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Reliability Based Design Optimization on Qualitative Objective with Limited Information

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ABSTRACT

Reliability based design optimization (RBDO) algorithms typically assume a designer's prior knowledge of the objective function along with its explicit mathematical formula and the probability distributions of random design variables. These assumptions may not be valid in many industrial cases where there is limited information on variable variability and the objective function is subjective without mathematical formula.

A new methodology is developed in this research to model and solve problems with qualitative objective functions and limited information about random variables. Causal graphs and design structure matrix are used to capture designer's qualitative knowledge of the effects of design variables on the objective. Maximum entropy theory and Monte Carlo simulation are used to model random variables' variability and derive reliability constraint functions. A new optimization problem based on a meta-objective function and transformed deterministic constraints is formulated, which leads close to the optimum of the original mathematical RBDO problem.

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The developed algorithm is tested and validated with the Golinski speed reducer design case. The results show that the algorithm finds a near-optimal reliable design with less initial information and less computation effort as compared to other RBDO algorithms that assume full knowledge of the problem.

1. INTRODUCTION

Design optimization helps to minimize costs or maximize performances of to-be developed artifacts and systems. Optimization algorithms normally search for the optimal values of design variables under a group of constraints. Design variables include controllable design variables and uncontrollable surrounding parameters [1].

Reliability based design optimization (RBDO) algorithms solve optimization problems with random design variables and probability constraints. The majority of RBDO algorithms operate under the assumption that users have prior knowledge of random variable distributions and mathematical formulas of the objective function and constraints [2]. This prior knowledge is often not available in most industrial applications. Therefore, RBDO methodologies with insufficient information have been developed [3]. The term “insufficient information” was restrained to indicate the lack of information about probability distributions of random variables.

In this research, the authors propose an extended scope of limited information assumption to include both the objective function and the distributions of random design variables, in order to address a wider range of real-world RBDO problems.

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Limited information of the objective function means the designer has limited knowledge of the objective function and its mathematical formula. However, he/she normally has adequate experience in the qualitative logical relationships between the design variables, intermediate variables, and the objective function. An example of this case is the customer satisfaction maximization problem, where the objective function is qualitative human perception with no proved mathematical expression. However, designers can involve focus groups and gain an understanding of the causal relationship between design variables such as product features and the customer satisfaction objective. The RBDO involving qualitative objective function has not been investigated in the literature to the best of authors' knowledge.

Limited information on random design variables' variability means that the probability distribution of design variables is unknown due to either limited available historical data [4] or non-standard variability shown by historical records.

There are various approaches in literature to deal with limited information of variable variability, including the following:

- Assume all design variables and functions of them are normally distributed. Many RBDO algorithms have been developed based on this assumption [5,6].
- Apply interval analysis using the known range of each design variable [7–12].
- Apply possibility based design optimization where the possibility theory is utilized to derive alternative possibility constraint formulas [4,13,14].
- Apply evidence theory where the epistemic design variables variability is described by determining a basic probability assignment (BPA) for each

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interval of the variable range. BPA expresses the degree of evidence supporting the claim that the variable lies in the corresponding interval [15].

- Apply Bayesian RBDO where design variables are divided into epistemic and aleatory based on the available knowledge. A number of trials are performed on the aleatory variable to construct a probability table. The reliability of each constraint is then calculated for each value of the probability table [16].

In this research, the authors exploit the available information about variable variability using Shanon's maximum entropy theory to represent variables randomness with bounded uniform and triangular distributions.

If constraints are expressed as functions of random and bounded design variables, they would be random and bounded functions as well. There is no general formula for probability distributions of functions of triangular and uniform random variables. This problem has received limited attention in the literature probably due to the mathematical complexity. Archived research in this area is limited to simple special cases like summing a number of uniform random variables [17], summing two triangular random variables [18], and the product of two triangular random variables [19]. Alternatively, the authors in this research explore the probability distribution characteristics of constraint functions using Monte Carlo simulation. These characteristics are used to construct constraint functions' approximate cumulative distribution formulae, which are used to transform probabilistic constraints into a deterministic form.

In this research, causal graphs [20] are used to model the available knowledge of logical relationships between design variables and the objective function. A meta-

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objective formula is derived to represent the known logical relationships. The developed meta-objective formula combined with the transformed constraints represents a meta-alternative deterministic formulation of the RBDO problem.

This paper is composed of five sections. In Section 2, meta-objective function development from qualitative knowledge is described. In Section 3, transforming RBDO probabilistic constraints into a deterministic form is explained. In Section 4, a case study of the developed algorithm applied to Golinski gear reducer is shown. The results are compared to a number of previously developed RBDO algorithms that assume full knowledge of the problem. Section 5 includes the conclusion and prospective applications of the developed algorithm.

