Employing Knowledge on Causal Relationship to Assist Multidisciplinary Design Optimization

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With the increasing design dimensionality, it is more difficult to solve multidisciplinary design optimization (MDO) problems. Many MDO decomposition strategies have been developed to reduce the dimensionality. Those strategies consider the design problem as a black-box function. However, practitioners usually have certain knowledge of their problem. In this paper, a method leveraging causal graph and qualitative analysis is developed to reduce the dimensionality of the MDO problem by systematically modeling and incorporating the knowledge about the design problem into optimization. Causal graph is created to show the input–output relationships between variables. A qualitative analysis algorithm using design structure matrix (DSM) is developed to automatically find the variables whose values can be determined without resorting to optimization. According to the impact of variables, an MDO problem is divided into two subproblems, the optimization problem with respect to the most important variables, and the other with variables of lower importance. The novel method is used to solve a power converter design problem and an aircraft concept design problem, and the results show that by incorporating knowledge in form of causal relationship, the optimization efficiency is significantly improved. [DOI: 10.1115/1.4042342]

Keywords: dimension reduction, multidisciplinary design optimization, causal graph, dimensional analysis

1 Introduction

Multidisciplinary design optimization (MDO) has been widely applied in engineering to improve the performance of complex systems, which are often high-dimensional problems. When the dimensionality increases, the optimization search space grows exponentially, which makes sampling in the vast space intractable. With the high-fidelity simulation models, such as computational fluid dynamics and finite element analysis involved in MDO, the computational cost of finding an optimal solution rises significantly. Therefore, reducing the dimensionality of the problem is a natural strategy to tackle this challenge. One of the strategies is to decompose the complex system into several subsystems of lower dimension [1]. Different decomposition strategies, including concurrent subspace optimization (CSSO) [2], collaborative optimization (CO) [3], and bilevel integrated system synthesis (BLISS) [4], have been developed in the last decades. These methods have been employed in many different engineering applications [5–9]. However, the number of discipline function calls in those methods is still large. In Ref. [7], CO and CSSO were tested with several numerical benchmarks and the results show that even for low-dimensional problems, CO and CSSO need thousands of discipline function calls. BLISS was used to solve an aircraft concept design problem. For different variations of BLISS methods, although the number of system analysis was reduced to be around 10, the number of total discipline calls is around 400 and especially BLISS/RS2 required more than 1000 discipline calls [4]. For MDO problem with time-consuming simulation models, the cost of running an optimization using such distributed MDO strategies is often unacceptable.

A common feature of existing decomposition strategies is that they usually treat the design model as a black-box function, which means that the designer or practitioner does not know any information about the design problem. In practice, however, practitioners usually have some knowledge of the design problem but this knowledge is not utilized when solving MDO problems. Generally, in order to perform MDO, one needs to gather the following information: variables involved in the problem and the input–output relationships between them, the unit of each variable and sometimes the mathematical functions of the problem. By analyzing the existing knowledge about an MDO problem, some hidden valuable information can be extracted, which can help to reduce the dimensionality of the optimization problem. For example, if one can find that the objective is a monotonic function with respect to some design variables in a simply bounded design space, the values of such design variables can be determined and the dimensionality of the optimization problem can thus be reduced. Therefore, how to systematically incorporate knowledge and information of the design problem into optimization becomes an interesting research topic in order to improve the efficiency of MDO. This paper develops an approach for extraction and exploitation of latent knowledge for solving MDO problems.

The dimensional analysis concept modeling (DACM) framework [8,9] was developed to gather and organize knowledge associated with an engineering problem during the conceptual design phase. In this paper, the causal graph techniques of the DACM framework are employed to capture the hidden information supporting reduction of the dimensionality of MDO problems. The variables and objectives of the optimization model are identified before optimization but there is no mathematical expression of the

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input–output relationship. A causal graph is constructed based on practitioners' understanding of the MDO problem, which illustrates the cause–effect relationship between variables and objective(s). It has been defined in Ref. [8] that variables without contradictions are those variables having a monotonic effect on the objectives, whose values can be determined at its lower or upper bounds. On the other hand, variables with contradictions are those influencing an objective through different routes in the causal graph with conflicting effects. For instance, increase of these variables may cause the increase of the objective via one route but the drop of the objective via another route. In this work, a qualitative analysis algorithm is developed to separate these two groups of variables. The values of variables without contradictions can be determined without resorting to optimization. Thus, the dimensionality of the original problem can be reduced. A design structure matrix (DSM) [9]-based method is developed to automatically find the variables leading to no contradiction. A second simplification of the MDO problem is conducted to separate between the variables influencing significantly on the MDO problem and variables having a limited impact on the MDO problem. The impact of each variable is calculated and used to divide the entire optimization problem into two subproblems, one with important variables and the other with less important variables. As a result, the dimensionalities of both subproblems are reduced as compared with the original problem. Note that if all the design variables contain contradictions and are deemed important, the proposed method cannot reduce the dimensionality of the problem.

This paper is organized as follows: Details of the proposed dimension reduction method are described in Sec. 2. A numerical problem is employed to present the process of the dimension reduction method. The proposed dimension reduction method is applied to a power converter design problem and an aircraft concept design problem in Sec. 3. Section 4 concludes the paper and discusses about future research.

2 Dimension Reduction Method

A causal relationship-assisted dimension reduction method is developed in this section. By building a causal graph, input–output relations between variables in the numerical model are illustrated. Dimensional analysis with qualitative analysis provides a method to detect source of contradictions. This is supporting the reduction of dimensionality before performing optimization. The DSM, constructed according to the causal graph, is employed to automatically find the variables leading to no contradiction. Calculation of the impact of variables helps to divide the optimization problem into subproblems. The details of the proposed method are described in this section.

2.1 Overall Process. The overall process of our proposed dimension reduction method includes constructing causal graph, performing qualitative analysis, removing variables, calculating the weight of each link, simplifying causal graph, and performing two-stage optimization. The steps of the method are described as follows:

Step 1. Construct a causal graph based on cause–effect relationship. A causal graph is an oriented graph showing the causal relations between variables. Figure 1 is an example of a causal graph. In the graph, the nodes represent variables, the arrows give the input–output relations, and labels "+1" and "−1" represent how the input influences the output. For example, "+1" on the arrow from A to C means that C increases when A increases. It should be noted that the input and output in one link should have monotonic relation. This can be achieved by defining design space carefully. Additionally, the more elaborate the causal graph is, the simpler the causal relations will be in each link and the easier it will be to achieve monotonic relations for each link. Also, values of the variables in an engineering problem are usually larger than zero, which helps to avoid nonmonotonic links to some extents.

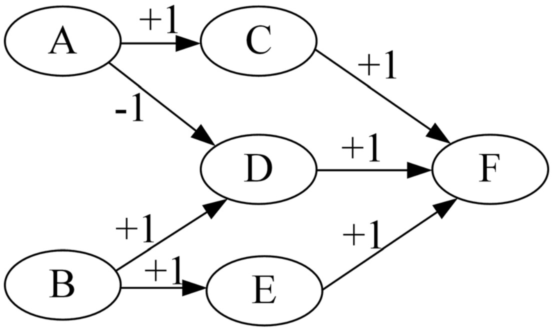

Fig. 1 Causal graph example

Coatanea et al. [8] give a process of constructing a causal graph. First, all the fundamental variables are listed and located in a functional structure that represents the functional flow of the system. Then, the causal rules are employed to define the causality for each variable. Finally, all the variables are linked together to form the causal graph. The causal graph should not miss important links, and this requirement can be satisfied if the designers are familiar with the design problem. Once the causal graph is constructed, the links in the causal graph can be checked by giving a perturbation on each design variable. If the causal graph can reflect the changes on each intermediate variable and objective, the causal graph can be regarded as correct.

