta x_i} = -\alpha_{ij} \frac{y_i}{x_i}. \tag{4}$$

3.2. Causal network construction

Causal network (graph) construction, which is based on the basic principles of functional modelling and bond graphing, is one of the important components that the DACM framework incorporates. It is introduced in the framework to solve the issue of selecting sensible combinations of dimensionless primitives for a set of variables in a specific engineering context (Coatanéa *et al.* 2016). The dimensionless primitives created using the Vashy–Buckingham theorem suffer from such fundamental limitation. In the framework, the causal network facilitates backward propagation of the qualitative objectives back to the design and optimization variables. In addition, the framework utilizes the network to structure the correlations between variables and to display the importance levels of the variables in the impact level computation process. The causal networks are derived in such a way that nodes with different shape and/or colours that represent different variable categories are connected by links with arrowheads. The arrowheads of the links direct to those variables that are affected by the variables at the foot nodes. Mathematical models with addition/subtraction relations are shown by +/− signs at the intermediate positions of the links, respectively.

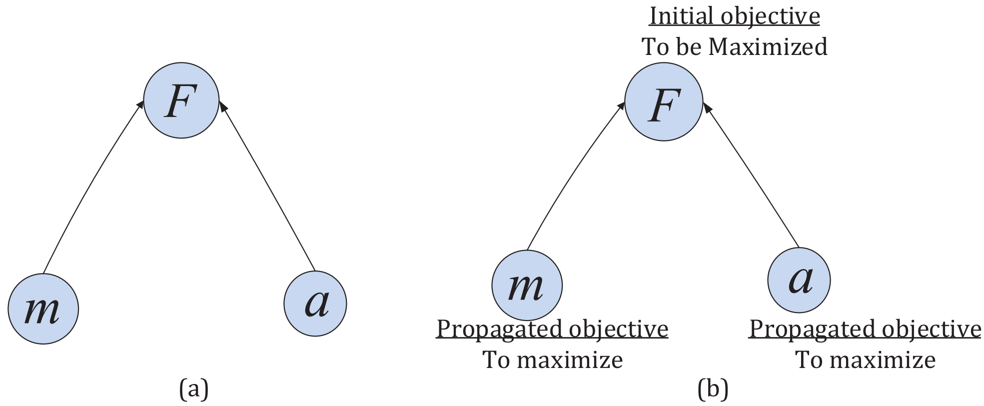

Figure 2. Simple causal network (a) causal relation between force, mass and acceleration, (b) objective propagation.

In order to demonstrate the process better, consider the causal relation in the famous Newton's second law expression, $F = m \cdot a$, discussed above. Force (F) is measured in newtons (N), mass (m) is measured in kilograms (kg) and acceleration (a) is measured in metres per second squared (m/s^2). To understand the objective propagation principle, the dimensionless product of the relation between F,m and a, Equation (5), is constructed, which shows how the causal network and the dimensionless primitives are related:

$$\pi_F = Fm^{-1}a^{-1}. \tag{5}$$

A causality is not embedded in such equations but in practice engineers usually consider certain causalities in the equations. This causality is usually influenced by how certain systems behave. For example, if a system uses a mass displacement with a certain acceleration to generate a force, then it embeds a certain causality associated with the function and behaviour of the system. The causality network for such behaviour is built as illustrated in Figure 2(a). Assuming an initial objective of maximizing the force and employing the relation in the partial derivative, Equation (4), the relation returns a result as given by Equation (6):

$$\begin{aligned} \frac{\delta F}{\delta m} &= 1\frac{F}{m} \\ \frac{\delta F}{\delta a} &= 1\frac{F}{a}. \end{aligned} \tag{6}$$

The result shows a direct relation of the objective variable and the impacting variable. Thus, the objective propagation is straightforward, as shown in Figure 2(b). Therefore, the framework proposes the utilization of dimensional analysis theory and its concepts in the causal graph construction, discussed above, to construct the network, hence to backward propagate the qualitative objectives of the objective functions.

3.3. Impact level computation

Weighting impact levels of the variables towards the objective variable(s) in the system is the other crucial procedure in the customized DACM framework-based decomposition process. It is used to rank the impact levels of each design variables and paths on the performance/objective variables. As discussed in Step 7 of Section 2, the impact level calculation helps to identify less important variables so that their values can be determined through qualitative evaluation. Therefore, as discussed in Section 4, the variables are then removed from the causal network and will not be considered as optimization parameters, at least in the single-step optimization framework. This leads to the decomposition of the problem variables into two clusters: important and less important variable clusters. Design-of-experiment methodologies are employed in the sampling and impact level computation process. In the current study, the well-known Taguchi method is utilized for the impact level calculations. This method is relatively simple to use compared to other variance methods, such as ANalysis Of VAriance (ANOVA) (Ghani, Choudhury, and Hassan 2004; Phadke 1989). The Taguchi method utilized follows a two-level design-of-experiment approach. For instance, assume $y = f(x)$, where x is a set of a total number of n input variables. Implementing the L8 two-level orthogonal array design sample allocation method for n variables, according to the Taguchi method, the effect of any of the variables x_i $(i = 1, 2, 3, \ldots, n)$ on y is computed using the expression given in Equation (7):

$$\text{Effect}_{x_i \text{ on } y} = \frac{\sum_{j=0,\ldots,t/2}^{\text{for } x_i \text{ at high level}} y \text{ at level 2}}{t/2} - \frac{\sum_{j=0,\ldots,t/2}^{\text{for } x_i \text{ at low level}} y \text{ at level 1}}{t/2}, \tag{7}$$

where n is the number of variables and t is the number of experiments. The weight of the effect of x_i on y is then computed using Equation (8):

$$\text{Weight}_{x_i \text{ on } y} = \frac{\text{Effect}_{x_i \text{ on } y}}{\sum_{k=1}^{n} \text{Effect}_{x_k \text{ on } y}}. \tag{8}$$

The weights computed using Equation (8) show the importance levels of each link between two causally dependent variables in a causal network. Thus, based on the limits set by the designer, the weights calculated are used to cluster the variables in different categories.