2. Meta-objective function formulation

In this section, the proposed algorithm for deriving the meta-objective function is explained with application to a simple example, given the qualitative logical relationships between design variables and the objective function.

1- Logical relationship could be modeled by the strength of the causal effect between two entities. Since we assume that there is no available quantitative information about such a relationship, an expert's subjective evaluation would be used. A popular 5-level Likert scale [21–25] is employed in this research to quantify the effect as shown in Figure 1:

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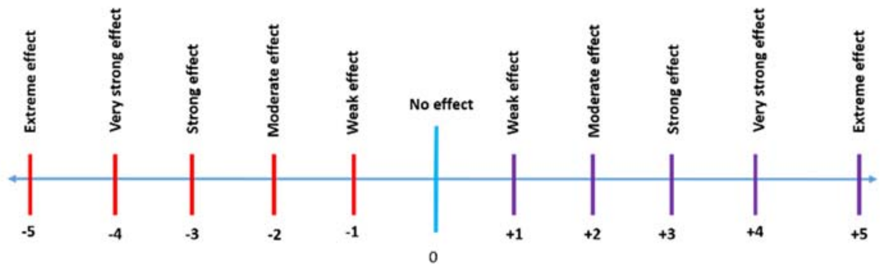

Figure 1 The 5- Level Likert scale

Where "5" indicates an extreme effect, "0" indicates no effect, and numbers in-between indicates effect strengths between the two extremes. The positive sign indicates a positive effect and the negative sign indicates a negative effect. It is assumed that all effects are monotonic in the known design range similar to the typical assumption of 2-level Design of Experiments [26].

During this step, the cause-effect relationships between variables and the objectives are defined in the form of a causal graph [27]. A simple example explaining the combined application of Likert scale and a causal graph is shown in Figure 2 where designer experience indicates that x_1 has an extremely positive effect on x_3; x_2 has a strong negative effect on x_3; x_1 has a very strong positive effect on objective Y; and x_3 has a moderate positive effect on Y.

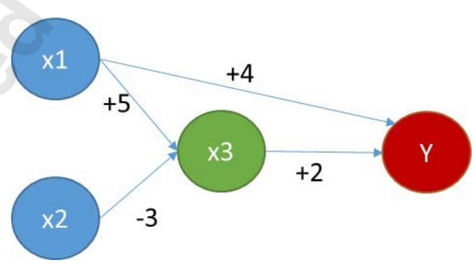

Figure 2 An example of causal graph.

2- The above-indicated effects are represented in a design matrix $\boldsymbol{A}$ as shown in Figure 3 where the diagonal elements $a_{i,i}$ corresponding to design variables x_i

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(x_1 and x_2 in this case) are set to 1 and all other diagonal elements are set to 0. Elements $a_{i,j}$ represent the effect of variable x_i on variable x_j.

	x_1	x_2	x_3	y
x_1	1	0	5	4
x_2	0	1	-3	0
x_3	0	0	0	2
y	0	0	0	0

Figure 3 Design matrix derived from causal graph for the example.

3- Each design variable x_i affects response y directly and indirectly through paths emanating from x_i leading to y. Therefore, the total effect of x_i on response Y is quantified by the sum of weight products of all paths originated from x_i leading to the response y.

For example, in Figure 2 , x_1 affects y through two paths. Therefore, its effect$=$ $[+4] + [(+5) \times (+2)] = 14$. While x_2 affects y through one path, its effect on Y is $[(-3) \times (+2)] = -6$.

Mathematically, the total effect of design variables on y could be calculated by a sequence of matrix multiplications of design matrix **A** by the effect vector $\boldsymbol{X}$ until

$$\boldsymbol{AX} = \boldsymbol{X} \tag{1}$$

Where the effect vector is set initially to be the last column of matrix $\boldsymbol{A}$ (corresponding to the target response y) and it is updated at each step with the matrix multiplication result as shown in Figure 4. From Eq. (1), the final effect vector $\boldsymbol{X}$ is $\boldsymbol{A}$'s Eigen vector corresponding to Eigen value 1.

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In case there is no feedback loop, the design matrix will assume a triangular form. Therefore, it has Eigen values equal to the values of its diagonal elements [28], 0 and 1 in this case. Consequently, the loop in Figure 4 is convergent.