Step 2. Perform a qualitative analysis. The causal graph is used to detect variables with or without contradictions. By multiplying the labels on the arrows of one route, the relation between the input and the final output of the route can be detected. For the example in Fig. 1, if multiplying "+1" on the arrow from A to C by the "+1" on the arrow from C to F, the multiplication result is "+1," which means that F is monotonically increasing with respect to A. If all the relations between design variables and the objectives are calculated, contradictions of the variable can be found according to the multiplications. In Fig. 1, A influences F via C or D (i.e., −1 via D and +1 via C). A is generating a contradictory influence on F if we consider both routes and multiplications.

On the other hand, F is a monotonically increasing function with respect to B no matter if it traverses through D or E. Therefore, A has a contradiction and B is a variable without contradictions. The vector of design variables is represented as $\boldsymbol{x}$. After qualitative analysis, the design variables can be divided into two parts, variables with contradictions ($\boldsymbol{x}_c$) and variables without contradictions ($\boldsymbol{x}_{uc}$). This qualitative analysis can be performed by checking the causal graph manually. However, in optimization, all the steps are desired to be executed automatically. Thus, a DSM-based qualitative analysis method is proposed to fulfill the requirement and will be described in Sec. 2.2 in more detail.

Step 3. Determine values of $\boldsymbol{x}_{uc}$ and remove them from the causal graph and the set of design variables. After qualitative analysis, the way design variables in $\boldsymbol{x}_{uc}$ influence the objective can be confirmed. Thus, $\boldsymbol{x}_{uc}$ can be regarded as a constant variable set at its lower (or upper) bounds. Taking Fig. 1 as an example, B has no contradiction and thus decrease of B leads to the decrease of the objective F. If a minimum F is desired, B should be set at its lower bound value. Thus, the optimal value of B is determined before optimization. Now, the design variable set of the optimization problem becomes $\boldsymbol{x}_c$ only. Variables without contradictions can thus be removed from the causal graph and from the optimization variable set.

Step 4. Calculate the weight of each link. The causal graph can be further simplified by considering the weight of each link. In this step, following the DACM methodology, the Taguchi method is used to calculate the weight of each link [8]. Section 2.3 describes the approach in detail. Before calculating the weights, the range of every variable, including design variables and state

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variables, is required. There exist two methods to determine those ranges. First, for engineering problems, the recommended range of variables can be found from references and it can be used in the sensitivity analysis. Second, the range can be determined by sampling a certain number of random points and calculating the responses of the samples. The maximal and minimal values can be used as the upper and lower bounds, respectively.

Step 5. Simplify the causal graph according to calculated weights. The link whose weight is lower than a threshold is regarded as a low importance link and is removed from the causal graph. The threshold can be selected according to the weights obtained from step 4. The main principle of threshold selection is that the threshold should not be too high to miss important links, nor too low to be ineffective. A higher threshold will let one regard more variables as unimportant variables then may increase the number of iterations in the two-stage optimization, and removing more variables from the important variable set may reduce the accuracy of the optimization. Generally, the threshold is no larger than 15%. In this paper, 10% is selected based on the different case studies that we have tested during the development of the approach. It provides a good balance between number of iterations and accuracy of the optimization. The value can be adjusted if needed.

After removing those deemed less important links, some variables may not affect the objective at all. Those removed variables are represented by $\boldsymbol{x}_{un}$. On the other hand, contradictions of some variables (represented by $\boldsymbol{x}_{unc}$) may disappear due to removal of the less important links. Then, in a similar way to step 3, values of such variables can be determined according to the qualitative analysis. Thus, the design variables $\boldsymbol{x}_c$ (variables with contradictions) can be divided into two parts, the kept variables with contradictions $\boldsymbol{x}_{ke}$, and less important variables $\boldsymbol{x}_{re}$, which includes both $\boldsymbol{x}_{un}$ and $\boldsymbol{x}_{unc}$.

Step 6. Use a two-stage optimization process to obtain the final optimal solution. The original optimization problem is divided into two subproblems: one with respect to $\boldsymbol{x}_{ke}$ and the other with respect to $\boldsymbol{x}_{re}$.

Then the two optimization problems are optimized separately. Results of the two optimization problems are combined together to form the final optimal solution. Details of the two-stage optimization process are shown in Sec. 2.4.

2.2 Qualitative Analysis Based on Design Structure Matrix.

The qualitative analysis process is designed to find the variables without contradictions and it has to be executed automatically for optimization. Thus, a novel DSM-based qualitative analysis method is developed in this section.

In the DSM-based qualitative analysis, two matrices, [A] and [A1], are built according to the causal graph, where [A] shows the input–output relations between each pair of variables and [A1] gives [A] and direction of each link. For both [A] and [A1], the first rows (columns) refer to design variables, the last row (column) is for the objective, and the intermediate variables are in between. [A] and [A1] are n-by-n matrices, where n is the number of entities including design variables, intermediate variables, and the objective. For convenience and consistency with DACM nomenclature, we refer to these entities as variables with the understanding that the global objective is located in the last row (column) for both [A] and [A1]. Matrix [A] uses "1" to represent the links between two variables. If i is the input of j, then $a_{ij} = 1$; otherwise, $a_{ij} = 0$. In matrix [A1], the number "+1" and "−1" are used to represent the relationship between the input and output. If variable j decreases with i increasing, $a_{ij} = -1$; otherwise, $a_{ij} = +1$. We assume that the optimization problem is a single objective problem. Thus, the last column of [A] and [A1] shows the objective and its direct inputs. It is also possible to consider a multi-objective problem. By checking the absolute values of elements in the last column of [A1] and [A], variables without contradictions can be detected. Details of DSM-based qualitative analysis method are shown as follows: Sec. 2.5 gives a numeric example for better explanation.

Assume that the number of design variables is $nVar$ and the number of intermediate variables is $nInt$, then $n = nVar + nInt + 1$, for a single objective problem.

Step 1. Find coupled variables. In practice, if a "1" appears under the diagonal in DSM, one can recognize that there is a feedback link. However, these feedback links do not necessarily represent a loop and those links that do not represent loops should be moved above the diagonal to simplify the DSM. This can be accomplished via a simple strategy modifying [A]. The rows of [A] are checked one-by-one. If there is a "1" element under the diagonal, i.e., $a_{ij} = 1, i > j$, variable j is re-ordered to be before i. The number of "1" elements under the diagonal (i.e., nf) is counted after one movement, which is compared with the smallest number of "1"s under the diagonal (nf*). If $nf < nf^*$, $nf^* = nf$ and the sequence of the variables is recorded. If the value of nf^* does not change for a given time (i.e., five iterations), the modification will stop and the sequence with the smallest number of feedback is used to reconstruct [A] and [A1] and to obtain [A'] and [A1']. The location of the "1" element under the diagonal is used to give the coupled variables. For example, if $a_{ij} = 1$ and $i > j$, then variables i and j are coupled. The coupled variables are stored in a 2-by-nf^* matrix FB, each column of which is shown as one pair of coupled variables.

Step 2. Calculate the number of links in the longest route. To detect the contradictions of the ith design variable, all the routes from ith design variable to the objective should be identified. The longest route should be determined and the other shorter routes are checked at the same time. In some cases, the coupling relations make it difficult to find the longest route because of the presence of the feedback loop. Thus, the longest route is considered in this paper to be obtained by going through each feedback link once and only once.