4. Customized DACM-framework-based optimization frameworks

This section presents the proposed optimization frameworks developed based on the theories and concepts discussed in the previous sections. The customized DACM-framework-based optimization frameworks provide mechanisms that utilize the simplified or both simplified and decomposed models, and thereby enable the computational costs that might be incurred by the optimization process of the original models to be reduced. The mechanisms also enable better optimization outcomes to be returned, particularly on expensive high-dimensional problems. As described in the previous sections, some optimization/design variables are cut out from the optimization parameter list based on their characteristics. This is done as a result of the objective propagation process and based on their impact level towards the objective. Two different optimization frameworks that are based on the simplified and decomposed models are presented in this section. The first framework is based on a simplified model after the first stage simplification process, while the second framework is based on models after the simplification process followed by the decomposition process, *i.e.* after the first- and second-stage processes.

The optimization frameworks presented in this section are called (a) single-step optimization, and (b) two-step optimization frameworks. These frameworks are summarized and highlighted in Figures 3(a,b), respectively. The single-step optimization framework computes the optimal solution utilizing the simplified model as a function of the variables retained from the simplification process only, while the two-step optimization sequentially utilizes models as a function of both the variables remaining from the simplification process as well as the less important variables cut out as a result of the impact level computation in the second stage decomposition process. In this study, the general optimization problem is formulated as given in Equation (9) and programmed in MATLAB's scripting tool. The GA optimization tool is utilized in both frameworks. Figure 3 shows optimization flowcharts. In the flowcharts, a general optimization problem $f(x_u, x_r)$ is assumed, where x_u represents the important variables remained un-removed after the simplification process and x_r represents the less important variables removed in the second stage decomposition process. The upper and lower

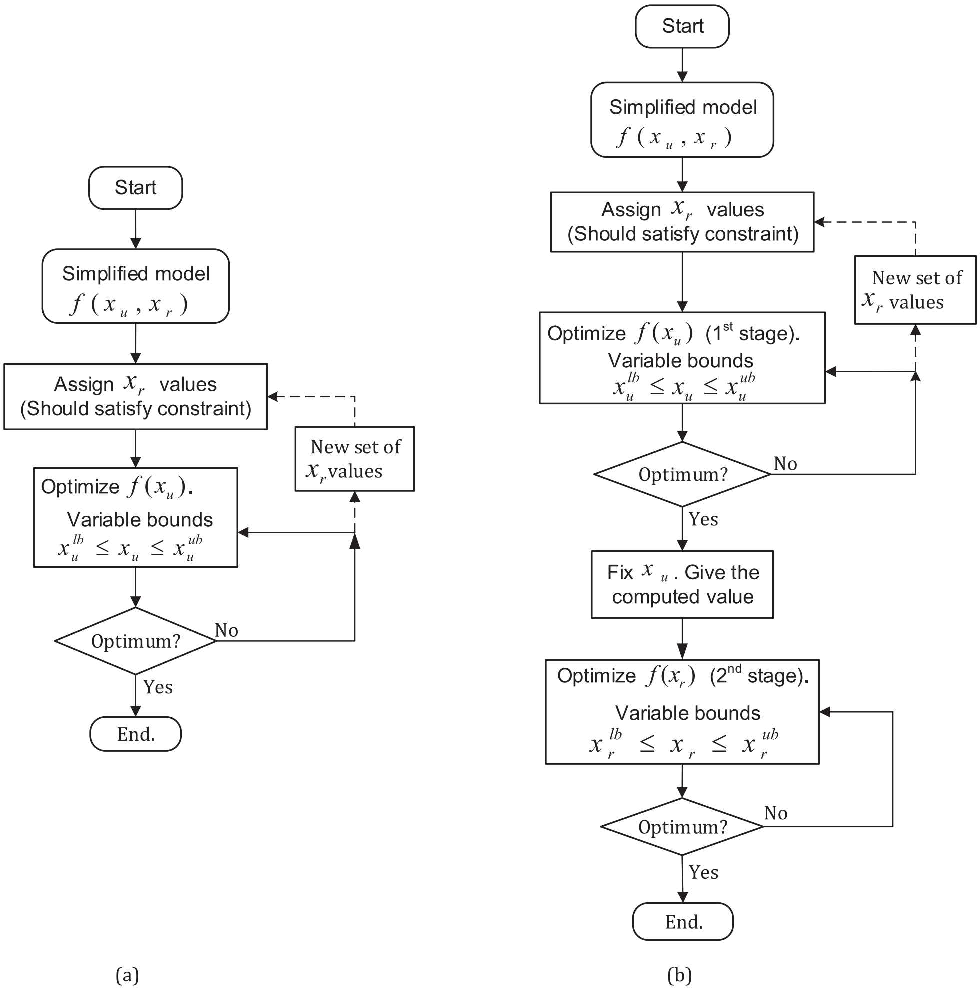

Figure 3. Customized DACM based optimization frameworks (a) single-step and (b) two-step.