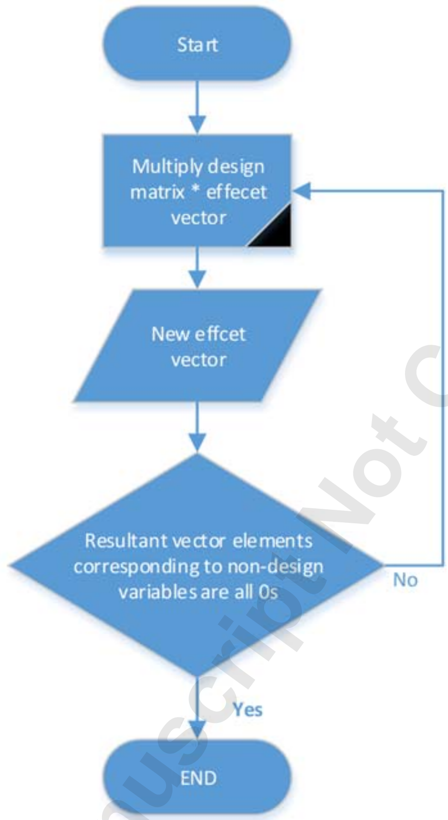

Figure 4 Eigen vector calculation steps.

Applying the above algorithm to our example:

$$\begin{bmatrix} 1 & 0 & 5 & 4 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 14 \\ -6 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 5 & 4 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 14 \\ -6 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 14 \\ -6 \\ 0 \\ 0 \end{bmatrix}$$

4- The final effect vector is normalized based on the absolute values of its elements, therefore the normalized effect vector is:

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$$\begin{bmatrix} 0.7 \\ -0.3 \\ 0 \\ 0 \end{bmatrix}$$

5- Assuming the design variable ranges are known to the designer, normalized variables $\grave{x}_\imath$ are formulated such that the range of all normalized variables is $[0,1]$.

6- The meta-objective function is formulated as follows:

$$\acute{y} = \frac{1}{n-1}(\sum_{i=1}^{k} a_i^{1-\acute{x}_i} + \sum_{i=k+1}^{n} \|a_i\|^{\acute{x}_i} - 1) \quad (2)$$

Where:

x_1 to x_k	Positive effect design variables
x_{k+1} to x_n	Negative effect design variables
a_i	Normalized effect (weight) corresponding to x_i, where $\sum_{i=1}^{n} \|\|a_i\|\| = 1$
n	Number of design variables

Eq. (2) is constructed in a way such that $\acute{y}$ satisfies the following conditions:

- The range is $[0,1]$.
- The maximum corresponds to the situation that all positive effect design variables are at their maximum value (1) and all negative effect design variables are at their minimum value (0).
- The minimum corresponds to the situation that all positive effect design variables are at their minimum value (0) and all negative effect design variables are at their maximum value (1).

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Applying Eq. (2) to the above example,

$$\begin{aligned} \acute{y} &= \frac{1}{2-1}\left(0.7^{1-\acute{x}_1} + 0.3^{\acute{x}_2} - 1\right) \\ &= \left(0.7^{1-\acute{x}_1} + 0.3^{\acute{x}_2} - 1\right) \end{aligned} \tag{3}$$

It is to be noted that that $\acute{y}$ value is not related to the value of actual response variable y but both of them are optimized (maximized/minimized) at the same values of design variables under the assumed conditions. Therefore, the current approach does not calculate the optimum objective function value but it determines the optimal design variable settings.

The calculated meta-objective optimum value has no physical meaning. However the meta-objective function is constructed in a way so that the function is optimized at the same setting of design variables that optimize the actual objective function. Moreover, since its range is in [0 1] for all problems, its global unconstrained optimum is already known (0 in case of minimization and 1 for maximization). In the presence of constraints, the constrained optimum will shift away from these optima, from which one can assess the effect of different constraints on the attainable design optimum.

3. Limited information about variable variability

In this section, we explain how the triangular distribution can be used to approximately model the probability distribution of constraint functions in case there is limited information about design variables and parameters variability. Consequently, we explain an algorithm for transforming the probabilistic constraints into deterministic forms using the triangular distribution cumulative formula.

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RBDO constraints take the form $pr(g(X) \leq 0) \geq \emptyset(-\beta)$ where β is the reliability index [29–31]. Since design variables $x_i's$ are random variables, $g(X)$ would be a random function. Precise calculation of $pr(g(X) \leq 0)$ requires knowledge of $g(X)$'s probability distribution which is unattainable in most practical cases.

In this research, we assume random variables are of the following two types:

- Design variables: the designer can set the variable at a desired setting. However, the actual value changes randomly around the set value within a known range/tolerance.
- Design parameters: they are surrounding uncontrolled parameters. However, the designer knows the range of its variation.

According to the maximum entropy theory [32], the probability distribution which best represents a current state of knowledge is the one with the largest entropy. Ref. [33] stated that "the maximum entropy distribution is uniquely determined as the one which is maximally noncommittal with regard to missing information, and that it agrees with what is known, but expresses maximum uncertainty with respect to all other matters." Based on this theory, we assume random design variables follow a triangular distribution with the mode at the set value and minimum and maximum at the lower and upper limits respectively; we also assume that design parameters follow a uniform distribution within the known range.