The number of links in the longest route contains two parts: one is the number of links ($nNoC$) in the longest route without feedback and others are the number of links ($nC_i, i = 1, 2, ., nf^*$) in all loops. Summing $nNoC$ and $nC_i, i = 1, 2, ., nf^*$ together, the final number of links ($nMax$) can be obtained as follows:

$$nMax = nNoC + \sum_{i=1}^{nf^*} nC_i \tag{1}$$

To count the number of links in the route without feedback, the "1" elements under the diagonal in [A'] are turned to "0" to obtain matrix [Anoc]. Warfield [10] used the number of multiplications to represent the links between two variables. It reported that multiplying matrix [A] by itself k times and if $a_{i,n}$ is nonzero, it means that variable i influences the objective through a route with $k + 1$ links. [Anoc] is multiplied by itself, and when the objective column contains nonzero elements, the number of multiplication ($mNoc$) is recorded. After multiplying $n - 1$ times, the largest $mNoc$ gives the number of links, i.e., $nNoC = mNoc + 1$.

After multiplying a DSM matrix itself several times, if nonzero elements exist in the diagonal of the DSM matrix, it means there is at least one loop in the problem and a nonzero element means that this variable goes through the loop once and back to itself. Therefore, once a loop has been detected, by counting the times of multiplications before nonzero elements appear in the diagonal, the number of links in the loop can be identified [10].

For the ith coupling loop, the two coupled variables are $FB_{1,i}$ and $FB_{2,i}$. The variables from $FB_{1,i}$ to $FB_{2,i}$ are used to construct a small DSM with one coupling loop, [C$_i$]. Between $FB_{1,i}$ and $FB_{2,i}$, there are $nL = FB_{2,i} - FB_{1,i}$ links. The matrix [C$_i$] is multiplied by itself $nL + 1$ times, and when $c_{1,1} = c_{nL+1,nL+1} = 1$, the number of multiplication (mC_i) is recorded. After $nL + 1$ times of multiplication, the largest mC_i gives the number of links in the coupling loop, i.e., $nC_i = mC_i + 1$.

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Step 3. Find the variables without contradictions. After obtaining the number of links *nMax*, matrices [A’] and [A1’] are multiplied by themselves $(nMax - 1)$ times to check the contradictions.

In general, at the kth multiplication, if $a_{i,n}^{k}$ $(i = 1, \ldots, nVar)$ is nonzero, which means that variable i has impact on the objective through $k+1$ links, the absolute value in the objective column $|a_{i,n}^{k}|$ and $|a1_{i,n}^{k}|$ are compared. The value $|a_{i,n}^{k}|$ shows that there are $\left|a_{i,n}^{k}\right|$ routes from variable i to the objective, which contain $(k+1)$ links. If $|a_{i,n}^{k}| \neq |a1_{i,n}^{k}|$, it means that there is at least one route through which the objective changes in directions as compared with other routes. Thus, the variable i has contradictions. If $|a_{i,n}^{k}| = |a1_{i,n}^{k}|$ for all the multiplications, the sign of nonzero $a1_{i,n}^{k}$ in every multiplication is checked. If the signs of $a1_{i,n}^{k}$ are different, that means through different routes, the objective changes in directions. Therefore, the variable i has contradictions. Otherwise, if the sign of $a1_{i,n}^{k}$ are the same for all the multiplications, variable i has no contraction. After multiplying [A’] and [A1’] by themselves *nMax* times, variables without contradictions can be picked out. The sign of $a1_{i,n}^{k}$ indicates the relations between variable i and the objective. Assuming that the objective is to be minimized, if the sign of $a1_{i,n}^{k}$ is “+,” it means that the variable i should be set at the lower bound value; otherwise, the upper bound value should be selected.

2.3 Weight Calculation.

For the impact of each design variable on the objective is different, a practitioner often focuses on important variables. By selecting important variables, the problem dimensionality can be reduced further. Thus, the weight of each link in the causal graph is calculated in the proposed method, and the original optimization problem is divided into two subproblems according to weights of the links. Several methods have been developed to calculate the weights, including analysis of variances [11], principle component analysis [12], and so on. Taguchi method [13,14], one of the design of experiment tools, offers a simple and systematic approach to calculate the impact of each input on the output. In this paper, a two-level Taguchi approach to compute impact is selected to calculate the weight of each link Assume that an equation is as follows:

$$y = f(\boldsymbol{x}), \quad \mathbf{x} = \{x_1, x_2, \ldots, x_t\} \tag{2}$$

There are t inputs of y in Eq. (2): y can represent for example an intermediate variable and x_i represent the variables influencing y. First, the sample points are generated according to the Taguchi orthogonal arrays. In this paper, it is assumed the boundary of design variables and the intermediate variables are appropriately selected so that the output is monotonic or nearly monotonic with respect to each input. Therefore, a two-level Taguchi design has the capability to capture the impact of each input. The two-level Taguchi orthogonal array selected is shown in Table 1.

Table 1 The Taguchi orthogonal array for $t = 7$

	Column						
Experiment number	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

“1” means that the variable takes the value of the lower bound and “2,” the upper bound. For the equation shown in Eq. (3), eight sample points are generated according to Table 1 and the responses are calculated at each sample. The symbol i represents the columns of the table. The effect of $x_i (i = 1, 2, \ldots, nVar)$ to y can be calculated as follows:

$$\text{effect}_{x_i - y} = \frac{\overset{\textit{for } x_i \text{ at level high}}{\sum\limits_{j=1 \textit{ to } m}} y_{\text{Level2}}}{m/2} - \frac{\overset{\textit{for } x_i \text{ at level low}}{\sum\limits_{j=1 \textit{ to } m}} y_{\text{Level1}}}{m/2} \tag{3}$$

where m is the number of experiments. In this case, $m = 8$. Then, the effect is normalized by Eq. (4) and the normalized effect is the weight of link x_i to y.

$$\text{weight}_{x_i - y} = \frac{\text{Effect}_{x_i - y}}{\sum\limits_{k=1}^{n} \text{Effect}_{x_k - y}} \tag{4}$$

For problems of variables equal to or less than seven, as shown in Table 1, only eight samples are needed. One sample corresponds to one system analysis, or a complete simulation of the whole system. With these samples, one can perform the Taguchi computation of weights for each link, and thus the added cost of function evaluations is eight. For problems of larger scale, the added expense is determined by the specific orthogonal array that one chooses. The size of the orthogonal array is dependent on the number of variables in the problem. Note that although only the influence of each single input is calculated, the cross effect of the inputs is considered in Taguchi method because the sampling array is designed to consider the cross effect by employing as small number of sample points as possible.

Each link represents the influence of a variable on another variable and instead of calculating the importance of a variable to the final objective, the weight of every single link in the causal graph is estimated. By removing the links with low importance, the causal graph can be simplified. Then, another qualitative analysis is performed on the simplified causal graph to find variables without contradictions as well as variables that have no links to the objective. Optimal values of these variables can thus be determined and removed from the set of important optimization variables.