bounds of the variables are given by $x_u^{lb} \leqslant x_u \leqslant x_u^{ub}$ and $x_r^{lb} \leqslant x_r \leqslant x_r^{ub}$:

$$\begin{aligned} \text{Function:} \quad & Y = f(x_r, x_u) \\ \text{Objective:} \quad & \text{Max} \quad Y \\ \text{Find:} \quad & (x_r, x_u). \end{aligned} \tag{9}$$

During the optimization process, the GA algorithm generates an initial population from the variables' design space, while cross-over and mutation techniques are used to generate the populations of the next generations until they converge to the global optimum value. Unlike the two-step optimization, the single-step optimization is expected to reduce the computational cost by a large amount. However, in some cases, it might compromise the magnitude of the optimization results.

5. Case study: cross-flow turbine

In this study, the proposed framework is employed to enhance the theoretical performance of cross-flow turbine design utilizing existing empirical equations based on reports presented by the master minds behind the original design of such a turbine type (Mockmore and Merryfield 1949). Improving the power output is the objective of the optimization process. The power output is given as a function of selected geometric variables. Optimization using the original and simplified models is carried out to measure the efficiency of the proposed optimization framework. In this case, the number of function evaluations is used to measure the performance of the methods.

The cross-flow turbine under consideration is a micro cross-flow hydro turbine. The configuration of such turbines is designed in such a way that the water entering the inlet passes through a nozzle and strikes the turbine's rotor, as illustrated in Figure 4. The rotor converts the fluid power to mechanical power. As such turbines are assumed to operate at atmospheric pressure, they are considered to be of impulse turbine type. Thus, the theoretical power transfer computation is carried out through changes in the fluid's kinetic energy. There are two stages of power generation in cross-flow turbines. At the first stage, the water jet leaving the nozzle strikes the rotor from the outer periphery and then leaves the blade. Thereafter, it crosses the inner space until it strikes the blade again at the second stage from the inner periphery before it exits the turbine through the outlet.

In the theoretical analysis, the fluid velocity leaving the nozzle, V_{in}, is calculated from a given fluid head using Equation (10):

$$V_{\text{in}} = C_v\sqrt{2gH}, \tag{10}$$

where H is the head of the fluid at the inlet of the nozzle, g is the acceleration due to gravity, and C_v is a dimensionless constant that accounts for losses across the nozzle. As shown in Figure 5(a), the blades in the rotor are placed around the periphery of circular discs. The size and shape of the profile of these blades and the radial size of the supporting discs are the important design parameters of the rotor that determine the turbine's power generating capacity (Desai and Aziz 1994; Sammartano *et al.* 2013). Once the water jet has left the nozzle, the rotor design is responsible for mechanical power generation. The entire turbine is represented by mathematical models that are developed employing

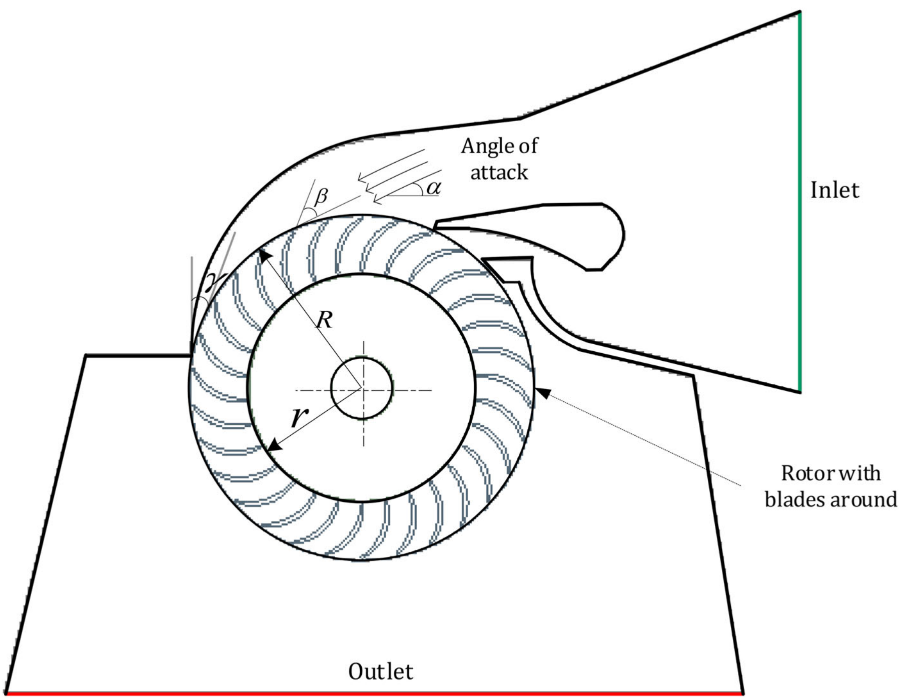

Figure 4. T15-300a cross-flow turbine geometric model.