Constraint functions, in this case, would be random functions of triangular and uniform random variables. There is no general explicit mathematical approach to deduce

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the probability distribution of such functions. Therefore, the authors used a Monte Carlo simulation to investigate the characteristics of the constraint functions' distribution.

Three symmetric triangular and two uniform random variables are generated; $x_1 \sim Tri(1,3,5)$, $x_2 \sim Tri(6,7,8)$, $x_3 \sim Tri(9,12,15)$, $x_4 \sim unif(15,20)$, and $x_5 \sim unif(20,30)$. The probability distributions of four arbitrary functions of these random variables are explained in Figure 5. For each function, the maximum, minimum, and function value corresponding to triangular variables' modes and the mid-point of uniform variables are indicated. The functions' distributions reveal that:

- Functions variability is bounded and has a single mode.
- The mode is approximately at the indicated function value corresponding to triangular variables' modes and uniform random variables' mid-points.

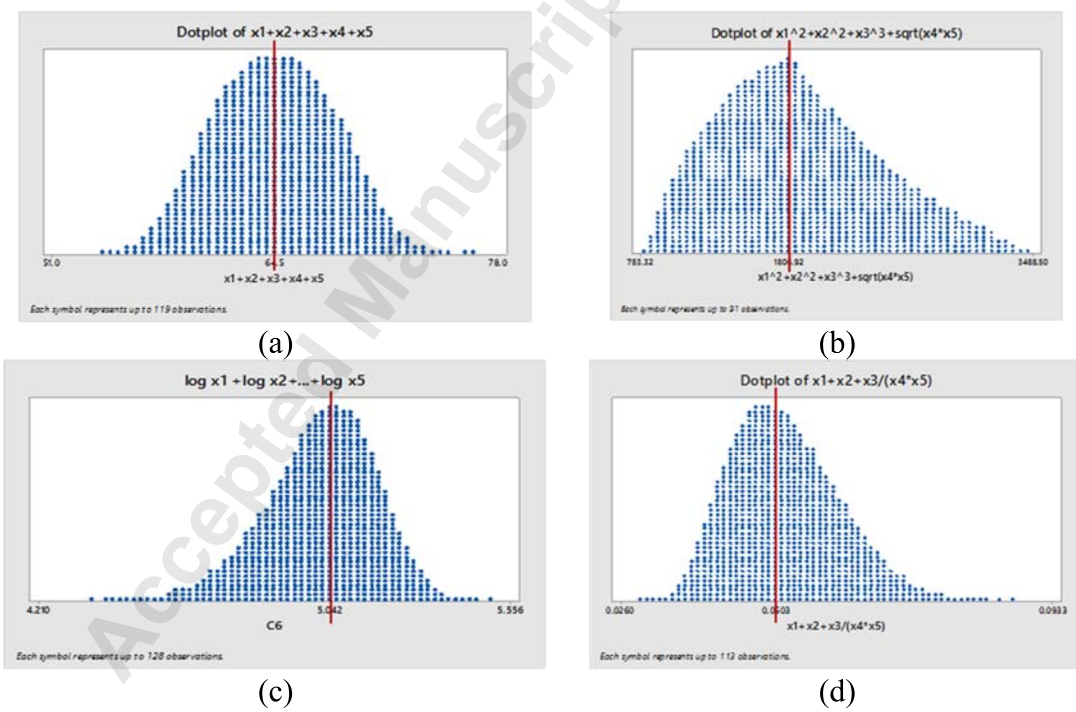

Figure 5 Probability density functions of various functions of assumed random variables.

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According to the maximum entropy theory and given the above two observations, the authors assume that constraint functions approximately follow a triangular distribution with the following parameters:

- The minimum a_f is the function value corresponding to all variables with positive effect are at their minimum value x_{i_a} and all variables with negative effect are at their maximum value x_{i_b}.
- The maximum b_f is the function value corresponding to all variables with positive effect are at their maximum value x_{i_b} and all variables with a negative effect are at their minimum value x_{i_a}.
- The mode c_f is the function value where all the design variables are at their mode x_{i_c} and design parameters at their mid-point, $\frac{x_{i_a}+x_{i_b}}{2}$.

These assumptions are validated by comparing the assumed triangular cumulative distribution curve against the actual simulated cumulative distribution curve for the four arbitrary functions as shown in Figure 6. The figure shows that the triangular distribution is an acceptable approximation for the actual probability distribution.