2.4 Two-Stage Optimization Process.

After the second simplification, design variables are divided into two parts, the important variables $\boldsymbol{x}_{ke}$ and the less important variables $\boldsymbol{x}_{re}$. Then, two optimization problems are constructed shown as Eqs. (5) and (6) and optimized sequentially

Problem 1

$$\begin{aligned} &\textit{find } \boldsymbol{x}_{ke} \\ &\min f(\boldsymbol{x}_{ke}, \boldsymbol{x}_{re}, \boldsymbol{x}_{uc}) \\ &s.t. \qquad g(\boldsymbol{x}_{ke}, \boldsymbol{x}_{re}, \boldsymbol{x}_{uc}) \leq 0 \\ &\qquad\qquad \boldsymbol{x}_{ke}^{lb} \leq \boldsymbol{x}_{ke} \leq \boldsymbol{x}_{ke}^{ub} \end{aligned} \tag{5}$$

Problem 2

$$\begin{aligned} &\textit{find } \boldsymbol{x}_{re} \\ &\min f(\boldsymbol{x}_{re}, \boldsymbol{x}_{ke}, \boldsymbol{x}_{uc}) \\ &s.t. \qquad g(\boldsymbol{x}_{re}, \boldsymbol{x}_{ke}, \boldsymbol{x}_{uc}) \leq 0 \\ &\qquad\qquad \boldsymbol{x}_{re}^{lb} \leq \boldsymbol{x}_{re} \leq \boldsymbol{x}_{re}^{ub} \end{aligned} \tag{6}$$

In both problems, $\boldsymbol{x}_{uc}$ is fixed at the value determined by the qualitative analysis. When optimizing problem 1, $\boldsymbol{x}_{re}$, $\boldsymbol{x}_{unc}$ are fixed at the determined values, while the value of $\boldsymbol{x}_{ke}$ is fixed at the

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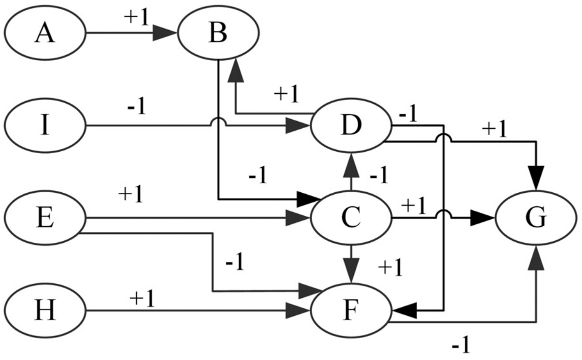

Fig. 2 Causal graph of a numerical example

optimal solution obtained from problem 1 when optimizing problem 2. In the following tests, MATLAB function *fmincon*(.) is employed to solve the optimization problem. Other optimization methods can also be used to solve the subproblems. The stopping criterion is checked at the end of each problem. If the optimum is not found yet after optimizing problem 2, the sequential optimization process will be performed again. For the purpose of comparing the efficiency with other methods, we need to fix the quality of the solution. Therefore, if the relative difference between the optimal results from problem 1 (or problem 2) $f_1^*(or\ f_2^*)$ and the given optimal results f^* is less than a given tolerance (i.e., 10^{-4}), the optimization process terminates. The relative difference is defined as follows:

$$\varepsilon = \frac{|f^* - f_1^*|}{f^*} \tag{7}$$

The sequential optimization method may be stuck into a suboptimum when dealing with multimodal problems. In the proposed decomposition method, it should be noted that the unimportant variables include two categories, the variables without contradictions and the variables having less impact on the objective. For the variables without contradictions, the optimal solution can be accurately determined according to the qualitative analysis results. By separating the rest of variables into unimportant and important variables using knowledge also reduces the risks of falling into a suboptimum.

2.5 Numerical Example. A simple numerical problem is employed in this section to explain how the proposed method works. The expression of the problem is as follows:

$$\begin{aligned}
&Find\ \mathbf{x} = [A, E, H, I]\\
&\min\ G = 10DF^{-2} + 100C^2\\
&where, F = 2C^{1.8}D^{-2}E^{-2.2}H^{2.5}\\
&\qquad\qquad D = 2I^{-1.5} - C^4\\
&\qquad\qquad C = 0.5E^{0.3}B^{-1.2}\\
&\qquad\qquad B = 2AD\\
&s.t. \qquad 1 \le A, E, H, I \le 2
\end{aligned} \tag{8}$$

Step 1. The causal graph (Fig. 2) is constructed according to Eq. (8). The design variables are drawn at the left side and the objective is located at the right side. As shown in the causal graph, a coupling loop involved B, C, and D exists in the problem. The labels "+1" and "−1" are assigned above the arrows according to each equation. Take the equation $C = 0.5E^{0.3}B^{-1.2}$ as an example, C increases when E increases or when B decreases. Thus, a "+1" is located above the arrow from E to C and a "−1" is added above the arrow from B to C.

Table 2 Matrix [A] for the numerical example

	A	*E*	*H*	*I*	*B*	*C*	*F*	*D*	*G*
A	0	0	0	0	1	0	0	0	0
E	0	0	0	0	0	1	1	0	0
H	0	0	0	0	0	0	1	0	0
I	0	0	0	0	0	0	0	1	0
B	0	0	0	0	0	1	0	0	0
C	0	0	0	0	0	0	1	1	1
F	0	0	0	0	0	0	0	0	1
D	0	0	0	0	**1**	0	**1**	0	1
G	0	0	0	0	0	0	0	0	0

Table 3 Matrix [A1] for the numerical example

	A	*E*	*H*	*I*	*B*	*C*	*F*	*D*	*G*
A	0	0	0	0	+1	0	0	0	0
E	0	0	0	0	0	+1	−1	0	0
H	0	0	0	0	0	0	+1	0	0
I	0	0	0	0	0	0	0	+1	0
B	0	0	0	0	0	−1	0	0	0
C	0	0	0	0	0	0	+1	−1	+1
F	0	0	0	0	0	0	0	0	−1
D	0	0	0	0	**+1**	0	**−1**	0	+1
G	0	0	0	0	0	0	0	0	0

Step 2. Qualitative analysis based on design structure matrix is performed to find the design variables without contradictions. The two matrices [A] and [A1] are constructed as shown in Tables 2 and 3. The first four columns refer to design variables and the last column *G* shows the objective. For example, *B* is the output of *A* and the input of *C*, the elements (*A*, *B*) and (*B*, *C*) are "1" in [A]. The labels "+1" and "−1" above the arrows in the causal graph are used to construct [A1]. The process of DSM-based qualitative analysis is presented step-by-step.

Step 2.1. The coupled variables are found in this step. As shown in Fig. 2, there is one loop involving *B, C*, and *D*. In Table 2, nonzero elements exist under the diagonal (boldfaced). For detecting the loop, the sequence of the columns in matrix [A] is changed. First, the element (D, B) is detected and to remove the "1" under the diagonal, variable *D* is moved to the front of *B*. Then, the modified [A] (named as [A']) is shown in Table 4, and the number of "1"s under diagonal is one in the modified matrix, i.e., $nf^* = 1$. Repeating this step five times, the nf^* does not change during the repeating process. Thus, the new sequence of the variables and objective is

$$\mathrm{Se}q = [A, E, H, I, D, B, C, F, G] \tag{9}$$

The modified [A'] and [A1'] are listed in Tables 4 and 5. There is one loop ("1" in boldface) detected through this step and the pair of the coupled variables are *D* and *C*.