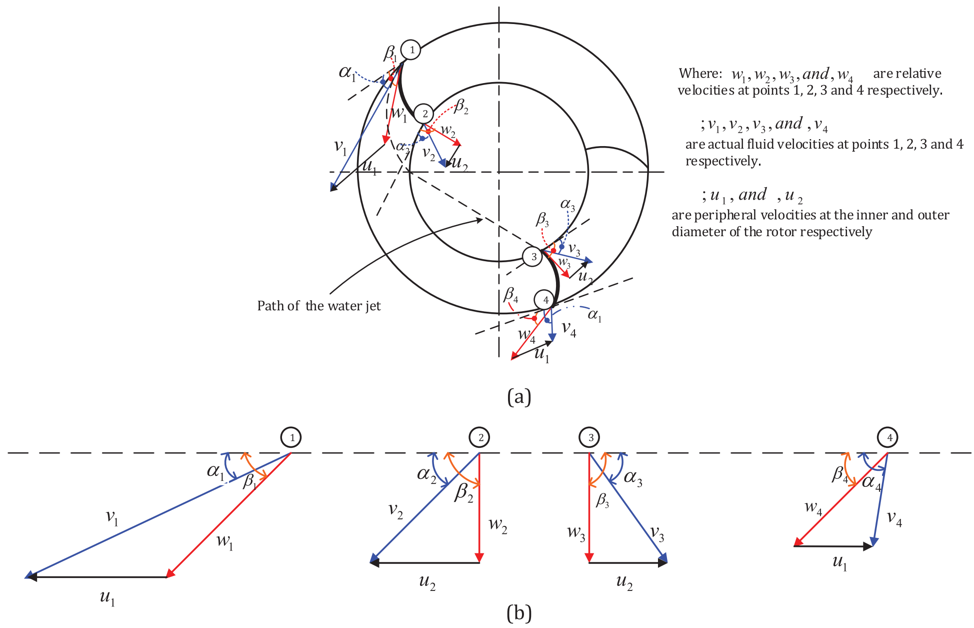

Figure 5. (a) Blade profile and trajectory of the water jet crossing the turbine's rotor; (b) velocity triangles at each point.

the concept of Euler's turbo-machinery equation. They are derived based on the following three basic assumptions:

- the turbine is a pure impulse turbine,
- the head difference between the inlet and exit due to the height difference is negligible, and
- the gravitational load on the stream inside the turbine is negligible.

When the water stream issued by the nozzle reaches the outer periphery of the rotor, it will have a velocity magnitude of V_{in} (from Equation 10) and it makes an angle of attack α from the tangent to the periphery. The extracted velocity triangles at points 1, 2, 3 and 4, are shown in Figure 5(b).

In the analysis, the magnitude and direction of the relative velocities are dependent on the geometry of the rotor. In the derivation, it is assumed that, at the entry of the rotor, the angle that the relative velocity makes with the tangent to the periphery should be equal to the blade angle, *i.e.* $\beta_1 = \beta$. The theoretical velocity and torque equations at the 1st and 2nd stages are then given by Equations (11)–(18). From the velocity triangles, the velocity components of the fluid at each location of the rotor are computed using the relations given by Equations (11)–(14):

$$V_{u1} = V_{\text{in}} \cos \alpha_1 \tag{11}$$

$$V_{u2} = V_2 \cos \alpha_2 = U_2 = \omega r \tag{12}$$

$$V_{u2} = U_2 = U_3 = U_{u3}, \tag{13}$$

where $\alpha_1 = \alpha$ is the angle of attack; U_2 and ω are the rotor's peripheral and angular velocities at its entry, respectively. $V_{\text{in}} = V_1$, where V_1–V_4 represent the actual fluid jet velocities at locations 1–4, respectively. From the velocity diagram, and by applying the cosine law, the relation for V_{u4}, is expressed by Equation (14). In this derivation, it is taken that $W_4 \approx \phi W_1$, which is based on one of the commonly used assumptions for the turbine type (Durgin and Fay 1984; Pereira and Borges 2014).

$$V_{u4} = \phi(\sqrt{V_1^2 + U_1^2 - 2V_1U_1\cos\alpha_1})\cos\beta_1 - U_1, \tag{14}$$

where ϕ is an empirical coefficient that accounts for losses inside the blades between points 1 and 4. A study by Mockmore and Merryfield (1949) reported an approximate value of $\phi \approx 0.98$. The same article used a limiting expression, Equation (15), in the design to avoid back pressure while the water jet is cross-flowing across the rotor:

$$z^2 + z\tan^2\beta - \tan^2\beta = 0, \tag{15}$$

where z is the square of the radius ratio, $(r/R)^2$, and from the study in the article, it is also indicated that, for optimum efficiency, the value of z should fall in the range 0.6–0.69.

Based on Euler's turbo-machinery equation, which is based on the conservation of impulse from Newton's 2nd law, Equation (16), the theoretical power of cross-flow turbines is computed. It was assumed that the empirical coefficient, ϕ, also accounts for the loss due to the entrained fluids by some of the blades.

$$F = \rho Q\int_2^1 \mathrm{d}v = \rho Q(V_1 - V_2). \tag{16}$$

Employing the expression in Equation (16) and multiplying it by the corresponding torque arm, the torque transfers of the two stages are given by Equations (17) and (18):

$$T_{12} = \rho Q(RV_{u1} - rV_{u2}) \tag{17}$$

$$T_{34} = \rho Q(RV_{u3} - rV_{u4}), \tag{18}$$

where T_{12} and T_{34} are the torques at the 1st and 2nd stages, and ρ and Q are the density and volume flow rates of the fluid, respectively.