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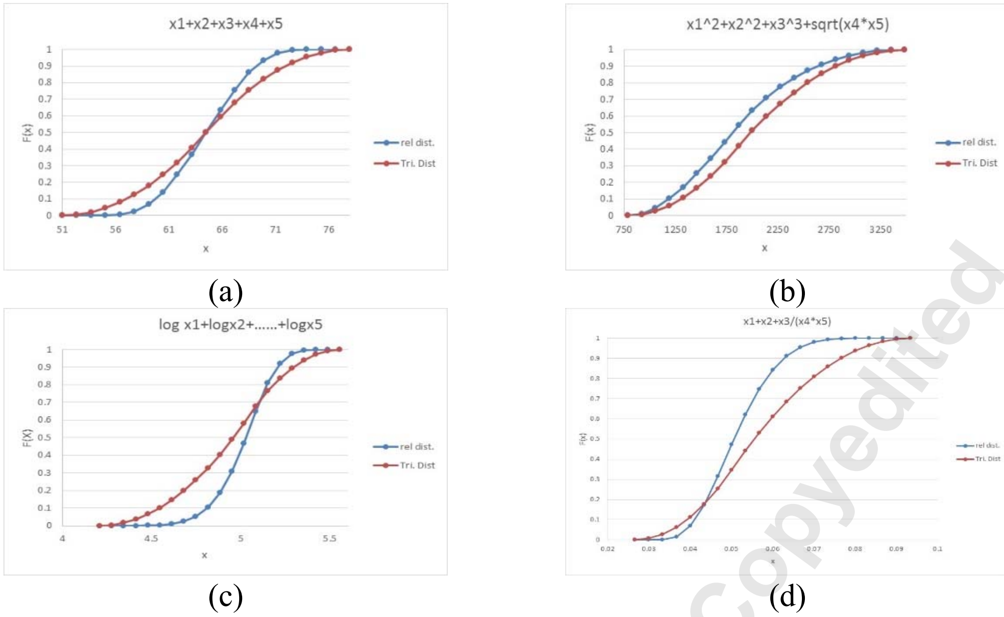

Figure 6 Cumulative distributions comparison between the actual and triangular distribution.

Following the standard cumulative density function for a triangular distribution, $P(G(X) \leq 0)$ can be expressed as shown in Eq. (4):

$$P(G(X) \leq 0) = \begin{cases} 0 & 0 \leq G(X)_a \\ \dfrac{(-G(X)_a)^2}{(G(X)_b - G(X)_a)(G(X) - G(X)_a)} & G(X)_a \leq 0 \leq G(X) \\ 1 - \dfrac{(G(X)_b)^2}{(G(X)_b - G(X)_a)\big(G(X)_b - G(X)\big)} & G(X) \leq 0 \leq G(X)_b \\ 1 & 0 \geq G(X)_b \end{cases} \quad (4)$$

Where $G(X)_a$ and $G(X)_b$ are the lower and upper limits of $G(X)$ respectively. They are calculated by replacing x_i in $G(X)$ by either "$x_i + d_i$" or "$x_i - d_i$" according to the direction of effect of x_i on $G(X)$ as explained in Table 1 where d_i is the half variability range of the random variable x_i.

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Table 1 $G(X)_a$ and $G(X)_b$ formulation method.

	Effect of x_i on $G(X)$	
	Increasing	Decreasing
$G(X)_a$	$x_i - d_i$	$x_i + d_i$
$G(X)_b$	$x_i + d_i$	$x_i - d_i$

Assuming $x_i{}'$s have a monotonic effect on $G(X)$ within the design range, the effect direction is determined by the constraint formula. In cases of complicated formulas, calculating the value of constraint function corresponding to the design variables' upper and lower limits can determine the effect direction.

The last step is replacing each x_i in $G(X)$ formula with its equivalent in terms of $\acute{x_i}$ using Eq. (5) such that $G(\acute{X})$ becomes a function of positive variables only. Effect direction of any design variable would not be affected by such variable transformation in the constraint formulas.

$$x_i = x_{i_a} + \acute{x_i}\,(x_{i_b} - x_{i_a}) \tag{5}$$

Once all constraints are transformed into the deterministic form using Eqs. (4) and (5), the optimization problem could be solved like traditional deterministic optimization problems with any suitable solver.

The developed approach is motivated by following observations in cases of limited information on design variables' variability:

- Design variables are practically not unbounded with continuous distributions. Their variability is typically within a pre-known expected range.
- Some RBDO research assumes $G(X)$ is normally distributed if all $x_i{}'s$ are normally distributed [5,6]. This assumption may not be valid in all cases.

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- Some RBDO techniques require a starting design point which is assumed to be reliable [34]. This assumption may not be guaranteed especially in early design stages.
- RBDO normally involves double loop optimization, which demands intensive computation. Recently, research shows that this drawback has been considerably alleviated [31,34–36]. The proposed approach only involves single loop deterministic optimization and the required computational is fundamentally reduced.

4. Case study

Golinski speed reducer [37] as shown in Figure 7 has been used as a case study to test many RBDO algorithms with weight minimization and physical constraints.