Table 4 Modified matrix [A'] for the numerical example

	A	*E*	*H*	*I*	*D*	*B*	*C*	*F*	*G*
A	0	0	0	0	0	1	0	0	0
E	0	0	0	0	0	0	1	1	0
H	0	0	0	0	0	0	0	1	0
I	0	0	0	0	1	0	0	0	0
D	0	0	0	0	0	1	0	1	1
B	0	0	0	0	0	0	1	0	0
C	0	0	0	0	**1**	0	0	1	1
F	0	0	0	0	0	0	0	0	1
G	0	0	0	0	0	0	0	0	0

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Table 5 Modified matrix [A1'] for the numerical example

	A	E	H	I	D	B	C	F	G
A	0	0	0	0	0	+1	0	0	0
E	0	0	0	0	0	0	+1	−1	0
H	0	0	0	0	0	0	0	+1	0
I	0	0	0	0	+1	0	0	0	0
D	0	0	0	0	0	+1	0	−1	+1
B	0	0	0	0	0	0	−1	0	0
C	0	0	0	0	**−1**	0	0	+1	+1
F	0	0	0	0	0	0	0	0	−1
G	0	0	0	0	0	0	0	0	0

Table 6 Matrix [Anoc] for the numerical example

	A	E	H	I	D	B	C	F	G
A	0	0	0	0	0	1	0	0	0
E	0	0	0	0	0	0	1	1	0
H	0	0	0	0	0	0	0	1	0
I	0	0	0	0	1	0	0	0	0
D	0	0	0	0	0	1	0	1	1
B	0	0	0	0	0	0	1	0	0
C	0	0	0	0	0	0	0	1	1
F	0	0	0	0	0	0	0	0	1
G	0	0	0	0	0	0	0	0	0

Table 7 Matrix [C] for the numerical example

	D	B	C
D	0	1	0
B	0	0	1
C	1	0	0

Step 2.2. The number of links in the longest route is counted in this step. The "1" element in (C, D) in matrix [A'] is turned to be "0" to construct the matrix [Anoc] (as shown in Table 6). [Anoc] is multiplied by itself eight times and at the first, second, third, and fourth times, the objective column contains nonzero element. Thus, $nNoC = 5$.

In the example, only one coupling exists, so one matrix [C] is built. As shown in Table 4, "1" showing the feedback appears in the element (C, D). Thus, variables from D to C in Table 4 (i.e., D, B, and C) are used to construct the matrix [C], which is shown in Table 7. In matrix [C], the two coupled variables, D and C, are located at the first and third columns, so $nL = 3 - 1 = 2$. [C] is multiplied by itself three times and at the second multiplication, $c_{1,1} = c_{nL+1,nL+1} = 1$, which means $nC = 3$. Thus, the total number of links ($nMax$) in the longest route is $nMax = nNoc + n\mathrm{C} = 5 + 3 = 8$. As one can see from Fig. 2, the longest path is A-B-C-D-B-C-D-F-G with eight steps.

Step 2.3. The variables without contradictions are detected in this step because $nMax = 8$, [A'] and [A1'] are multiplied by themselves seven times. Table 8 illustrates that for the four design variables, the values in the objective column in [A'] and [A1'] after each multiplication.

Table 9 Taguchi sampling table of objective function

	Inputs			
Experiment number	C	D	F	G
1	0.0525	0.714	0.0026	1056213
2	0.0525	0.714	0.0026	1056213
3	0.0525	1.9848	0.8635	26.89465
4	0.0525	1.9848	0.8635	26.89465
5	0.3388	0.714	0.8635	21.05431
6	0.3388	0.714	0.8635	21.05431
7	0.3388	1.9848	0.0026	2936106
8	0.3388	1.9848	0.0026	2936106

In Table 8, G is the element value in the objective column in [A'] while $G1$ gives the values in the objective column in [A1'] in every multiplication. As shown in Table 8, the absolute values of G and $G1$ are the same for every variable in every multiplication. Now, if checking the first and second multiplications for variable E, it can be found that in the first multiplication, $G1$ has the sign "+" while in the second multiplication, $G1$'s sign is "−", which means that through two steps the increasing of E can increase G while the increasing of E may decrease G when going another route with three links. Therefore, a contradiction exists in variable E. The same applies to A and I when checking the second and the third multiplications. On the other hand, for H, only at the first multiplication, the values in (H, G) and (H, $G1$) are nonzero and the absolute values are the same, which means that H only influences objective G through one route with two links. Therefore, H has no contradiction. Thus, A, E, and I are variables containing contradictions while H is without contradictions, i.e., $\boldsymbol{x}_{uc} = H$ and $\boldsymbol{x}_c = [A, E, I]$.

Step 3. After qualitative analysis, one variable (H) is found without contradictions. Since the sign of element $(H, G1)$ is "−," H should be set at the upper bound value because G is to be minimized. The other three variables (A, E and I) will go through the next steps.

Step 4. The weight of each link is calculated using the Taguchi method. The objective function $G = 10DF^{-2} + 100C^2$ is taken as an example to show the process. Because D, F, and C are intermediate variables, their ranges are decided by calculating the responses of 50 random sample points. In this case, the ranges of D, F, and C are [0.7140, 1.9848], [0.0026, 0.8635], and [0.0525, 0.3388]. Next, the Taguchi table is constructed as shown in Table 9 and the response of each sample is constructed.

By calling Eqs. (3) and (4), the weight of the three inputs links (i.e., D to G, F to G, and C to G) are 24.2%, 51.6%, and 24.2%, respectively. Using the same method to calculate the weight of all the links and using the weight to replace the "1" element in matrix [A], the weighted matrix [$\mathrm{A_w}$] is constructed as shown in Table 10.

Step 5. The causal graph is simplified according to the weight and the variable sets, $\boldsymbol{x}_{ke}$ and $\boldsymbol{x}_{re}$are detected in this step. In this case, the threshold is selected as 10%. Comparing the weights of each link with the threshold, the links E -> C and C -> D are removed and the simplified causal graph is shown in Fig. 3.

From Fig. 3, it can be found that the coupling loop is decoupled because the link between C and D is cut. In the simplified graph,

Table 8 Element values in the objective column in [A'] and [A1']

Multiplication No.	1		2		3		4		5		6		7	
	G	$G1$	G	$G1$	G	$G1$	G	$G1$	G	$G1$	G	$G1$	G	$G1$
A	0	0	1	−1	2	2	1	1	1	−1	2	2	1	1
E	2	2	2	−2	1	−1	1	1	2	−2	1	−1	1	1
H	1	−1	0	0	0	0	0	0	0	0	0	0	0	0
I	1	1	1	1	1	−1	2	2	1	1	1	−1	2	2

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Table 10 Weighted matrix [A_w] for numerical example

	A	E	H	I	B	C	F	D	G
A	0	0	0	0	15.0%	0	0	0	0
E	0	0	0	0	0	3.6%	25.5%	0	0
H	0	0	0	0	0	0	25.6%	0	0
I	0	0	0	0	0	0	0	96.1%	0
B	0	0	0	0	0	96.4%	0	0	0
C	0	0	0	0	0	0	27.3%	3.9%	24.2%
F	0	0	0	0	0	0	0	0	51.6%
D	0	0	0	0	85.0%	0	21.6%	0	24.2%
G	0	0	0	0	0	0	0	0	0

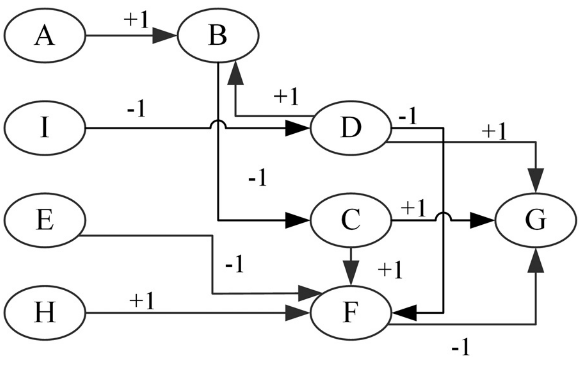

Fig. 3 Simplified causal graph for the numerical example

Table 11 Optimization results of the original problem and decomposed problem

	x^*	f^*	# of SA	Variance of # of SA
Original	[1.365,1,2,2]	7.9735	91	[60,131]
Decomposed	[1.365,1,2,2]	7.9735	41	[38,48]

the variables without contradictions are detected through a qualitative analysis. In this case, variable E is found without contradictions and the objective G decreases with E decreasing. Therefore, the kept variables needed to be optimized $\boldsymbol{x}_{ke} = [A, I]$ and the less important variable $x_{re} = E$.