The total theoretical output power, P_{out}, is then given by Equation (19):

$$P_{\mathrm{out}} = (T_{12} + T_{34})\omega. \tag{19}$$

The independent design variables from the mathematical models are summarized in Table 1. The minimum and maximum bounds of the turbine diameters are obtained from the configuration design manual of T15-300a, one of the T-series cross-flow turbine design model (ENTEC 2014; Protel Multi Energy 2012). Considering the design geometry, roughly half of the blade's height (±0.051 m) is used to determine the bounds from the actual value, except for the upper bound of the outer diameter. The actual outer diameter ($D = 2R$) and inner diameter ($d = 2r$) values from the turbine design document are 0.302 and 0.204 mm, respectively. Moreover, a 10 m water head at a rotational speed of 350 rpm and a flow rate of 1 m^3/s are retained as initial design conditions in the entire computation.

Additionally, similar parameter values in the GA tool setting are used in all runs and the same workstation is employed for the computation in all cases.

Table 1. Lower and upper bounds of the optimization parameters.

Variable	Lower bound	Upper bound
α (degrees)	15	23
β (degrees)	20	30
D (m)	0.251	0.302
d (m)	0.153	0.257

6. Results and discussion

This section discusses the optimization results from both the original and the customized DACM assisted optimization approach presented. After carrying out a dimensional analysis of the individual expression equations in the theoretical analysis (see Table A1 in Appendix), the variable types are identified and associated with node shapes and colours based on the concept in Step 2 of Section 2. The list of variables, their category, associated nodes and colour designations are shown in Table 2.

Moreover, a causal graph example using the empirical equation of one of the dependent variables, T_{12} in Equation (17), is shown in Figure 6. This graph gives a clear view of how the overall system causal network is constructed. In the graph, T_{12a} and T_{12b} represent the first and the second terms in Equation (17), respectively.

6.1. Simplification result based on customized DACM framework

Based on the procedures, the causal graph of the model is constructed and the qualitative objective is propagated backward as shown in Figure 7. Thereafter, the independent variables with and without contradictions are identified based on the procedures in Steps 3 and 4 of Section 2. In this case, it has been observed that two independent variables (α, β) have no contradicting objectives. Thus, the only two independent optimization variables remaining are the two geometric parameters (D, d), which

Table 2. Variable categories, nodal shape and colour designations.

Category	List of variables	Nodes and colour
Objective variable	P_{out}	light blue ellipse
Exogenous variables	g, H, Q, ρ, ω	grey circle
Dependent variables	$V_{\text{in}}, V_{u1}, V_{u2}, V_{u3}, V_{u4}, V_{u4a}, V_{u4b}, T_{12}, T_{12a}, T_{12b}, T_{34}, T_{34a}, T_{34b}, T_{\text{tot}}$	light grey square
Independent variables	α, β, D, d	grey hexagon
Constant state variables	C_v, ϕ	green circle
Connecting variables	$\cos\alpha, \cos\beta$	light grey triangle
Constraint variable	z	yellow circle

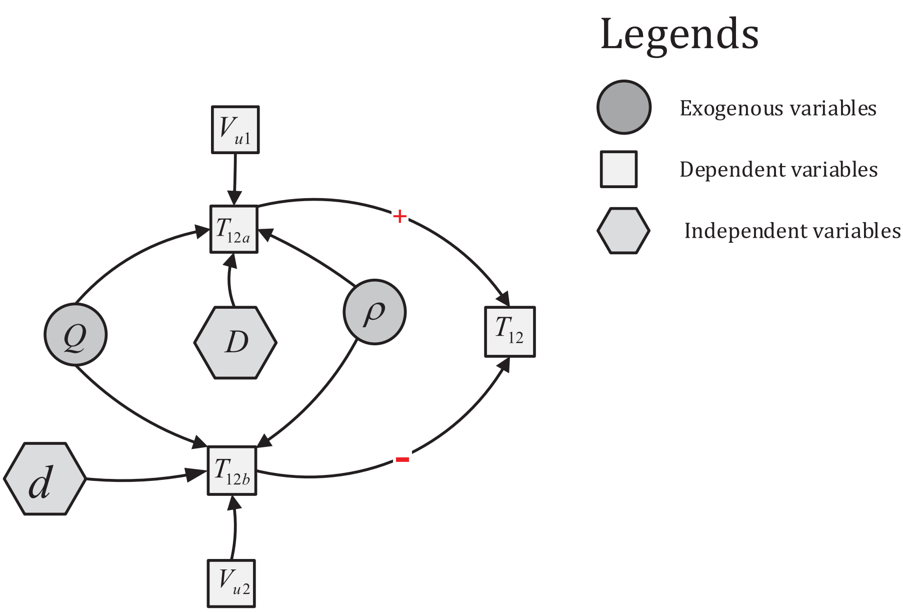

Figure 6. Causal graph example.