In this research, we assume the designer does not know the objective function formula. We will construct the causal graph with relational weights according to assumed logical relationship knowledge. The reliability based optimization problem will be solved using the proposed approach. The results are compared with the reported results from [29] which assumes full knowledge of the objective function formula.

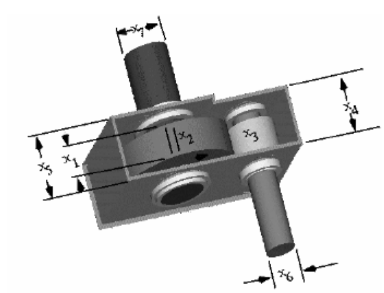

Figure 7 Golinski speed reducer configuration.

The design variables are:

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x_1 Gear width

x_2 Gear module

x_3 No. of teeth of the pinion

x_4 Shaft 1 length

x_5 Shaft 2 length

x_6 Shaft 1 diameter

x_7 Shaft 2 diameter

Although x_2 and x_3 are discrete and integer variables respectively, we assume all the variables are continuous for simplicity with known design range and variability as shown in Table 2 (we use the same values given in [29] for fair comparison):

Table 2 Variables design range and variability for the speed reducer case.

Variable	Design range	Variability
x_1	[2.6,3.6] cm	$Tri(d_i - 0.015, d_i, d_i + 0.015)$
x_2	[0.7,0.8] cm	$Tri(d_i - 0.015, d_i, d_i + 0.015)$
x_3	[17,28]	$Tri(d_i - 0.015, d_i, d_i + 0.015)$
x_4	[7.3,8.3] cm	$Tri(d_i - 0.015, d_i, d_i + 0.015)$
x_5	[7.3,8.3] cm	$Tri(d_i - 0.015, d_i, d_i + 0.015)$
x_6	[2.9,3.9] cm	$Tri(d_i - 0.015, d_i, d_i + 0.015)$
x_7	[5.0,5.5] cm	$Tri(d_i - 0.015, d_i, d_i + 0.015)$

Assuming that we don't know the mathematical expression between the variables and the objective, the logical relationship is represented by a causal graph shown in Figure 8. The indicated causal weights are proposed by the authors based on general mechanical engineering knowledge. For example, it is assumed that the pinion diameter is positively

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strongly affected by both the pinion module, x_2 and the number of teeth, x_3. Therefore these two links assume $+3$ weight. While the pinion shaft diameter x_6 extremely negatively affects the pinion mass, therefore this link weight is set to -5. We used nine intermediate variables to express the logical relationships between the design variables and the objective function.