Step 6. The two-stage optimization problem is constructed as follows:

Problem 1

$$\begin{aligned} &find\ \boldsymbol{x}_{ke} = [A, I] \\ &\min\ G = f(\boldsymbol{x}_{ke}, E, H) \\ &1 \le \boldsymbol{x}_{ke} \le 2 \\ &where, E = 1, H = 2 \end{aligned} \tag{10}$$

Problem 2

$$\begin{aligned} &find\ x_{re} = E \\ &\min f(x_{re}, \boldsymbol{x}_{ke}, H) \\ &1 \le x_{re} \le 2 \\ &where, \boldsymbol{x}_{ke} = [A^*, I^*], H = 2 \end{aligned} \tag{11}$$

The optimal value $f^* = 7.9735$ is set to be the stopping criterion value. MATLAB function *fmincon*(.) is employed to perform the optimization and the results are shown in Table 11. The starting point of the original problem is randomly generated in the design space. For the two-stage optimization problem, the starting points of the two stages are the same as that in the original problem. For example, if the starting point in the original problem is $x_0 = [A_0, I_0, E_0, H_0] = [1.2, 1.3, 1.4, 1.5]$, then the starting point for problem 1 will be $\boldsymbol{x}_{ke,0} = [A_0, I_0] = [1.2, 1.3]$ and the starting point for problem 2 will be $x_{re,0} = E_0 = 1.4$. Since the optimal value is reached at the first stage optimization, the second stage optimization is not run in this case. f^* is the optimal value and SA stands for system analysis. The optimization is repeated 11 times so the median is an actual tested value. The median number of SA is shown in Table 11. The optimal value and the optimal points are the results in the run with the median number of SA.

Table 12 Comparison of two thresholds (10% and 20%)

Threshold	x^*	f^*	# of SA	Variance of # of SA
10%	[1.365,1,2,2]	7.9735	41	[38,48]
15%	[1.365,1,2,2]	7.9735	55	[41,65]

As shown in Table 11, the number of system analysis for the two-stage optimization is 41, including eight system analyses in weight calculation, which is 45% of the number of analysis used in optimization of the original problem. That is because the four-dimensional problem is reduced to a two-dimensional problem.

To test the influence of the threshold, the threshold is selected as 15%. Then, link A -> B is removed from Fig. 3 as well, which means A has less impact on the final objective. Thus, the kept variables $x_{ke} = I$ and the less important variable $\boldsymbol{x}_{re} = [A, E]$. Using *fmincon*(.) function to optimize the decomposed problem, the results are shown in Table 12.

As shown in Table 12, the number of SA when using 10% threshold is smaller than that with 15% threshold. When using 10% as threshold, after optimizing the important variable $\boldsymbol{x}_{\boldsymbol{ke}}$ the optimum is reached and the optimization process is terminated. However, when selecting 15% as threshold, optimizing x_{ke} cannot reach the target value because one of the important variable A left as an important variable. As a result, the unimportant variable $\boldsymbol{x}_{re}$ needs to be optimized, which increases the number of SA. Therefore, missing important variables will lead to more function calls. To avoid remove important variables mistakenly, a smaller threshold is preferred, i.e., 10%.

3 Engineering Case Studies

3.1 Power Converter Design Problem. A power converter design problem [15,16] is used to test the performance of the proposed dimension reduction methodology. The design problem has six design variables, as shown in Table 13. The upper and lower bounds defined in Ref. [16] are used in this paper. The objective of the problem is to minimize the weight of the power converter as shown in Eq. (12). The formulation of the problem is defined as follows and all constant values are taken from [15]

$$\min y_1 = W_c + W_w + W_{cap} + W_{hs} \tag{12}$$

Table 13 Design variables in power converter design

Variables	Name	Description	Lower bound	Upper bound
x_1	C_w	Core center leg width (m)	0.001	0.1
x_2	*Turns*	Inductor turns	1.0	10
x_3	A_{cp}	Copper size (m^2)	7.29×10^{-8}	1.0×10^{-5}
x_4	$L_f/PINDUC$	Inductance (H)	1.0×10^{-6}	$1.0 \times 10-^{5}$
x_5	C_f	Capacitance (F)	1.0×10^{-5}	0.01
x_6	w_w	Core window width (m)	0.001	0.01

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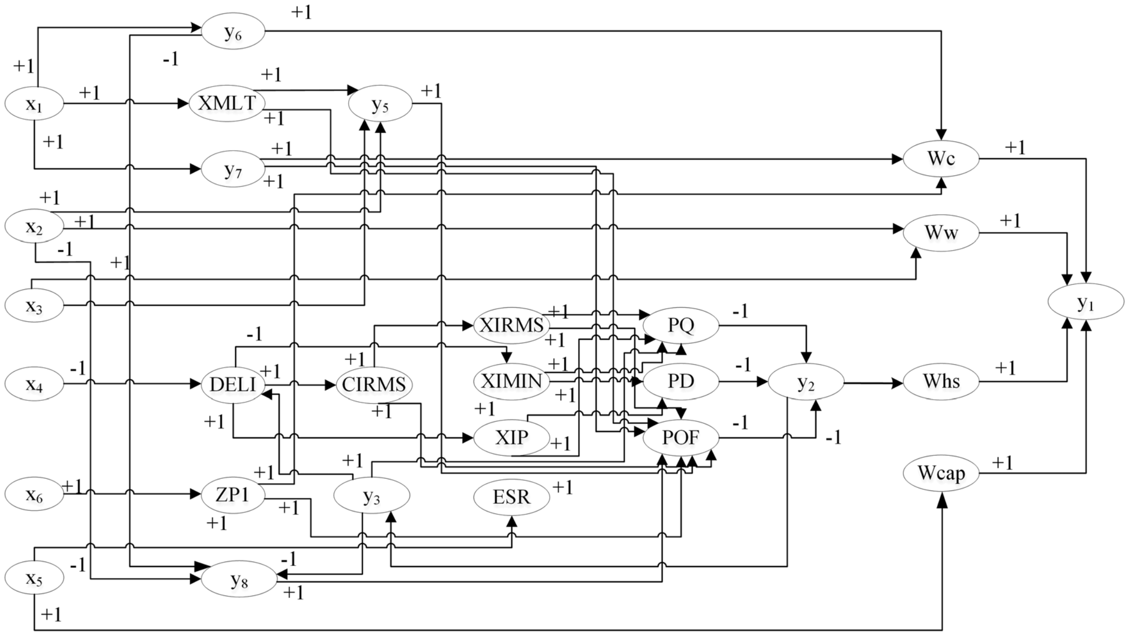

Fig. 4 Causal graph of the power converter problem

where, $W_c = \left|DIy_6(ZP_1 + y_7)\right|$, $ZP_1 = 2(1 + K_2)x_6$, $W_w = \left|(XMLT)\ (DC)x_2x_3\right|$, $XMLT = 2x_1(1 + K_1)FC$, $W_{cap} = |DK_5x_5|$, and $W_{hs} = \left|\frac{PO}{KH}\left(\frac{1}{y_2} - 1\right)\right|$.