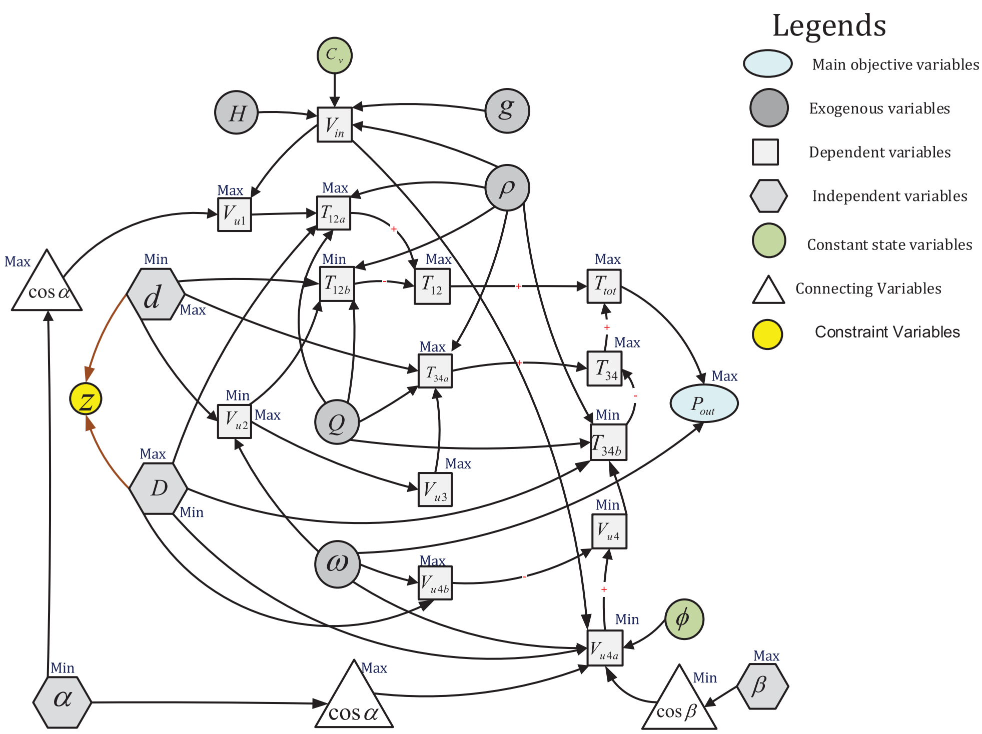

Figure 7. Objective propagated causal graph.

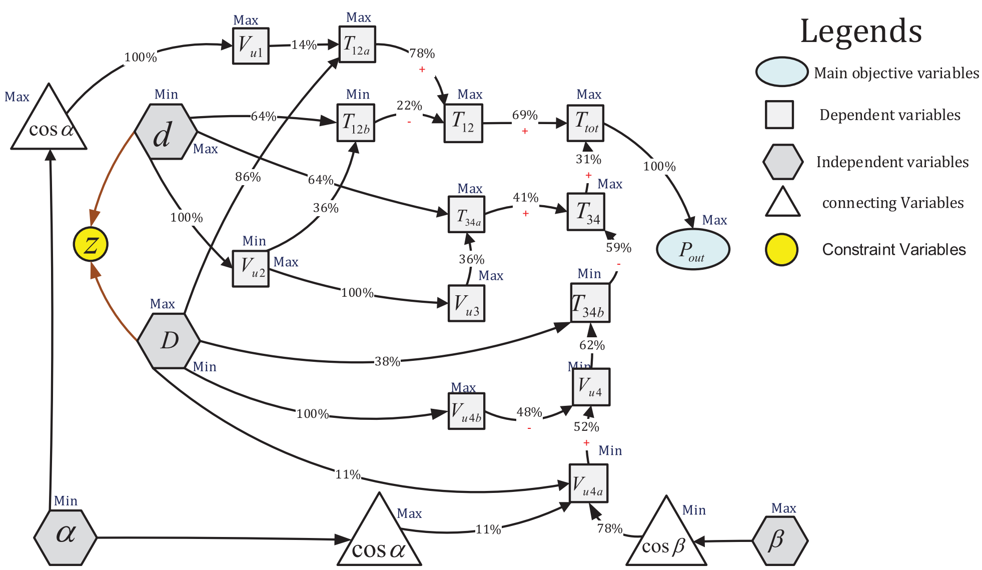

Figure 8. Weighted causal graph based on the experimental design method.

show contradictions. The functions that represent the theoretical model are thus simplified (1st stage). Initial simplification of the causal graph with weighted links is illustrated in Figure 8.

Depending on the complexity of the problem, one can further simplify problems by identifying and removing variables with little impact on the objective. The weights of the variables in the system

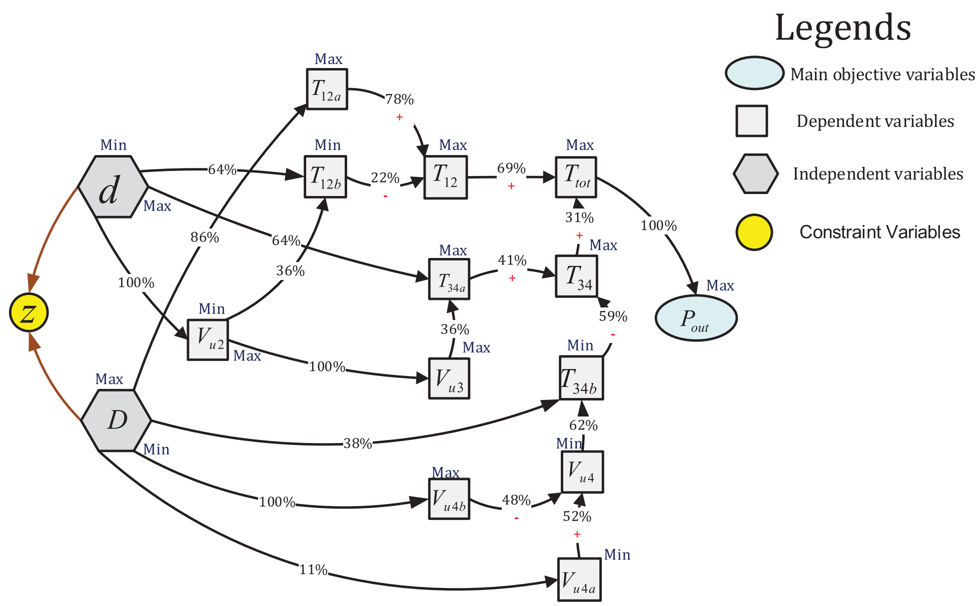

Figure 9. Final simplified causal graph.

are computed by applying the Taguchi method. After carrying out impact level computation, the remaining variables are maintained, because according to the values they are quite as important as the others. Therefore, the final simplified causal graph from the customized DACM approach is shown in Figure 9.