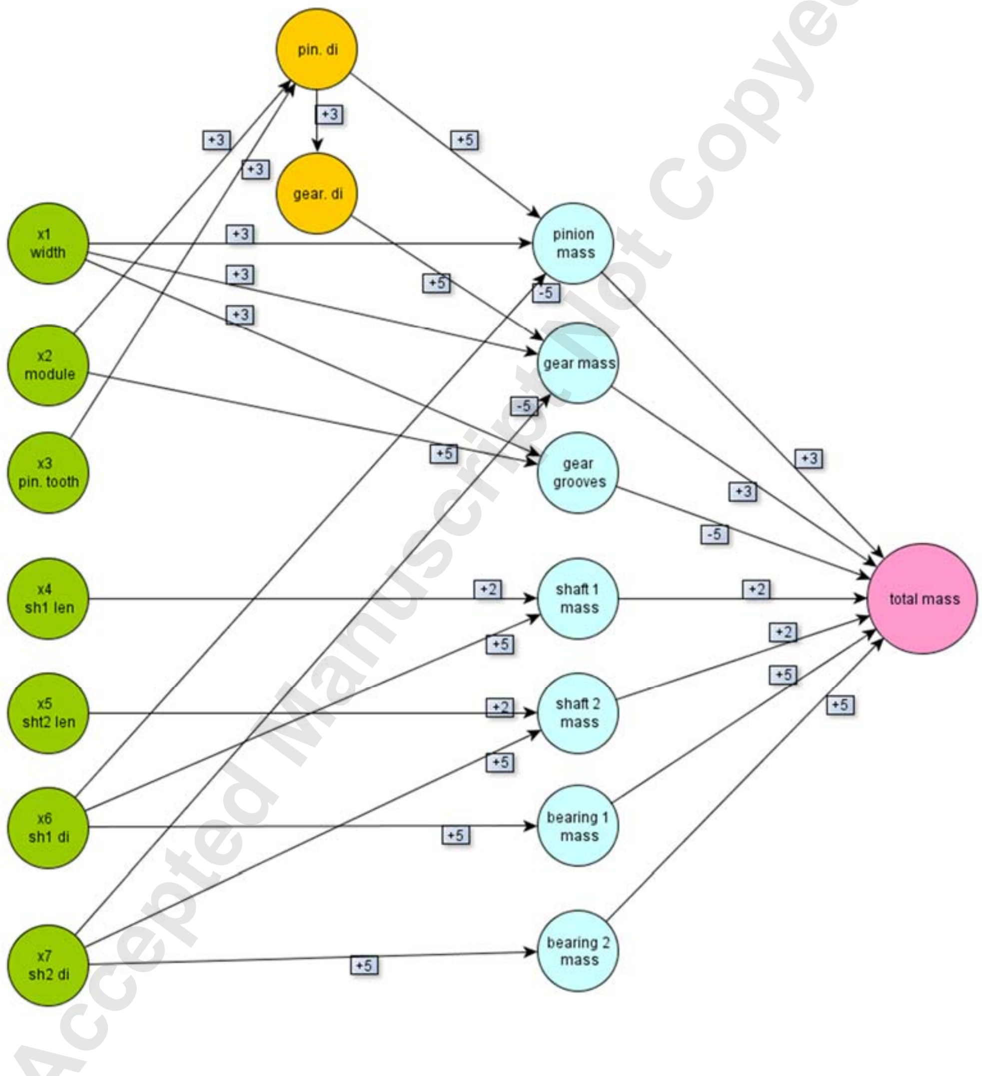

Figure 8 Causal graph for speed reducer weight reduction problem.

The causal graph is represented by a 17×17 (7 design variables$+$ 9 intermediate variables$+$ 1 objective function) design matrix as shown in Figure 9.

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									pinion P. Diam.	gear P. Diam.	pinion mass	gear 2 mass	empty gear vol.	shaft 1 mass	shaft 2 mass	bearing 1 mass	bearing 2 mass	total mass
		x1	x2	x3	x4	x5	x6	x7	z1	z2	y1	y2	y3	y4	y5	y6	y7	f
width	x1	1	0	0	0	0	0	0	0	0	3	3	3	0	0	0	0	0
module	x2	0	1	0	0	0	0	0	3	0	0	0	5	0	0	0	0	0
pin. no. teeth	x3	0	0	1	0	0	0	0	3	0	0	0	0	0	0	0	0	0
shaft 1 length	x4	0	0	0	1	0	0	0	0	0	0	0	0	2	0	0	0	0
shaft 2 length	x5	0	0	0	0	1	0	0	0	0	0	0	0	0	2	0	0	0
shaft 1 diam	x6	0	0	0	0	0	1	0	0	0	-5	0	0	5	0	5	0	0
shaft 2 diam	x7	0	0	0	0	0	0	1	0	0	0	-5	0	0	5	0	5	0
pinion P. Diam.	z1	0	0	0	0	0	0	0	0	3	5	0	0	0	0	0	0	0
gear P. Diam.	z2	0	0	0	0	0	0	0	0	0	0	5	0	0	0	0	0	0
pinion mass	y1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3
gear 2 mass	y2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3
empty gear vol.	y3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-5
shaft 1 mass	y4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
shaft 2 mass	y5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
bearing 1 mass	y6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5
bearing 2 mass	y7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5
total mass	f	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 9 Design matrix for the speed reducer.

Applying the algorithm shown in Figure 4 to the design matrix yields the effect column shown in Table 3 (the second column):

Table 3 Variable weights before and after normalization.

Variable	Effect column	Normalized Weight
X1	3	0.007772
X2	155	0.401554
X3	180	0.466321
X4	4	0.010363
X5	4	0.010363
X6	20	0.051813
X7	20	0.051813
X8	0	0
. . .	0	0
X17	0	0

Normalizing the resultant effect column yields the weights shown in Table 3 as the third column.

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Applying Eq. (2) to the resultant weights yields the following meta-objective function:

$$\acute{y} = \frac{1}{7-1}\Big(0.007772^{1-\grave{x}_1} + 0.401554^{1-\grave{x}_2} + 0.466321^{1-\grave{x}_3} + 0.010363^{1-\grave{x}_4} + 0.010363^{1-\grave{x}_5} + 0.051813^{1-\grave{x}_6} + 0.051813^{1-\grave{x}_7} - 1\Big) \quad (6)$$

Where $\grave{x}_1$ to $\grave{x}_7$ are the normalized design variables formulated as shown in Eq. (5).