Electrical design state analysis duty cycle

$$y_3 = \frac{EO}{\left(\frac{y_2EI}{2(XN)}\right)} \tag{13}$$

Minimum duty cycle

$$y_4 = \frac{EO}{\left(\frac{y_2EIMAX}{2(XN)}\right)} \tag{14}$$

Inductor resistance

$$y_5 = \frac{XMLTx_2(RO)}{x_3} \tag{15}$$

Core cross-sectional area

$$y_6 = K_1x_1^2 \tag{16}$$

Magnetic path length

$$y_7 = \frac{\pi}{2}x_1 \tag{17}$$

Inductor value

$$y_8 = \frac{(EO + VD)(1 - y_3)}{y_6x_2(FR)} \tag{18}$$

Loss design state analysis:

$$y_2 = \frac{PO}{PQ + PD + POF + PXFR} \tag{19}$$

The proposed dimension reduction method is employed to solve the six-dimensional multidisciplinary design optimization problem. It is to be noted that this problem entails mathematical expressions, which are used to build the causal graph as shown in Fig. 4. In most engineering problems, one does not have equations and thus should use their knowledge to construct a causal graph. By employing the qualitative analysis, it can be found that all variables contain contradictions. To further simplify the causal graph, the less important links are removed according to the weights and two-stage optimization is constructed as shown in Eqs. (20) and (21).

Problem 1:

$$\begin{aligned} &\mathit{find}\ \boldsymbol{x}_{ke} = [x_1, x_2, x_5]^{\mathrm{T}} \\ &\min\ y_1 = f(\boldsymbol{x}_{ke}, \boldsymbol{x}_{re}) \\ &\qquad \boldsymbol{x}_{ke}^{lb} \le \boldsymbol{x}_{ke} \le \boldsymbol{x}_{ke}^{ub} \end{aligned} \tag{20}$$

Problem 2:

$$\begin{aligned} &\mathit{find}\ \boldsymbol{x}_{re} = [x_3, x_4, x_6]^{\mathrm{T}} \\ &\min\ y_1 = f(\boldsymbol{x}_{re}, \boldsymbol{x}_{ke}) \\ &\qquad \boldsymbol{x}_{re}^{lb} \le \boldsymbol{x}_{re} \le \boldsymbol{x}_{re}^{ub} \end{aligned} \tag{21}$$

When optimizing problem 1 for the first time, the design variables $\boldsymbol{x}_{re}$ are fixed at the given value determined by the qualitative analysis. According to the previous qualitative analysis results, the upper bounds of x_3 and x_4 and the lower bound of x_6 should be selected. In this case, $x_3 = 1e^{-5}$, $x_4 = 1e^{-5}$, and $x_6 = 0.001$.

The MATLAB function *fmincon*(.) is employed to optimize the two problems. The starting point is generated randomly in the design space for the original problem. The starting points for the problems 1 and 2 in the two-stage optimization are the same as the starting point for the original problem. The original problem with six design variables is optimized first. The optimal result of the original problem is used as the stopping criterion for the two-stage optimization. Both optimizations are repeated 11 times and the median number of SA and the optimal results in that run are shown in Table 14.

For the two-stage optimization, after optimizing problems 1 and 2 once, the optimal value reaches 0.9866. The number of SA in the two-stage optimization is 210 including eight system analyses in sensitivity analysis, which is only 23% of that used in the

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Table 14 Optimization results for the power converter problem

	x^*	f^*	# of SA	Variance of # of SA
Original	$[0.003, 3.605, 1 \times 10^{-5}, 1 \times 10^{-5}, 8 \times 10^{-5}, 0.001]$	0.9864	887	[636,1442]
Decomposed	$[0.003, 3.586, 1 \times 10^{-5}, 1 \times 10^{-5}, 1 \times 10^{-4}, 0.001]$	0.9866	210	[186,557]

Table 15 Comparison of optimization results with a fixed number of SA for the power converter problem

	x^*	f^*	Variance of f^*	# of SA
Original	$[0.0028, 3.618, 9 \times 10^{-6}, 8 \times 10^{-6}, 1 \times 10^{-4}, 0.001]$	1.0024	[0.9887,1.0222]	250
Decomposed	$[0.0030, 3.384, 1 \times 10^{-5}, 1 \times 10^{-5}, 1 \times 10^{-4}, 0.001]$	0.9865	[0.9864,0.9893]	250

original optimization. The significant reduction of SA is due to the reduction of dimensions. In the original problem, six design variables need to be optimized. Although all the variables contain contradictions at the beginning, three variables with weak contradictions are selected from the original design variable set in the second simplification and the six-dimensional problem are divided into two lower-dimensional problems with three variables each. The reduction of the dimensions improves significantly the optimization efficiency. Although the description seems tedious, the qualitative analysis and dimension reduction are automatically conducted using the developed algorithm and code.

To illustrate the efficiency of the proposed method, the decomposed problem is compared with the original problem with the same number of function evaluations. In this case, the number of function calls is fixed at 250 for both problems. Note that for the decomposed problem, the maximum number of function calls for problem 1 is set as 250. If problem 1 optimization is terminated before 250 function calls, problem 2 will continue to run to reach to 250 function evaluations. This test is also repeated 11 times for both methods and the results are shown in Table 15.

When fixing the number of function evaluations, optimizing decomposed problem can obtain better results than optimizing the original problem. For the original problem, using 250 function calls cannot obtain the optimal results. However, for the decomposition problem, problem 1 usually needs about 200 SAs to find the optimal solution for the important variables. Then, around 50 function evaluations are used in problem 2 to obtain the final optimal results. To summarize, using the proposed dimension reduction method can help to achieve better results when the number of function evaluations is fixed.

3.2 Aircraft Concept Design Problem. The aircraft concept design problem [4] is used to test the performance of the proposed method. There are ten design variables (listed in Table 16) and three coupled disciplines (structure, aerodynamics, and propulsion). The objective of the problem is to maximize the range computed by the Breguet equation. The causal graph is shown in Fig. 5. By employing the proposed method, it can be found that variable h has no contradiction and the upper bound of h is desired. Then, the original problem is divided into two optimization problems,

Table 16 Design variables in aircraft concept design

	Variables	Description	Lower bound	Upper bound
1	M	Mach number	1.4	1.8
2	T	Throttle setting	0.1	1.0
3	S_{REF}	Wing surface area (ft^2)	500	1500
4	AR	Aspect ratio	2.5	8.5
5	t/c	Thickness/chord ratio	0.01	0.09
6	λ	Wing taper ratio	0.1	0.4
7	Λ	Wing sweep (deg)	40	70
8	x	Wingbox x-section area (ft^2)	0.9	1.25
9	h	Altitude (ft)	38,000	60,000
10	C_f	Skin friction coefficient	0.75	1.25

Problem 1

$$\begin{aligned} &\textit{find } \boldsymbol{x}_{ke} = [M, T, S_{REF}, t/c, \Lambda, x, C_f]^{\mathrm{T}} \\ &\max\ R(\boldsymbol{x}_{ke}, \boldsymbol{x}_{re}, \boldsymbol{x}_{uc}) \\ &\textit{s.t. } g(\boldsymbol{x}_{ke}, \boldsymbol{x}_{re}, \boldsymbol{x}_{uc}) \le 0 \\ &\qquad \boldsymbol{x}_{ke}^{lb} \le \boldsymbol{x}_{ke} \le \boldsymbol{x}_{ke}^{ub} \end{aligned} \tag{22}$$

Problem 2

$$\begin{aligned} &\text{find } \boldsymbol{x}_{re} = [\lambda, AR]^{\mathrm{T}} \\ &\max\ R(\boldsymbol{x}_{re}, \boldsymbol{x}_{ke}, \boldsymbol{x}_{uc}) \\ &\textit{s.t. } g(\boldsymbol{x}_{re}, \boldsymbol{x}_{ke}, \boldsymbol{x}_{uc}) \le 0 \\ &\qquad \boldsymbol{x}_{re}^{lb} \le \boldsymbol{x}_{re} \le \boldsymbol{x}_{re}^{ub} \end{aligned} \tag{23}$$

When optimizing problem 1 at the first time, the design variables $\boldsymbol{x}_{re}$ and $\boldsymbol{x}_{uc}$ are fixed at the given values that are determined by the qualitative analysis. In this case, $h = 60{,}000$ and AR = 2.5. For λ, because it has no impact on the objective, λ is set to be the initial number 0.25. The details of how the proposed method performs in the aircraft concept design can be found in authors' conference paper [17].