6.2. Optimization results based on the customized DACM simplification framework

Based on the 1st stage simplified model, the optimization process is given as a function of the remaining two geometric variables only, since the variables that describe the entering fluid property have no contradictions. The simplified optimization problem of the theoretical hydraulic power model in the single-step optimization method has the form shown in Equation (20):

$$\begin{aligned} \text{Function}: &\quad P_{\text{out}} = f(D, d) \\ \text{Objective}: &\quad \text{Max} \quad P_{\text{out}} \\ \text{Find}: &\quad (D, d). \end{aligned} \tag{20}$$

This implies that, in this single-step optimization, the values of the two variables (α, β) are determined qualitatively according to the propagated objectives and the values of the other two (D, d) are searched in the optimization process from their given design spaces. The performance optimization results, employing the global optimization tool GA in MATLAB, are illustrated in Table 3.

Table 3. Optimization results of the original model and a model from the single-step approach.

	Design variables				P_{out}		Computation results
Model	α	β	D	d	Targ. Val.	Obj. Func. Val.	No. of Func. Evals
Original	15	25.232	0.3020	0.2423	5,7891	57,891.0	10,971.8
Single step	15	30	0.3009	0.2139	57,891	63,660.1	2,132.8

In the case study, the optimization result from the original model is used as a target value, stopping criterion, in the single-step optimization. Keeping similar settings in all computations, the optimization results in Table 3 are the average result values from 10 independent runs in each case. The results show that the number of function evaluations in the single-step optimization of the simplified model is reduced to less than one fifth of the total number of function evaluations of the original model. Moreover, the single-step optimization returned a better objective value than the optimization result from the original model. Some of the reasons could be that: (i) in the single-step model, the variables without contradiction are qualitatively given the possible optimum values from the bounds based on the propagated objectives; and (ii) the number of optimization parameters is reduced, which gives the tool a better exploration capacity on the remaining parameters.

7. Conclusions

The article presents optimization approaches to how to simplify and decompose expensive high-dimensional optimization problems using a customized Dimensional Analysis Conceptual Modelling (DACM) framework. It presents two different optimization frameworks that use the customized DACM framework. It also highlights the benefits of the customized DACM assisted optimization frameworks introduced using a case study.

In the case study, the DACM assisted optimization returned better outcomes in simplifying the problem and reducing the number of function evaluations, thereby reducing the associated computational costs. In both the original and the simplified models of the case study, the well-known metaheuristic global optimization tool, the genetic algorithm, is employed in MATLAB. The total number of function evaluations in the single-step optimization of the simplified model has been reduced to about one fifth of the total number of function evaluations of the optimization of the original model. Moreover, it returned an average objective value better than the average value obtained from optimizing the unsimplified original model. Thus, the customized DACM based simplification and optimization enables the associated computational costs to be reduced while returning a better objective result. An automated framework for the approach is therefore recommended for expensive high-dimensional optimization problems.

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Disclosure statement

No potential conflict of interest was reported by the authors.

References

Barenblatt, G. I. 1996. *Scaling, Self-Similarity, and Intermediate Asymptotics: Dimensional Analysis and Intermediate Asymptotics*. Cambridge, UK: Cambridge University Press.

Bhashkar, R., and A. Nigam. 1990. "Qualitative Physics Using Dimensional Analysis." *Artificial Intelligence* 45: 73–111. doi:10.1016/0004-3702(90)90038-2

Bridgman, Percy Williams. 1963. *Dimensional Analysis*. New Haven, CT: Yale University Press.

Coatanéa, E., R. Roca, H. Mokhtarian, F. Mokammel, and K. Ikkala. 2016. "A Conceptual Modeling and Simulation Framework for System Design." *Computing in Science and Engineering* 18 (4): 42–52. doi:10.1109/MCSE.2016.75

Desai, V. R., and N. M. Aziz. 1994. "Parametric Evaluation of Cross-Flow Turbine Performance." *Journal of Energy Engineering* 120: 17–34. doi:10.1061/(ASCE)0733-9402(1994)120:1(17)

Durgin, W. W., and W. K. Fay. 1984. "Some Fluid Flow Characteristics of a Cross-Flow Type Hydraulic Turbine." In *Proceedings of the AMSE Winter Annual Meeting on Small Hydro Power Fluid Machinery*, 77–83.

https://www.frenchriverland.com/some_fluid_flow_characteristics_of_a_cross_flow_type_hydraulic_turbine-_durgin_&_fay.htm.

ENTEC. 2014. "Entec—Model T-15—High-Efficiency Cross-Flow Turbines." Expert Agriculture. St Gallen, Switzerland: Entec AG Consulting and Engineering.