This optimization problem has 11 reliability constraints as follows [29]:

$$\Pr[g_i(X) > 0] \leq \emptyset(-\beta)$$

Where:

$$g_1 = \frac{27}{X_1 X_2^2 X_3} - 1 \qquad g_2 = \frac{397.5}{X_1 X_2^2 X_3^2} - 1$$

$$g_3 = \frac{1.93 X_4^3}{X_2 X_3\ X_6^4} - 1 \qquad g_4 = \frac{1.93 X_5^3}{X_2 X_3\ X_7^4} - 1$$

$$g_5 = \frac{\sqrt{\left(\frac{745 X_4}{X_2 X_3}\right)^2 + 16.9 \times 10^6}}{0.1 X_6^3} - 1100$$

$$g_6 = \frac{\sqrt{\left(\frac{745 X_5}{X_2 X_3}\right)^2 + 157.5 \times 10^6}}{0.1 X_7^3} - 850$$

$$g_7 = X_1 X_3 - 40 \qquad g_8 = 5 - \frac{X_1}{X_2}$$

$$g_9 = \frac{X_1}{X_2} - 12 \qquad g_{10} = \frac{1.5 X_6 + 1.9}{X_4} - 1$$

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$$g_{11} = \frac{1.1X_7 + 1.9}{X_5} - 1$$

The reliability constraints are transformed into deterministic forms using Eq. (4). For ease of understanding, transformation of g_1 constraint is shown here. All other constraints are transformed similarly.

All of X_1 , X_2, and X_3 have a negative effect on g_1. Therefore, according to Eq. (4) and Table 1, $Pr[g_1(X) \leq 0]$ is written as follows:

$$P(g_1(X) \leq 0) = \begin{cases} 0 & 0 \leq g_1(X)_a \\ \dfrac{(-g_1(X)_a)^2}{(g_1(X)_b - g_1(X)_a)(g_1(X) - g_1(X)_a)} & g_1(X)_a \leq 0 \leq g_1(X) \\ 1 - \dfrac{(G(X)_b)^2}{(g_1(X)_b - g_1(X)_a)\big(g_1(X)_b - g_1(X)\big)} & g_1(X) \leq 0 \leq g_1(X)_b \\ 1 & 0 \geq g_1(X)_b \end{cases} \quad (7)$$

Where:

$$g_1(X)_a = \frac{27}{(X_1 + 0.015)(X_2 + 0.015)^2(X_3 + 0.015)} - 1$$

$$g_1(X) = \frac{27}{X_1 X_2^2 X_3} - 1$$

$$g_1(X)_b = \frac{27}{(X_1 - 0.015)(X_2 - 0.015)^2(X_3 - 0.015)} - 1$$

The derived deterministic meta-optimization problem with the objective function as shown in Eq. (6) and constraints derived similarly to Eq. (7) is solved using an evolutionary optimization algorithm. The resultant optimum normalized design variables are transformed back to the actual design space. For comparison, the actual objective function value corresponding to the derived optimum settings is calculated using the

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quantitative formula given in [29]. Table 4 summarizes the optimization results with a comparison to the results of different RBDO methods.

Table 4 Optimization results comparison.

RBDO technique	Objective	Design variables						
		X_1	X_2	X_3	X_4	X_5	X_6	X_7
Reliability index approach	3,038.58	3.577	0.7	17	7.3	7.754	3.365	5.302
Performance measure approach+	3,039.97	3.578	0.7	17	7.3	7.764	3.366	5.302
Single loop single vector	3,048.45	3.589	0.7	17	7.3	7.783	3.369	5.307
Sequential optimization and reliability assessment	3,040.02	3.578	0.7	17	7.3	7.764	3.366	5.302
Convex linearization	3,040.61	3.58	0.7	17	7.3	7.764	3.366	5.302
Globally convergent method of moving asymptotes	3,045.84	3.591	0.7	17	7.3	7.762	3.367	5.308
Qualitative limited information (this work)	3,109.37	3.585	0.7	17.37	7.3	7.761	3.365	5.301

The results show that the developed qualitative limited information method arrives at the similar optimal design. Note since this method assumes continuous for X_3 (the number of teeth), the optimal objective value seems a bit off. After setting this value to 17, the objective function value is almost the same as the other methods.

The accuracy of the derived method depends on the quality of the designer knowledge of the logical relationship between variables. In this work, we assume the designer is rational, which means, (s)he will not assume a positive effect for a design variable that actually has a negative effect or vice versa. The sensitivity of the proposed method to the quality of prior knowledge and the variability of design variables are to be investigated in future work.

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5. Conclusion

Reliability Based Design Optimization (RBDO) problems with qualitative objective function and limited information on variable variability are solved in this study utilizing causal graphs, design structure matrix, and maximum entropy theory concept. The designers' qualitative knowledge is used to develop a meta-objective function. Random variables and constraint functions are modeled based on the maximum entropy theory as uniform and triangular random variables with known bounds and estimated mode.

The developed algorithm is validated by a case study of the speed reducer design. The results are compared to different RBDO techniques. The comparison shows that the developed approach is accurate in calculating the optimum design point even when compared to published results in the literature with known analytical objective function and variable distributions.

The developed algorithm opens the door for the application of RBDO in situations where only qualitative information is available. Such application thus could potentially extend to non-engineering fields such as sociology, marketing, and so on.

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