The MATLAB function *fmincon*(.) is employed to optimize the two problems. The starting points are selected randomly in the design space. The original problem with ten design variables is optimized first. The optimal result of the original problem is used as the stopping criterion in two-stage optimization. Each optimization is run 11 times and the results are shown in Table 17.

As shown in Table 17, it can be found that after decomposition, the two-stage optimization reaches the same optimal value with 210 function evaluations, which is half of the function calls used in the original optimization. In the original problem, ten design variables need to be optimized. After employing the causal knowledge to analyze the problem, one can find that there exists one monotonic variable. Then, the ten-dimensional problem turns to be a nine-dimensional problem. After simplification, the nine-dimensional problem is divided into two problems with seven and two variables, respectively. The reduction of the dimensions improves significantly the efficiency of the optimization.

Then, the two-stage optimization is compared with the original optimization with a fixed number of SA. In this case, the maximum function evaluations are set to be 180 for both optimizations. Each optimization is run 11 times and the median results are shown in Table 18. It can be found that with the fixed number of function evaluations, the result of the two-stage optimization is much better than the optimal value obtained from the original problem.

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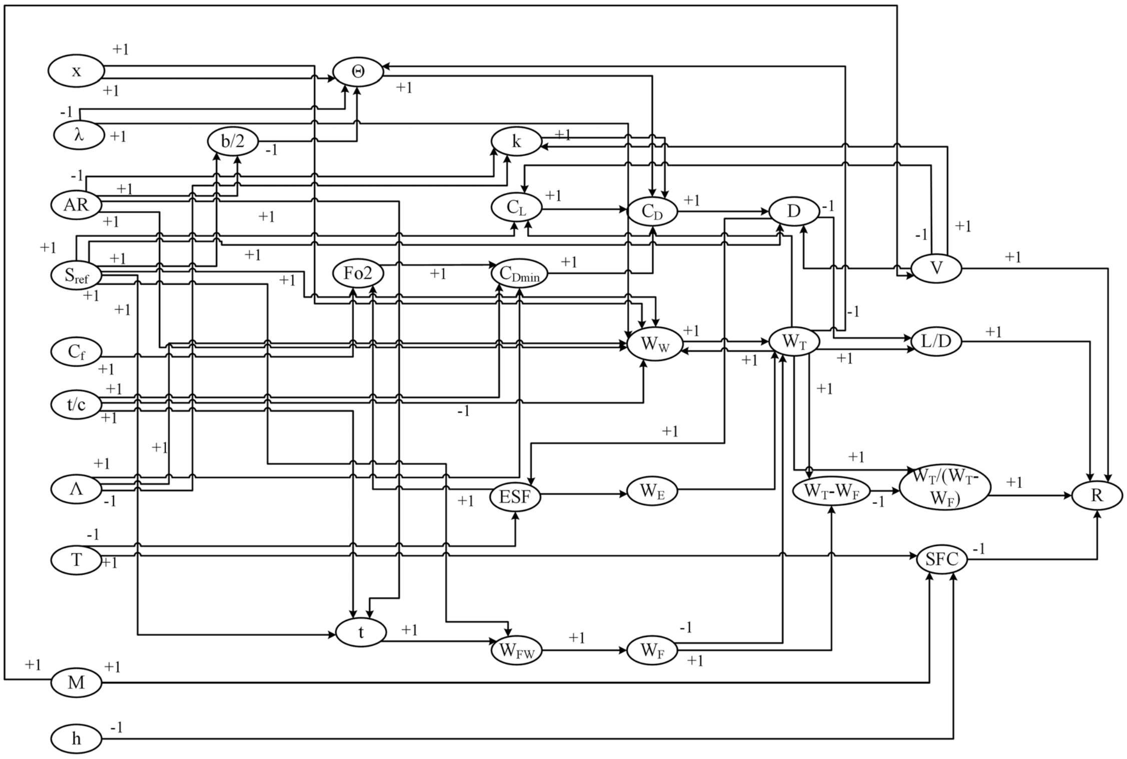

Fig. 5 Causal graph of the aircraft concept design problem

Table 17 Optimization results of aircraft concept design

	x^*	f^*	# of SA	Variance of # of SA
Original	[1.4, 0.265, 1500, 2.5, 0.09, 0.1, 70, 0.9, 60,000, 0.75]	4459	453	[420,724]
Decomposed	[1.4, 0.265, 1500, 2.5, 0.09, 0.1, 70, 0.9, 60,000, 0.75]	4459	210	[166,231]

Table 18 Comparison of optimization results with a fixed number of SA for the aircraft problem

	x^*	f^*	Variance of f^*	# of SA
Original	[1.4, 0.366, 870, 2.5, 0.09, 0.14, 70, 0.9, 50535, 0.75]	1885	[951,4413]	180
Decomposed	[1.4, 0.263, 1500, 2.5, 0.09, 0.1, 70, 0.9, 60,000, 0.75]	4458	[4458,4459]	180

4 Conclusion

This work proposed a dimension reduction method using causal graph and qualitative analysis, and this method is used to solve an MDO problem to improve the optimization efficiency. Causal graphs are constructed to show the input–output relationships between variables. To find the variables without contradictions automatically, a novel DSM-based qualitative analysis method is developed. Then, the values of the variables without contradictions can be determined before optimization and the dimensionality of the problem can be reduced. Taguchi method is employed to calculate the weight of each relationship and the original problem is divided into two subproblems, one with important variables and the other with less important variables. Thus, the number of variables in each subproblem is reduced compared with the original problem. Finally, the two subproblems are optimized sequentially to obtain the optimal solution. The proposed method is employed to solve a power converter design problem and an aircraft concept design problem from literature, and the results are compared with those obtained by optimizing the original problem. With the same optimal value, the efficiency of the proposed method is significantly higher as compared to optimizing the original problem. On the other hand, with the same number of function calls, the proposed method arrives at a better optimal solution. Nevertheless, the method can reach its limit if in the optimization problem all variables have contradictions and no simplifications can be made with the approach developed in the article. It is to be noticed that the only added function calls by the proposed method are for the weight calculation; the total number of these calls is limited according to the problem dimension and corresponding orthogonal array. Other steps of the proposed method are only the analyses of the causal graph with matrix operations. The associated cost of those operations is negligible and the operations are automated. Thus, the dimension reduction method can be performed as a pre-analysis before launching the optimization.

This paper offers a novel way to reduce the dimensionality of MDO problems and support decomposition of the problem by systematically incorporating the knowledge of the design problem in form of causal–effect relationships between variables. Those causal graphs are used in a preprocessing stage to support the optimization process and to improve the optimization efficiency.

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Future work will develop other ideas leveraging this work to define and capture other type of knowledge and optimization approaches to contribute to design optimization.

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