Gen, Mitsuo, and Runwei Cheng. 2000. *Genetic Algorithms and Engineering Optimization*. Vol. 7 in the series *Engineering Design and Automation*. New York: Wiley InterScience.

Ghani, J. A., I. A. Choudhury, and H. H. Hassan. 2004. "Application of Taguchi Method in the Optimization of End Milling Parameters." *Journal of Materials Processing Technology* 145 (1): 84–92. doi:10.1016/S0924-0136(03)00865-3

Islam, M. F., and L. M. Lye. 2009. "Combined Use of Dimensional Analysis and Modern Experimental Design Methodologies in Hydrodynamics Experiments." *Ocean Engineering* 36 (3-4): 237–247. doi:10.1016/j.oceaneng.2008.11.004

Kahneman, Daniel. 2011. *Thinking, Fast and Slow*. New York: Farrar, Straus and Giroux.

Matz, W., and R. Krempff. 1959. *Le Principe de Similitude en Génie Chimique* [The Principle of Similarity in Chemical Engineering]. Paris: Dunod.

Maxwell, James Clerk. (1873) 1892. A Treatise on Electricity and Magnetism, 3rd ed., edited by J. J. Thompson. Vol. II. Oxford, UK: Clarendon Press.

Mockmore, C. A., and F. Merryfield. 1949. *The Banki Water Turbine*, Bulletin Series No. 25. Corvallis, OR: Engineering Experiment Station, Oregon State College, Oregon State System of Higher Education.

Paytner, H. M. 2000. "The Gestation and Birth of Bond Graphs." https://www.me.utexas.edu/ longoria/paynter/hmp/Bondgraphs.html.

Pereira, Nuno H. C., and J. E. T. Borges. 2014. "A Study on the Efficiency of a Cross-Flow Turbine Based on Experimental Measurements." Paper Presented at the 5th International Conference on Fluid Mechanics and Heat & Mass Transfer (Fluidsheat'14), Lisbon, Portugal, October 30-November 1. http://www.wseas.us/e-library/conferences/2014/Florence/MECH/MECH-07.pdf.

Phadke, M. S. 1989. "Quality Engineering Using Design of Experiments." In *Quality Control, Robust Design, and the Taguchi Method*, 31–50. Pacific Grove, CA: Wadsworth & Brooks/Cole Advanced Books & Software.

Protel Multi Energy. 2012. "Micro Hydro Unit." West Java, Indonesia: Protel Multi Energy. http://www.pme-bandung.com/7-electronic-load-controller-and-micro-hydro-turbines-mikro-hidro-piko-hidro-cross-flow-elc-hydro-micro-hydro-unit.

Ring, J. 2014. "Discovering the Real Problematic Situation: The First Aspect of Conceptual Design." *Insight (American Society of Ophthalmic Registered Nurses)* 17 (4): 11–14. doi:10.1002/inst.201417411

Sammartano, V., C. Aricó, A. Carravetta, O. Fecarotta, and T. Tucciarelli. 2013. "Banki–Michell Optimal Design by Computational Fluid Dynamics Testing and Hydrodynamic Analysis." *Energies* 6 (5): 2362–2385. doi:10.3390/en6052362

Shen, Q., and T. Peng. 2006. "Combining Dimensional Analysis and Heuristics for Causal Ordering." In *Rob Milne: A Tribute to a Pioneering AI Scientist, Entrepreneur and Mountaineer*, edited by A. Bundy and S. Wilson, Vol. 139 of the series *Frontiers in Artificial Intelligence and Applications*, 39–53. Amsterdam: IOS Press Ebooks.

Shim, T. 2002. *Introduction to Physical System Modelling Using Bond Graphs*. Dearborn, MI: University of Michigan-Dearborn Press.

The MathWorks. 2014. *Downloads. Latest Release*. Natick, MA: The MathWorks, Inc. https://www.mathworks.com/downloads/.

Wang, G. G., and S. Shan. 2006. "Review of Metamodeling Techniques in Support of Engineering Design Optimization." *Journal of Mechanical Design* 129 (4): 370–380. doi:10.1115/1.2429697

Warfield, J. N., and J. Ring. 2004. "Understanding Complexity: Thought and Behavior." *Insight (American Society of Ophthalmic Registered Nurses)* 6 (2): 44–45. doi:10.1002/inst.20046244a

A. Appendix. Variables' nomenclatures and their dimensional analyses

Table A1 shows the nomenclature of the variables and their dimensional analyses, wherein M represents mass, L represents length and T represents time in the dimensional analysis representation.

Table A1. Nomenclature of the variables and their dimensional analyses.

Parameters	Nomenclature	Dimensional analysis
Output power	P_{out}	ML^2T^{-3}
Fluid head at inlet	H	L
Fluid flow rate	Q	L^3T^{-1}
Density of fluid	ρ	ML^{-3}
Torque at 1st stage	T_{12}	ML^2T^{-1}
Torque at 2nd stage	T_{34}	ML^2T^{-1}
Fluid inlet (attack) angle	α	–
Angle between relative velocity W and peripherals U	β	–
Second stage blade outlet angle (blade back side tangent angle)	γ	–
Outer diameter of rotor disc	D	L
Inner diameter of rotor disc	d	L
Inlet velocity	V_{in}	LT^{-1}
Peripheral component of actual velocity	V_1	LT^{-1}
Rotational speed	ω	T^{-1}
Gravitational acceleration	g	LT^{-2}